Astronomy 210
Homework Set #9

Due in class: Friday, April 15
Total Points: 50 + 5 bonus

1. *How far can the eye see?* The human eye can see a star, under ideal conditions, as dim as an *apparent* visual magnitude $m_{\rm lim} \approx 6$.

   (a) **[5 points]** If a star has absolute magnitude M, show that the distance $d_{\rm lim}$ (in parsecs) at which its apparent magnitude is $m_{\rm lim}$ is given by

   $$d_{\rm lim} = 10^{1+(m_{\rm lim}-M)/5} \text{ pc} \tag{1}$$

   (b) **[5 points]** Using this, find the *maximum* distance $d_{\rm max}$, in parsecs, from which your naked eye can see a star:

   i. with absolute magnitude the same as the Sun, $M_\odot = 4.83$

   ii. with the absolute magnitude of the *most* luminous star–that is, the one with the largest *negative* absolute magnitude–found in James Kaler's table of the *brightest* stars in the sky, available via the course `links` page

   iii. with the absolute magnitude of the *least* luminous star–that is, the one with the largest *positive* absolute magnitude–found in the RECONS table of the *nearest* stars, available via the course `links` page

   Compare these results with the distance to the nearest star, and comment.

   (c) **[5 points]** Finally, comment on any possible bias that might affect the distribution of stars visible to the naked eye.

2. *The Sun, Neutrinos, and You.* As discussed in class, the Sun is powered by energy released in nuclear reactions. These reactions occur in chains which have the net effect of burning hydrogen to helium-4: $4p \to {}^4\text{He} + 2\nu + 2e^+$. The production of each new ^{4}He nucleus liberates $Q(4p \to {}^4\text{He}) = 26 \text{ MeV} = 2.6 \times 10^7 \text{ eV} = 4.2 \times 10^{-12}$ J worth of energy, and creates two neutrinos.

   (a) **[5 points]** Using charge conservation, deduce the electric charge of the neutrino from the first reaction in the $p-p$ chain, $p + p \to {}^2\text{H} + \nu + e^+$. Note that matter at the Sun's core is fully ionized, so that ^{2}H here is a deuterium nucleus, not a deuterium atom.

   (b) **[5 points]** Given the flux $F_\odot$ of sunlight at earth, find an expression for the solar energy E incident on a detector of area A over a time Δt. Next, find an expression for the number of neutrinos N_ν that were created in the sun in order to generate the energy E, and which therefore also pass through the detector in the same time interval. Then find an expression for the flux $\Phi_\nu = N_\nu/(A\,\Delta t)$ of solar neutrinos at earth, and show that $\Phi_\nu \propto F_\odot$. Finally, using $F_\odot = 1370 \text{ Watt/m}^2$, compute Φ_ν, in units of neutrinos $\text{m}^{-2}\ \text{s}^{-1}$.

   (c) **[5 points]** Estimate the area of your palm. Using this, calculate the number of neutrinos passing through your outstretched hand in one second (during the daytime).

   (d) **[5 points]** Comment on:

   i. How do you expect the number from part 2c to change when it is night?

   ii. What basic fact about solar energy is confirmed by the detection of solar neutrinos?

3. *Gravitational collapse: freefall.* In several stages of star's lives, gravity is not compensated by other forces, and the star collapses under its own weight. Here we will consider the simplest form of this, when there are *no* compensating forces at all. This is known as a freefall collapse.

   Consider a spherical gas cloud of uniform density ρ, and radius R.

   (a) **[5 points]** The gravitational acceleration $g(r)$ at any radius r inside a spherical mass distribution is given by the usual Newtonian result, but for the mass $M(r)$ at radii *inside* of r; this is sometimes known as the "enclosed mass." Find $g(r)$ for radii r inside the gas cloud, in terms of r as well as the (uniform) density ρ.

   (b) **[5 points]** Now imagine the cloud is released from rest. Assume for simplicity that gas at each radius falls with a *constant* acceleration equal to its initial acceleration $g(r)$, find the collapse time for material at each radius. Show that this estimate of the freefall collapse time is

$$\tau_{\rm ff}(r) = \frac{C}{\sqrt{G\rho}} \tag{2}$$

   where C is a numerical constant that will come out of your calculation; be sure to indicate the value of C.

   Using this result, calculate the freefall collapse time of an interstellar cloud that is forming a new star, with mass density $\rho = m_p n_{\rm H}$, where $n_{\rm H} \approx 10^8$ atoms/m^3 is the number of hydrogen atoms per unit volume, each of which have a mass equal to the proton mass $m_p = 1.6 \times 10^{-27}$ kg. Express your answer in years and comment.

   (c) **[5 points]** The result in (2) is independent of the cloud radius. Given this, explain how will the cloud's structure (density profile) evolve as it collapses.

   Go on to explain how the collapse would change if the cloud were spherical but had a density that is largest at the center and decreases towards the surface.

   (d) **[5 bonus points]** Our answer to part (b) was oversimplified. Show that particles released from rest at radius $r(0) = r_0$ at the initial time $t = 0$ will have a distance $r(t)$ which evolves according to

$$\left(\frac{dr}{dt}\right)^2 = \frac{2GM}{r_0}\left(\frac{r_0}{r} - 1\right) \tag{3}$$

   hint: consider energy conservation. Using this show that the collapse time $t_{\rm ff} = \int dt$ is given by an integral

$$t_{\rm ff} = \sqrt{\frac{3}{8\pi G\rho}} \int_0^1 \frac{du}{\sqrt{1/u - 1}} \tag{4}$$

   where $u = r/r_0$. Solve or look up the answer to the integral to get the exact expression for the freefall time. How does this compare to the simplified answer you got in part (b)?