Astronomy 210
Homework Set #8
Due in class: Friday, April 8
Total Points: 50 + 5 bonus

1. *The random walk of light inside the Sun.* Photons in the Sun collide and scatter frequently off of the charged particles (plasma) in the Sun. On average, travels only about $\ell = 1$ cm between collisions; this distance is the photon "mean free path." After each collision, the scattered photon moves in a new direction which is random and independent of the initial direction. This motion is an example of a "random walk."

   To get a feel for how this works, let's imagine a simplified problem in which photons start at an origin ($x = 0$) and move randomly along the x-axis, each time taking a step of length ℓ. For each step, there is a 50% chance of stepping to the right ($+x$ direction) and a 50% chance of stepping to the left ($-x$ direction). We will describe the photon's progress in terms of D_N, the net displacement after N steps.

   (a) [**5 points**] Explain why the average or "expected" value of D_N is zero (written as $\langle D_N \rangle = 0$). This value is the average value of D_N for a whole population of photons, each starting at the origin. If it helps, note that the photon's travels are similar to a sequence of repeated flips of a coin, with "heads" = right, and "tails" = left.

   (b) [**5 points**] Although $\langle D_N \rangle = 0$ for the whole population of photons, the average *absolute value* of an *individual* photon's displacement is *not* zero. The most convenient way to see this is to look at the positive quantity D_N^2, and it's average, the "mean square distance" $\langle D_N^2 \rangle$. By taking the average of the squares of two possible values for D_1, show that $\langle D_1^2 \rangle = \ell^2$.

   (c) [**5 points**] We can find value of $\langle D_N^2 \rangle$ at step N, given the value $\langle D_{N-1}^2 \rangle$ from the previous step, for $N > 1$. We either have $D_N = D_{N-1} + \ell$ or $D_N = D_{N-1} - \ell$. Given this, find the two possible values of D_N^2. The average of these two values is $\langle D_N^2 \rangle$. Note that by definition we should expect D_{N-1}^2 to average to $\langle D_{N-1}^2 \rangle$. Using this, solve for $\langle D_N^2 \rangle$ in terms of $\langle D_{N-1}^2 \rangle$.

   (d) [**5 points**] Using the results from (b) and (c), show that $\langle D_N^2 \rangle = N\ell^2$.

   (e) [**5 points**] If we now take the square root, to get back a distance, we find the "root mean square" distance traveled to be $D_{\rm rms} = \sqrt{\langle D_N^2 \rangle} = \sqrt{N}\,\ell$. Thus, after N steps, we expect our photon to have moved a distance $\sqrt{N}\,\ell$ from the origin. How many steps must the photon take to escape the Sun, i.e., to go a distance $D_{\rm rms} = R_\odot$, if each random step is $\ell = 1$ cm?

   (f) [**5 points**] How long a time interval does this escape take, if each step takes the time for a photon at speed c to go a distance $\ell = 1$ cm? Express this answer in years; 1 yr $= 3.16 \times 10^7$ sec.

   Comment on what would happen if, in some horrible industrial accident, the Sun stopped generating new photons in its core.

2. *A stellar lineup: comparing properties of stars.* We would like to know how the Sun compares to other stars. Some useful data is summarized in the table below.

Stellar Data

Star Name	Star Type	parallax p [arcsec]	apparent magnitude m_V	surface temperature T [Kelvin]
Barnard's star	main sequence	0.547	9.53	3400
Antares	supergiant	0.0053	1.06	3550
Sirus B	white dwarf	0.379	8.5	25000

(a) [**5 points**] A key physical parameter of a star is its radius. While we usually cannot measure this directly (i.e., geometrically) for most stars, we *can* infer it by realizing that (most) stars are basically blackbody emitters. Find an expression for a star's radius R given the star's luminosity L and surface temperature T, and assuming the star to be a blackbody emitter.

It also useful and convenient to compare other stars to the Sun. Find an expression for $R/R_\odot$ given the star's luminosity and temperature, and $L_\odot$ and $T_\odot$.

(b) [**5 bonus points**] Show that for two stars with luminosities L_1 and L_2, and corresponding absolute magnitudes M_1 and M_2, that

$$\frac{L_2}{L_1} = 10^{-(M_2 - M_1)/2.5} \tag{1}$$

(c) [**5 points**] The distance modulus relates absolute and apparent magnitudes (written here for the visible band) in this way: $\mu = m_V - M_V = -5 + 5\log_{10} d$, where d is the distance to the star in parsecs. Using this and the result from part (b), find an expression for the ratio $L/L_\odot$ of a stars luminosity to that of the Sun, in terms of the star's apparent magnitude m_V and μ. Take the Sun's absolute V-band magnitude as a known constant ($M_{V,\odot} = 4.83$).

(d) [**10 points**] Now let's apply the expressions you have found. You may find it convenient to use a spreadsheet or simple computer program to do this. Using the expressions above, and the data in the Table, compute the distance d and the ratio $L/L_\odot$ for each of the stars listed in table. How does the Sun's luminosity compare to that of other stars?

Go on to compute $R/R_\odot$ for each of the stars in the Table (note that $T_\odot = 5800$ K). How does the radius of the Sun compare to that of other main sequence stars? Are supergiants well-named? Are white dwarfs well-named?