Astro 210

Lecture 22

March 11, 2011

Announcements

- HW7 due
- Night Observing: **last chance** this week!
  first clear night today, Tue, or Wed will be *last* session
  due to time change, hours now **8–10 pm**
  report forms, info online
- Hour Exam 1: scores & stats posted last night
  exams handed out today

Last time: finished solar system tour

Today: build theory of solar system origin

The Age of the Earth & Solar System

Very useful, important to know
age of Earth, other solar system bodies

"gold-plated" method: radioactive dating

Radioactivity

recall: nucleus = collection of protons and neutrons
not all atomic nuclei are stable! some spontaneously decay!

why? rough rule of thumb:
nuclei "prefer" $\#n$ and $\#p$ roughly *equal*
if too many extra n or $p \to$ change to make more even

example: Carbon-14
$^{14}\text{C} = \boxed{\text{6p} \quad \text{8n}}$: 2 extra $n \to$ unstable, radioactive
decay to Nitrogen-14: $^{14}\text{N} = \boxed{\text{7p} \quad \text{7n}}$; has equal n and p, stable
how? change one $n \to p$
Q: why can't this be all that occurs in decay?
Q: how to predict when one ^{14}C *nucleus decays?*

radioactive decays: to try to balance n and p
decays can change $n \to p$, or $p \to n$
but note: electric charge(p) $\neq$ charge(n)
$\to$ need another charged particle–electron!
turns out: yet another particle too (neutrino)

for the case $n \to p$, really have

$$n \to p + e + \nu_e \tag{1}$$

where ν_e is a (electron-type) neutrino (more on these later)

note:

- at deeper level, decay is quark change $d \to u + e + \nu_e$
- decay produces have high kinetic energy $\to$ heat

Radioactive Decay Law

When will a given nucleus decay?
Trick question! In subatomic quantum world, decays are *random*!
cannot predict when individual particle will decay!
But *can* predict very accurately how a large sample will decay
`www: decay simulations`

The rule: starting with n_0 radioactive nuclei at time $t = 0$
decay rate $\propto$ number n of nuclei still alive
$dn/dt = -\lambda n$
w/ "decay constant" λ, depends on isotope

$$\frac{dn}{n} = -\lambda dt \qquad (2)$$

$$\ln\left(\frac{n}{n_0}\right) = -\lambda t \qquad (3)$$

$$n(t) = n_0 \, e^{-\lambda t} \qquad (4)$$

exponential decay law

fixed time for half of present sample to decay: *half-life*

rewrite:
$n = n_0\ 2^{-t/t_{1/2}} = n_0\ (e^{\ln 2})^{-t/t_{1/2}} = n_0 e^{-\lambda t}$
$\rightarrow$ half life and decay rate are inverses: $t_{1/2} = \ln 2/\lambda$

Radioactive Dating

radioactive material can be age-dated:

⋆ decay rate predictable: "clock"

⋆ both undecayed "parents" and decay "daughters" observable
$\rightarrow$ can infer amount of decay

⋆ some nuclei have very long $t_{1/2}$ $\rightarrow$ can measure very old ages

Example: Potassium–Argon dating *Demo: banana*

^{40}K is rare, unstable potassium isotope
decays to argon $^{40}\text{K} \rightarrow {}^{40}\text{Ar}$ with $t_{1/2} = 1.3 \times 10^9$ yr

Worked Example

Experiment: in rock, measure *ratio* $n(^{40}\text{Ar})/n(^{40}\text{K}) = 10.6$

assume $n(^{40}\text{Ar}) = 0$ at rock formation *Q: why*?
$\rightarrow$ what is age t of rock?

1. find $n_0(^{40}\text{K})$:

$$\frac{n(^{40}\text{Ar})}{n(^{40}\text{K})} = \frac{n_0(^{40}\text{K}) - n(^{40}\text{K})}{n(^{40}\text{K})} = 10.6 \qquad (5)$$

$\rightarrow n_0(^{40}\text{K})/n(^{40}\text{K}) = 11.6$

2. now get age:
$n_0(^{40}\text{K})/n(^{40}\text{K}) = 2^{t/t_{1/2}}$
$\Rightarrow \log_{10}(n_0/n) = t/t_{1/2}\log_{10} 2$

$$t = \frac{\log_{10}(n_0/n)}{\log_{10} 2}\ t_{1/2} = 4.6 \times 10^9\ \text{yr} \qquad (6)$$

Ages of Earth and the Solar System

Earth

^{40}K – ^{40}Ar clock "reset" whenever rocks melted *Q: why?*

- gives a range of dates for earth rocks
  Q: why does this make sense?
- oldest earth rocks give

$$t(\text{oldest rocks}) \approx 4.3 \times 10^9 \text{ yr } \leq t_{\text{earth}} \tag{7}$$

Solar System

radioactive dating show: **meteorites** oldest objects

strictly speaking: give *lower bound* to solar system age

practically: likely formed quickly → give SS age

∞

$$t_{\text{SS}} = 4.6 \times 10^9 \text{ yr} = 4,600,000,000 \text{ yr} = \boxed{\text{4.6 billion years!}} \tag{8}$$

Origin of the Solar System

theory building!
recall: geocentric/heliocentric theories...

Input SS data, laws of physics

Output: Model
sequence of events, predictions for evolution up to present

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patterns in the solar system to be explained by
a theory of solar system origin
Q: similarities? differences?

Solar System Data to be Explained

orbits, spins

- planet orbits in ecliptic plane
- rough spin/orbit alignments
- but some spins misaligned

Terrestrial/Jovian differences:

composition
location
size
spacing

debris: comets, asteroids

Theory of Solar System Origin: Protosolar Nebula

stars born in cold gas & dust clumps: molecular clouds
Q: what's dust, in astro context?
`www: HST Eagle Nebula`

Initial protosolar material a small parcel of larger cloud
- cold gas & dust
- spinning: net angular momentum $\neq 0$
  Q: why is $\vec{L} \neq 0$ a reasonable assumption?

For simplicity: imagine first a cold cloud
with *zero* spin
i.e., *zero* angular momentum

Q: forces on particles in cloud?
Q: response of particles to these forces?

Q: why is coldness important for this to work?

Gravitational Collapse

ignoring spin:
particles in cold cloud feel forces of

- gravity
- thermal pressure

but if cloud is cold: T low, pressure $P = \rho k T / m_{\text{particle}}$ small
$\rightarrow$ only important force is gravity

diagram: cloud $\rightarrow$ collapse
gravity $\rightarrow$ inward motion $\rightarrow$ denser $\rightarrow$ stronger gravity
$\rightarrow$ runaway!
"gravitational collapse"

Q: why doesn't collapse continue until all matter $\rightarrow$ point?

iClicker Poll: Contraction of a Spinning Swarm

Consider a swarm of particles, spinning around an axis

Which is easier?

A moving a particle *parallel* to spin
$\downarrow$ toward midplane

B moving a particle *perpendicular* to spin
$\leftarrow$ toward spin axis

C both motions equally easy

Nebular Collapse: Birth of Sun and Disk

indeed, most matter compressed $\rightarrow$ central "proto-Sun"

but real pre-stellar clouds are clumpy parts of larger nebulae
$\rightarrow$ turbulent motions
$\rightarrow$ clumps have random but nonzero spins: $\vec{L}_{\text{init}} \neq 0$

spin $\rightarrow$ axial but not spherical symmetry
describe with cylindrical coordinates (r, θ, z)

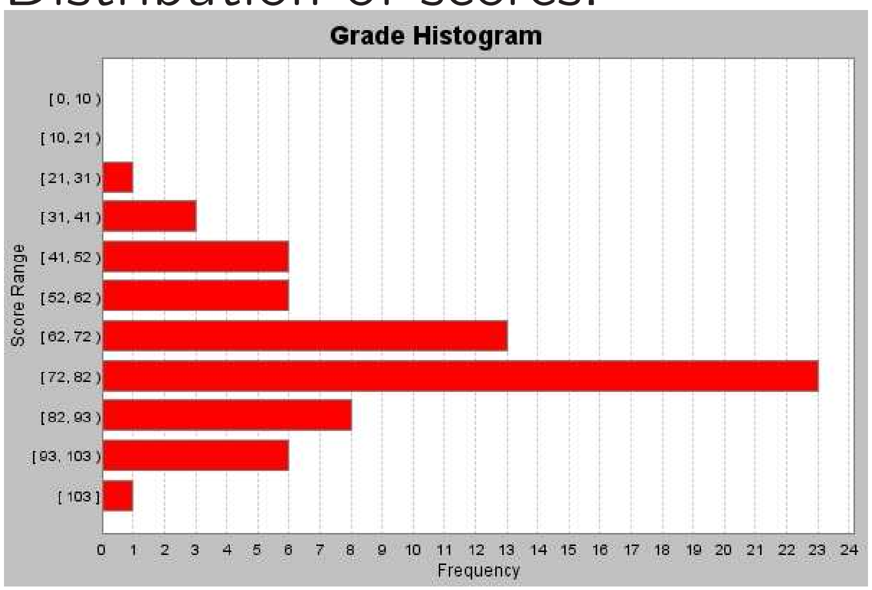
$\rightarrow$ collapse not spherical
angular momentum "centrifugal barrier" along R, but not z
$\Rightarrow$ collapse easier along z
$\Rightarrow$ protoplanetary disk
diagram: disk

disk $\rightarrow$ planet & debris orbit planes, spin axes

Hour Exam 1

- Scores and statistics posted on Compass.
- median = 74, std deviation = spread around mean = 16.8
- Solutions posted online.

Distribution of scores:



Recall:

- this exam worth 10% of final grade
- equivalent to 2 HW grades