Astro 210

Lecture 13

Feb 16, 2011

Announcements

- Hour Exam 1 in class Friday
  info online
- HW1 bonus problem due online Friday
- last Planetarium shows: tomorrow and Monday
  registration, report forms, info online
- Night Observing tonight, tomorrow, next week. *Dress warmly!*
  report forms, info online

Last time: structure of matter: atoms $\rightarrow$ nuclei + electrons
on atomic scales: quantum effects important
$\rightarrow$ wave/particle duality of light *and* matter!

Bohr Atom: Quantum Electrons Orbit Nucleus

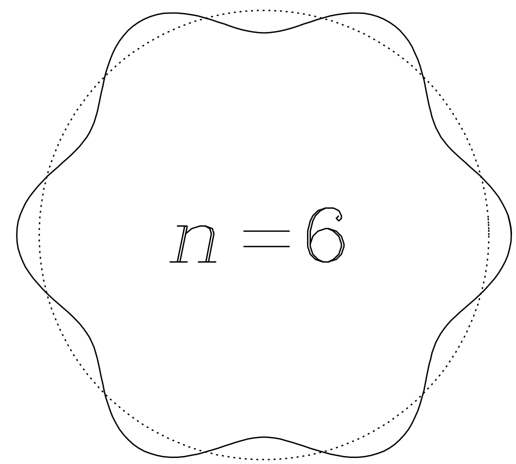
Ingredients:
- circular orbits
- electrons have de Broglie wavelengths $\lambda = h/p = h/m_e v$

- standing waves:

Demo: slinky

e orbit path length
  an integer multiple of λ:

$$2\pi r = n\lambda = n\frac{h}{m_e v} \quad (1)$$



$\rightarrow$ for each n, radii and speeds related

- Coulomb force provides centripetal accel:

Q: remind me–what is Coulomb force?

Coulomb force: electrical attraction between opposite charges
an inverse square law! same structure as gravity!

$$F_{\text{Coulomb}} = \frac{q_1 q_2}{r^2} = \frac{e^2}{r^2} \tag{2}$$

(cgs charge units: $e^2_{\text{cgs}} = k e^2_{\text{SI}} = e^2_{\text{SI}}/4\pi\varepsilon_0$)

Coulomb provides electron's centripetal acceleration:

$$m_e a_c = F_{\text{Coulomb}} \tag{3}$$

$$m_e \frac{v^2}{r} = \frac{e^2}{r^2} \tag{4}$$

another relation between r and v
$\rightarrow$ two equations, two unknowns $\rightarrow$ solution exists

The Bohr Atom: Results

Bohr: fit *integer* number $n \geq 1$ standing waves
into Coulomb-controlled circular orbits
⇒ only certain definite radii/speeds/energies allowed
⇒ "quantized" orbits

allowed radii:

$$r_n = n^2 \frac{\hbar^2}{e^2 m_e} \tag{5}$$

allowed speeds:

$$v_n = \frac{1}{n} \frac{e^2}{\hbar} \tag{6}$$

Q: so physically, higher n means?

Q: how is this similar to and different from ordinary non-quantum ("classical") circular Kepler motion?

Bohr orbit energies:

$$E_n = \frac{1}{2} m_e v_n^2 - \frac{e^2}{r_n} \quad (7)$$

$$= -\frac{1}{n^2} \frac{e^4 m_e}{2\hbar^2} \propto \frac{1}{n^2} \quad (8)$$

recall: negative energy $\rightarrow$ *bound* orbits

$\rightarrow$ electron bound to nucleus, takes energy to remove

diagram: energy level structure

Q: which level is most tightly bound?

Q: what about photon energies (lines)?

In transition, γ energy is **difference** between states:
if go from $n_{\rm hi} \to n_{\rm lo}$, with $n_{\rm hi} > n_{\rm lo}$, photon energy is

$$\begin{aligned} E_\gamma &= E_{n_{\rm hi}} - E_{n_{\rm lo}} && (9) \\ &= \frac{e^4 m_e}{2\hbar^2} \left(\frac{1}{n_{\rm hi}^2} - \frac{1}{n_{\rm lo}^2} \right) && (10) \\ &= \frac{e^4 m_e}{2\hbar^2} \, \frac{n_{\rm hi}^2 - n_{\rm lo}^2}{n_{\rm hi}^2 n_{\rm lo}^2} && (11) \end{aligned}$$

photon wavelength:

$$\lambda_\gamma(n_{\rm hi} \to n_{\rm lo}) = \frac{hc}{E_\gamma} = hc \frac{2\hbar^2}{e^4 m_e} \frac{n_{\rm hi}^2 n_{\rm lo}^2}{n_{\rm hi}^2 - n_{\rm lo}^2} \qquad (12)$$

define: Rydberg $R = \frac{4\pi c\hbar^3}{e^4 m_e} = 1.1 \times 10^7 \text{ m}^{-1}$

$$\Rightarrow \lambda_\gamma(n_{\text{hi}} \to n_{\text{lo}}) = \frac{1}{R} \frac{n_{\text{hi}}^2 n_{\text{lo}}^2}{n_{\text{hi}}^2 - n_{\text{lo}}^2} \tag{13}$$

put $n_{\text{lo}} = 2$: drop to 1st excited state

$$\Rightarrow \lambda_\gamma = \frac{1}{R} \frac{4n_{\text{hi}}^2}{n_{\text{hi}}^2 - 4} = 3.6 \times 10^{-7} \frac{n^2}{n^2 - 4} \text{ m} \tag{14}$$

$\Rightarrow$ Balmer's result! explained by quantum mechanics!

Lyman series: $n_{\text{lo}} = 1$
Balmer series: $n_{\text{lo}} = 2$
Paschen series: $n_{\text{lo}} = 3$

Transitions and Kirchoff

emission

dropping from $E_{\text{hi}} \to E_{\text{lo}}$: atom releases photon

how?

- spontaneous
- atoms collide ("de-excitation")

excitation

to jump from $E_{\text{lo}} \to E_{\text{hi}}$ atom must absorb energy

how?

- atoms collide
- γ absorption

absorption/re-emission of $\gamma \to$ Kirchoff 2 & 3 !

gas absorbs at characteristic λs (abs line spec)

gas re-emits at same λs but in all directions (emission line spec) all E_n are **bound** states, $E < 0$

also: unbound states, electron energy $E > 0$
then e free $\rightarrow$ ionized!

What is min E to ionize H from ground state ($n = 1$)?

$$\begin{aligned} E_{\text{ionize}} &= -E_1 = -\left(-\frac{Rch}{n^2}\right) = Rch \qquad (15) \\ &= 21.8 \times 10^{-19}\ \text{J} = 13.6\ \text{eV} \qquad (16) \end{aligned}$$

$E = 13.6$ eV binding energy of hydrogen

- "cost" to rip apart H$\rightarrow p + e$
- "payoff" when $p + e \rightarrow$H

Continuous Spectra and Blackbody Radiation

hot objects (e.g., stove burner) glow

- radiates! (Kirchoff)
- hotter $\rightarrow$ brigher, color change
- continuous spectrum

useful* to define an ideal substance:
a perfect absorber of light: **"blackbody"**
absorbs all λ, reflects none

*a useful idealization in the same way an "ideal gas" is useful: brings out essential physics, and a good approximation to behavior of many real substances

blackbody absorbs radiation $\rightarrow$ heats $\rightarrow$ re-emits according to T
"blackbody radiation" = thermal radiation
spectrum depends only on T
diagram: Flux F vs λ

Blackbody spectrum:

- $F > 0$ for all λ
- higher $T \to$ higher F at all λ
- **peak** at $\lambda_{\rm max}$
- for higher T, peak at smaller $\lambda_{\rm max}$

Q: why is this reasonable physically? Hint–photon energy!

Wien's law

$$\lambda_{\rm max} = \frac{2.9 \times 10^{-3} \ {\rm m\,K}}{T}$$

where T is in Kelvin: $T({\rm K}) = T(^\circ{\rm C}) + 273$

Ex: Sun's spectrum peaks in middle of visible range:
$\lambda_{\rm max,\odot} \simeq 500$ nm
Surface temperature is:

$$T_\odot \approx \frac{2.9 \times 10^{-3} \ {\rm m\,K}}{500 \times 10^{-9} \ {\rm m}} = 5800 \ {\rm K} \quad (17)$$

Observer's Scorecard

Blackbody spectrum & Wien's law are powerful tools:
get T from spectrum!

color $\leftrightarrow$ temperature

Q: are stars all the same color? what does this imply?

`www: objective prism spectra`

Q: compare bright stars in Orion: Betelgeuse, Aldebaran

iClicker Poll: Blackbody Radiation and You

What about people & animals?

Do people & animals emit blackbody radiation?

A yes, but flux too faint to see

B yes, but flux is not visible to naked eye

C no, living organisms cannot behave as blackbodies

D no, our skin traps radiation inside our bodies

`www: experiment says...`

Blackbodies: Total Flux

Total flux over all λ = sum of flux at each interval $\Delta\lambda$

$$F = \sigma T^4 \quad \text{Stefan–Boltzmann Law} \tag{18}$$

where

$$\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\ \text{K}^4} \quad \text{Stefan–Boltzmann constant} \tag{19}$$

flux units: [energy per per unit area per unit time]

Ex: the Sun

total solar **power** output

= rate per second of energy flow into space

= solar **"luminosity"** $= L_\odot = 3.85 \times 10^{26}$ W

Q: how to calculate this?

given $L_\odot$ and solar radius $R_\odot$

Q:how find Sun's blackbody temperature?

Use Stefan-Boltzmann to relate Sun's surface flux
to surface temperature:

$$F_{\odot,\text{surface}} = \frac{L}{\text{surf. area}} = \frac{L}{4\pi R_\odot^2} = 6.3 \times 10^7 \frac{\text{W}}{\text{m}^2} \quad (20)$$

$$\Rightarrow T^4 = \frac{F}{\sigma} \quad (21)$$

$$\Rightarrow T = \left(\frac{F}{\sigma}\right)^{1/4} = 5800 \text{ K} \quad (22)$$

check! this luminosity-based value *agrees*
with earlier color-based value using Wein's law
→ good consistency check, didn't have to agree
Q: what would disagreement mean?

And finally: flag thermodynamics(?)
Wein says: blue → $T \sim 8,000$ K
red → $T \sim 3,000$ K
Q: why doesn't a US flag burst into flame?
why aren't blue regions twice as hot as red?

Hour Exam 1: Review