Astro 210

Lecture 8

Feb 4, 2011

Announcements

- HW2 due
  apologies for the erratum
- HW3 available, due next Friday
  HW1 Q8 bonus still available
- **register** your iClicker; link on course webpage
- Planetarium: shows Mon & Tues of next two weeks
- if this is your first class: see me afterward!

Last Time: Two of the All-Time Greats

Galileo: Astronomer

Kepler's Laws

1. planet orbits are ellipses, with Sun at one focus
2. orbits sweep equal areas in equal times
3. $P_{\mathrm{yr}}^2 = a_{\mathrm{AU}}^3$

these completely and precisely characterize planet orbits

Galileo: Physicist

isolated and studied important special cases of motion

- free body *Q: what is one? what's the motion?*
- free fall *Q: what's that? what's the motion?*

Sir Isaac Newton 1642–1727 English

Newton's Laws of Motion – "T-Shirt Review"

Newton I

a free body = no net force (i.e., no acceleration)
motion: constant velocity $\rightarrow$ same speed **and** direction

Mathematically:
displacement $\vec{r}$, velocity $\vec{v}$, acceleration $\vec{a}$ are **vectors**
$\star$ **displacement** $\vec{r} = (x, y, z)$, distance $r = |\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$
$\star$ **velocity** $\vec{v} = d\vec{r}/dt$, speed $v = |\vec{v}|$
$\star$ **acceleration** $\vec{a} = d\vec{v}/dt$, magnitude $a = |\vec{a}|$

Note: time derivative of vector $\vec{v}(t) = [v_x(t), v_y(t), v_z(t)]$
is $d\vec{v}/dt \equiv \dot{\vec{v}} = [\dot{v}_x(t), \dot{v}_y(t), \dot{v}_z(t)]$
where "overdot" $= d/dt$

Newton I

a free body = no net force (i.e., no acceleration)
motion: constant velocity $\rightarrow$ same speed **and** direction

mathematically:
acceleration $\vec{a} \equiv \dot{\vec{v}} = 0$
$\Rightarrow$ velocity $\vec{v}_{\text{free}}(t) = \vec{v}_0 = const$

Newton I:
- encodes Galileo's "free body" behavior
- establishes existence of inertial frames

Newton II

acceleration is proportional to force,
and inversely proportional to body's mass
$\Rightarrow a = F/m$ or $F = ma$
or $F = dp/dt$, with $p = mv$ (**momentum**)
or in 3-D:

$$\boxed{\vec{p} = m\vec{v}} \tag{1}$$

$$\boxed{\vec{F} = d\vec{p}/dt} \tag{2}$$

Newton II is machine to *predict the future!*
Q: why? how? what needed for Newtonian fortunetelling?

Fortunetelling (and Archæology!) with Newton II:

$\dot{\vec{v}} = \vec{F}/m$: force changes speed

$\rightarrow$ after time interval δt, velocity changed by $d\vec{v} = \vec{F}\delta t/m$

$\rightarrow$ carries particle to new position

$\rightarrow$ where it feels new force

$\rightarrow$ which changes speed

...lather, rinse, repeat

So: if we know

- present position & speed (initial conditions)

then we can predict the future and reconstruct the past:

- determine the nature of the forces
- apply Newton II and turn mathematical crank
- solve particle trajectory for all time–past, present, future!

Newton III

"action-reaction"

jurisdiction: forces between objects

the rule:
when one body exerts force on another
the other body exerts force of equal magnitude
but opposite direction on the one

$$\vec{F}_{12} = -\vec{F}_{21} \tag{3}$$

$$1 \text{ on } 2 = -2 \text{ on } 1 \tag{4}$$

note magnitudes same: $|F_{12}| = F_{12} = F_{21}$

`www: jumpshot`

Newton Gravitation

Newton's **Law of Gravitation**

a force, gravity, exists between any two objects having mass

depends on masses M, m and

distance $\vec{r}$ between centers

diagram: 2-body forces

coordinates: centered at M; then force on m is

$$\vec{F} = -G\frac{Mm}{r^3}\vec{r} = -G\frac{Mm}{r^2}\hat{r} \tag{5}$$

where $r = |\vec{r}|$, and $\hat{r} = \vec{r}/r$ is a radial unit vector

G is Newton's constant: universal–applies everywhere!

but has to be determined experimentally *Q: how?*

expt: $G = 6.67 \times 10^{-11}\ \mathrm{m^3/kg\ s^2}$

∞

Q: find acceleration on earth surface?

gravitational acceleration on surface of earth $= \oplus$
for body of "test" mass m:

$$a = \frac{F}{m} = \frac{1}{m} G \frac{m M_\oplus}{R_\oplus^2} = G \frac{M_\oplus}{R_\oplus^2} \tag{6}$$

$$= 6.7 \times 10^{-11} \mathrm{m^3/kgs^2} \frac{6.0 \times 10^{24}\mathrm{kg}}{(6.4 \times 10^6 \mathrm{m})^2} = 9.8 \mathrm{m/s^2} \tag{7}$$

Note:

- test mass m cancels! as Galileo found experimentally!
- that is: inertial mass = gravitational coupling

  Not obvious! no reason why need to be identical

- mass m and weight F different things

iClicker Poll: Weightlesses in Space?

Consider an astronaut orbiting Earth on the Space Shuttle
Is she weightless?

A yes

B no

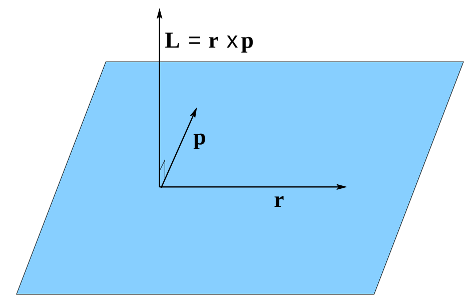
C depends on whether the rockets are firing

Note larger issue:
cosmic context requires rethinking "homegrown" intuition

Angular Momentum

For point mass, **angular momentum** defined as:

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} \tag{8}$$



i.e., using cross product

look at time change:

$$\begin{aligned} \frac{d}{dt}\vec{L} &= m\dot{r} \times \vec{v} + m\vec{r} \times \dot{v} & (9) \\ &= m\vec{v} \times \vec{v} + m\vec{r} \times \vec{a} & (10) \\ &= \vec{r} \times \vec{F} = \vec{\tau} \quad \text{torque} & (11) \end{aligned}$$

angular counterpart of Newton II:

- net (linear) force changes linear momentum
- net twisting force = torque changes angular momentum

Gravity and Angular Momentum

angular momentum changed by net torque

$$\frac{d}{dt}\vec{L} = \vec{r} \times \vec{F} \tag{12}$$

when force is due to **gravity**, torque:

$$\vec{\tau} = \vec{r} \times \vec{F} = -G\frac{mM}{r^3}\vec{r} \times \vec{r} = 0 \tag{13}$$

so if force is gravity, then

$$\frac{d}{dt}\vec{L} = 0 \tag{14}$$

and thus $\vec{L} = const$:

angular momentum is **conserved!**

Q: what about gravity force gauranteed this?

What Keeps the Earth in Orbit?

circular orbit $\rightarrow$ centripetal accel.

angular speed $d\theta/dt = \omega = 2\pi/P = const$

$$\boxed{\vec{a}_c = -\omega^2 \vec{r} = -\frac{v^2}{r}\ \hat{r}}$$

diagram: show $\vec{v}$, $\vec{r}$, $\vec{a}$

Newton II: acceleration demands net force

but Newton gravity supplies a force!

$\rightarrow$ Newtonian gravity is crucial and necessary ingredient
for understanding the dynamics of planetary motion
but have to see how the detailed predictions
compare with observation

Program:

- assume Newtonian gravity controls planetary motion
- that is, for any planet let $\vec{F}_{\text{net}} = \vec{F}_{\text{Sun}-\text{planet}}$
- input this into Newton's laws
- turn mathematical cranks $\rightarrow$ predict orbits
- compare predictions with observation

Solutions: Orbits

For attractive inv. sqare force, orbits are cross sections of cone:

- circle
- ellipse
- parabola
- hyperbola
- line

Circle eccentricity $e = 0$
at each point:
$F = ma = mv_{\rm c}^2/r$
$\Rightarrow GMm/r^2 = mv_{\rm c}^2/r$
$\Rightarrow$ circular orbits have speed $\boxed{v_{\rm c} = \sqrt{\frac{GM}{r}}}$

example: find circular speed 1 AU from Sun
$v_{\rm c} = 3 \times 10^4$ m/s

Kepler from Newton

Kepler I: Orbits are ellipses

Newton: bound orbits due to gravity are ellipses: check!

Kepler II: Equal areas in equal times

Newton: consider small time interval dt

move angle $d\theta = \omega dt$

sweep area

diagram: top view: path, $d\theta, \vec{r}, \vec{v}, \vec{v}_t$

$$dA = \frac{1}{2} r^2 d\theta = \frac{1}{2} r^2 \omega dt \tag{15}$$

but $\omega = v_\theta / r$, where $\vec{v_\theta} \perp \vec{r}$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{16}$$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{17}$$

finally, $r v_\theta = |\vec{r} \times \vec{v}| = |\vec{L}|/m$
Q: why?, so

$$dA = \frac{1}{2} \frac{L}{m} dt \tag{18}$$

Woo hoo! were' home free! *Q: why?*

But $L = const$ for radial force ($\vec{r} \times \vec{F} = 0$)
so

$$\frac{dA}{dt} = \frac{L}{2m} = const \tag{19}$$

Kepler II! $\rightarrow$ comes from ang. mom. cons.!