Astronomy 210
Homework Set #7

Due in class: Friday, March 17

1. *A Transiting Exoplanet.* NASA's *Kepler* space telescope is designed to find transiting planets, that is, planets whose orbit planes happen to be seen edge-on, so that the planet passes in front of its host star. The planet thus partially eclipses the star, an event known as a "transit." Here we consider the newly-discovered transiting exoplanet Kepler-7b. You will see that you can reproduce all of the professional results for this planet. Of course, show your work in your calculations.

   (a) [**5 points**] On the web, go to the Exoplanet page at `exoplanets.org`. In the table of planets, find the information on the planet Kepler-7b. From the known mass of the host star, and the orbital period, use Newton's form of Kepler's law is $a^3 = GMP^2/(4\pi^2)$, and to find the planet's semimajor axis a in AU. Your result should agree with the professional answer!

   (b) [**5 points**] Now assume the planet is in a circular orbit. You showed in HW4 that the star's and planet's distance from the COM, $r_\star$ and r_p, are related by $r_\star + r_\mathrm{p} = a$, and by $m_\star r_\star = m_\mathrm{p} r_\mathrm{p}$. Use these to show that the orbital speed of the star about the COM is

   $$v_\star = \frac{m}{M+m}\, v = \frac{m}{M+m} \frac{2\pi a}{P} \tag{1}$$

   where v is the speed of the planet *relative to the star.*

   (c) [**5 points**] The line-of-sight ("radial") velocity $v_\star$ is plotted in the planet's webpage, and the magnitude of $v_\star$ is denoted the "velocity semiamplitude." Using their best-fit value and eq. (1), compute the ratio m/M and then using the tabulated value for the host star's mass, compute the mass m of the planet; compare your result to Jupiter's mass.

   (d) [**5 points**] Now visit the *Kepler* mission's `http://kepler.nasa.gov` website. Find Kepler-7b on the "Discoveries" page, and click on its link. There you will see a plot of the star's brightness versus time (known as a "light curve"), which has a drop due to the transit. This drop lasts for a time which is measured to be $\Delta t = 0.22$ days. This is the time it takes for the planet to cross the face of the star, i.e., to move one stellar diameter. Find the speed v of the *planet* relative to the star based on Kepler's laws (note that this is different from the *star's* speed you found in part 1b). Using this and Δt, find the radius $R_\star$ of the star. Express your answer in meters, and compare to Sun's radius $R_\odot = 7.0 \times 10^8$ m as well as the value you found for a.

   (e) [**5 points**] Your estimate for the star's radius implicitly assumes we are seeing the system's orbit plane edge-on. Explain why this is assumed, and how an inclination angle for the plane away from precisely edge-on would affect the stellar radius estimate–would your result be an overestimate or an underestimate?

   (f) [**5 points**] The "depth" of the transit is a reduction δF of the flux of the star compared to its uneclipsed value F. The flux reduction is due to the disk of the planet covering some of the disk of the star. The data show $\delta F/F = 0.0068$. Using this result, compute the *ratio* $R_\mathrm{p}/R_\star$ of the planet's radius to the star's radius. Then use your value for the star's radius to find the planet's radius. Compare this to Jupiter's radius and the Earth's radius.

   (g) [**5 points**] Find the density of the planet, in kg/m^3. Compare to that of solar system terrestrial and Jovian planets. You should find Kepler-7b has a rather unusual density. What might cause this?

   (h) [**5 points**] If the planet has albedo A and radius R_p, and the host star has luminosity $L_\star$, find an expression for the luminosity L_p of the planet due to reflected light from its host star, and then calculate the ratio $L_\mathrm{p}/L_\mathrm{star}$ of the planet's luminosity to that of the host star, using $A = 0.2$.

(i) **[5 points]** The host star of Kepler-7b is very far from us–its distance is much much larger than the semimajor axis a that you found above. Given this, show that the ratio of the observed flux $F_{\rm p}$ from the planet, to the flux $F_\star$ from the star, is given by $F_{\rm p}/F_\star \approx L_{\rm p}/L_{\rm star}$. Explain why, if we wait 1/2 orbital period after a transit, the observed flux from the Kepler-7 system should again *decrease* in a "secondary" event, but now with a depth $\delta F/F \approx F_{\rm p}/F_\star$. *Kepler* can detect flux depths down to $(\delta F/F)_{\rm min} = 10^{-3}$. Can *Kepler* measure the secondary flux drop for Kepler-7b?

2. **[5 points]** In the vacuum of space, ice sublimes (i.e., evaporates directly from a solid to water vapor) in a relatively short time at a temperature of 175 K or higher. Take the albedo of ice as $A = 0.5$, and calculate the distance from the Sun at which the equilibrium blackbody temperature of an icy body is just equal to this sublimation temperature. Where in the solar system can icy bodies exist?