Astronomy 210 Spring 2011
Homework Set #4

Due in class: Friday, Feb. 25 Total Points: 50 + 5 bonus

1. *The Cooling Nighttime Moon.* This problem is wordy but is not difficult if you take it step by step. Each point on the Moon's surface is illuminated by the Sun for about two weeks, and then is dark for about two weeks. This leads to changes in the Moon's temperature. As we will later derive in class, the sunlight Moon has an average daytime temperature of about $T_{\rm d} = 330$ K.

   (a) **[5 points]** Although Kelvin units are the most useful for us in this course (and this problem!), to get a more intuitive feel, find the daytime lunar temperature in Fahrenheit. Recall that $T(\mathrm{K}) = T(°\mathrm{C}) + 273$, and that $T(°\mathrm{C}) = \frac{5}{9}[T(°\mathrm{F}) - 32]$. Comment on the implications of your answer for possible future moonbases.

   (b) **[5 points]** Find the peak wavelength of the blackbody radiation emitted from the sunlight Moon. In what part of the electromagnetic spectrum does the Moon thermally radiate?

   (c) **[5 points]** Find the blackbody flux F emitted from the daytime Moon. Express your answer in Watt/m^2.

   (d) **[5 points]** Now consider the nighttime half of the Moon. This region initially has temperature $T_{\rm d}$, but then cools because it loses energy due to its thermal radiation. Consider a region of the nighttime moon, of area A, with temperature T. Find an expression for the rate L at which this region loses energy (i.e., the power lost to heat).

   Calculate the value of L for a region of area $A = 1$ m^2 and at the initial temperature $T_{\rm d}$. Express your answer in Watts, and compare this result to the power of a typical lightbulb.

   (e) **[5 points]** Again for a surface region of area A and temperature T, find an expression for the thermal energy radiated over a time t.

   The unlit side of the Moon radiates thermally for a time about $t \approx 2$ weeks. For simplicity, assume that an area $A = 1\mathrm{m}^2$ radiates as a blackbody at temperature $T_{\rm d}$ for this entire time; find the total amount of energy $E_{\rm rad}$ radiated from this area over the 2 weeks. Express your answer in Joules.

   (f) **[5 points]** The top $h = 1$ meter of the Moon's surface has a density $\rho =$ (mass/volume) $\approx$ 3000 kg/m^3. Find an expression for the mass M of a region of lunar surface of area A and depth h.

   Further note that the heat energy $E_{\rm heat}$ stored in a mass M is given by $E_{\rm heat} = MCT$ where T is the temperature in Kelvin and C is the "heat capacity" which for the Moon's surface is about $C = 10^3$ Joule kg^{-1} K^{-1}.

   Find an expression for $E_{\rm heat}$ for a region of lunar surface of area A and depth h. Calculate $E_{\rm heat}$ for $A = 1\mathrm{m}^2$, $h = 1$ meter, and $T = T_{\rm d}$; express your answer in Joules.

   (g) **[5 points]** You have now found the energy $E_{\rm heat}$ stored in the lunar surface after being in the daytime, but you have also estimated the radiated energy $E_{\rm rad}$ during the night. Compare your two values. Do you expect the Moon's nighttime temperature to drop substantially, or just a little, compared to its daytime temperature? Comment on the implications for future moonbases.

   (h) **[5 points]** Of course, your answer to part (e) was too simplistic. In a more realistic treatment of this step, would you expect the radiated energy $E_{\rm rad}$ to be larger than or smaller than the value you calculated? How would this affect your answer to part (g)?

   (i) **[5 bonus points]** In fact, a more realistic calculation lets temperature be a smooth function $T(t)$ of time (with all other factors constant in time). Show that energy conservation implies $dE_{\rm heat}/dt = -L$. Use this to find an expression for dT/dt; show that this expression is independent of the area A. Solve for $T(t)$ given that $T(0) = T_{\rm d}$ at time $t = 0$. Using $h = 1$ m, to what temperature should the Moon cool in a time $t = 2$ weeks?

2. *Mapping Pluto.* [**5 points**] In July 2015, the NASA spacecraft *New Horizons* will visit Pluto, and revolutionize our understanding of it and its neighbors. Until then, the best view we have of Pluto is that of the *Hubble Space Telescope.*

   The primary mirror on *Hubble* has a diameter of 2.4 meters. Working at wavelength 400 nm, *Hubble* looks at Pluto, which presently lies at a distance of about 30 AU from Earth. Find the size (in km) of the smallest feature *Hubble* can resolve on Pluto's surface. Compare the size of the smallest resolved region with Pluto's radius of about 1150 km. Comment on how well *Hubble* can map Pluto.

3. *The Bohr Model of Hydrogen.* [**5 points**] In the Bohr model of the hydrogen atom, electrons orbit the nucleus in circular orbits of radius r. As discussed in class, it is also assumed that each orbit can only occur where the electron is a standing wave. That is, the electron's has a de Broglie wavelength $\lambda_{\rm deB} = h/p = h/(m_e v)$, and for a standing wave, the number of de Broglie wavelengths around the circumference must be exactly an *integer*; call this integer n. We thus have

   $$2\pi r = n\lambda_{\rm deB} \tag{1}$$

   Further assume that the Coulomb force provides the orbit's centripetal acceleration:

   $$F = \frac{e^2}{r_n^2} = m_e \frac{v_n^2}{r_n} \tag{2}$$

   Combining eqs. (1) and (2), show that the orbits—the radius and velocity pairs–can only come in quantized "states" with the values

   $$r_n = n^2 \frac{\hbar^2}{e^2 m_e} \quad \text{and} \quad v_n = \frac{1}{n}\frac{e^2}{\hbar} \tag{3}$$

   Also show that the energy can only take the quantized values

   $$E_n = \frac{1}{2} m_e v_n^2 - \frac{e^2}{r_n} = -\frac{1}{n^2}\frac{e^4 m_e}{2\hbar^2} \tag{4}$$