Astro 210
Lecture 17
Feb 28, 2011

Announcements

- HW5 due at start of class Friday
  Oops! – typo found in Problem 2(d)
  which should refer to the situation in 2(c)
  erratum and corrected question posted
- Night Observing this week – *Dress warmly!*
  report forms, info online

Last time: planet temperatures
planet T set by an **equilibrium**
Q: between which opposing effects?
Q: why are these effects exactly balanced?

Planetary Temperatures Calculated

Can get excellent estimate of planetary T from
(fairly) simple first-principles calculation!

key is energy (power) balance: $\boxed{\text{absorption} = \text{emission}}$
diagram: sun, planet. label $R_\odot$, d, R

$\boxed{\text{Absorption}}$
recall: if surface of area $S_{\rm surf}$ emit flux $F_{\rm surf}$
then radiated power = luminosity [energy/sec] is $L = F_{\rm surf} S_{\rm surf}$
Sun: $L_\odot = F_\odot S_\odot = 4\pi R_\odot^2 \sigma T_\odot^4$
at planet, flux is $F = L/4\pi d^2 = \sigma T_\odot^4 (R_\odot/d)^2$ [energy/area/sec]

....but we know not all incoming sunlight is absorbed!
Q: Why not?
2 *Q: What substance* would *absorb all incident sunlight?*
Q: What substance would absorb no *incident sunlight?*
Q how could we simply quantify all of this?

Not all sunlight absorbed...or else wouldn't see Earth from space!
some is *reflected*!
recall: perfect absorber is blackbody
perfect reflector: ideal mirror
real substances/planets: somewhere between

Define: **albedo**

$$A = \frac{\text{amount of light reflected}}{\text{incident light}} \tag{1}$$

ideal mirror: $A = 1$
blackbody $A = 0$
Earth surface (average value) $A_{\text{Earth}} \approx 0.4$

iClicker Poll: Sun, Shadows, and the Earth

Which of these is larger?

A The surface area of sunlit portion of Earth

B The area of Earth's shadow

C (a) and (b) are equal

Planetary Energy Balance: Absorption

albedo A is fraction of incident radiation *reflected*
$\rightarrow$ fraction absorbed is $1 - A$

absorbed flux is $F_{\rm abs} = (1 - A) L_\odot / 4\pi d^2 = (1 - A)\sigma T_\odot^4 (R_\odot / d)^2$
over sunlit surface of planet, energy absorbed per sec:

$$W_{\rm abs} = \text{area intercepting sunlight} \times F_{\rm abs} \quad (2)$$

$$= \pi R^2 \, (1 - A)\sigma T_\odot^4 \, \left(\frac{R_\odot}{d}\right)^2 \quad (3)$$

$$= (1 - A)\pi R^2 \frac{R_\odot^2}{d^2} \sigma T_\odot^4 \quad (4)$$

note: effective absorbing area is πR^2
$\rightarrow$ planet's cross section – i.e., area of shadow

Planetary Energy Balance: Emission

emitted flux: $F_{\text{emit}} = \sigma T^4$ (avg surface T)

what area emits?
diagram: side view

case I: slowly rotating, no atm: backside cool
$\rightarrow$ only dayside emits
case II: both sides hot $\rightarrow$ both sides emit

emitting area:

$$S_{\text{emit}} = \begin{cases} 2\pi R^2 & \text{slow rot} \\ \approx 4\pi R^2 & \text{fast rot} \end{cases} \quad (5)$$

$\rightarrow$ energy emitted per sec:

$$W_{\text{emit}} = S_{\text{emit}} \sigma T^4 \quad (6)$$

Planetary Temperatures: The Mighty Formula

In equilibrium: $\boxed{W_{\text{abs}} = W_{\text{emit}}}$

$$(1-A)\ \pi R^2 (R_\odot/d)^2\ \sigma T_\odot^4 = \begin{pmatrix} 2\pi R^2 \\ 4\pi R^2 \end{pmatrix} \sigma T^4 \quad \begin{cases} \text{slow rot} \\ \text{fast rot} \end{cases} \tag{7}$$

and so planet surface temperature is

$$T = \begin{bmatrix} (1-A)/2 \\ (1-A)/4 \end{bmatrix}^{1/4} \left(\frac{R_\odot}{d}\right)^{1/2} T_\odot \quad \begin{cases} \text{slow rot} \\ \text{fast rot} \end{cases} \tag{8}$$

note:

- planet T set by Sun surface T
- *independent* of planet radius R!
- drops with distance from Sun, but as $T \propto 1/\sqrt{d}$

Calculate for $T_\odot = 5800$ K, and d in AU:

$$T = \begin{cases} 332\ \text{K} \ \left(\frac{1-A}{d_{\text{AU}}^2}\right)^{1/4} & \text{slow rot} \\ 279\ \text{K} \ \left(\frac{1-A}{d_{\text{AU}}^2}\right)^{1/4} & \text{fast rot} \end{cases} \tag{9}$$

for solar system objects, with d_{AU} in AU

Example: *the Earth*
inputs: $d_{\text{AU}} = 1$ AU, try $A = 0$
atm: day and night temp roughly same $\rightarrow$ fast rot (case II)

$$T_{\text{average}} = 279\ \text{K} \sim 6^\circ\ \text{C} \sim 43^\circ\ \text{F} \tag{10}$$

pretty close! ... but a little low
but using $A \approx 0.4$ gives $T = 246\ \text{K} = -27^\circ\text{C}$: yikes!
but: haven't accounted for greenhouse effect,
small deviations from perfect blackbody emission, ...

Q: so what do we expect for the Moon?

Atmospheres: Gas Properties

gases on microscopic scale
a swarm of particles, for example atoms or molecules

- gas particles have empty space between them
  not packed together as in liquid or solid

- gas particles are in constant random motion
  collide with each other, container walls (if any)
  exchange energy & momentum $\rightarrow$ distribution of speeds

on macroscopic scales (i.e., how we see things)
particle motions perceived as *temperature*

iClicker Poll: Gas Particle Speeds

consider a parcel of gas:

- macroscopically, gas is at rest (not moving/blowing)
- at room temperature T

in this gas:

the average particle **velocity** $\vec{v}$ and **speed** $v = |\vec{v}|$ are:

A $\vec{v} = 0$ and $v = 0$

B $\vec{v} = 0$ and $v > 0$

C $\vec{v} \neq 0$ and $v = 0$

D $\vec{v} \neq 0$ and $v > 0$

average particle velocity vector vanishes: $\langle \vec{v} \rangle = 0$
why? *not* because particles are still
rather: equal numbers with $v_x > 0$ vs $v_x < 0$ $\rightarrow$ averages to zero
otherwise: gas would have net v_x, wouldn't be at rest!

note microscopic–macroscopic (particle–bulk) correspondence:
micro: equal probabilities for particle $\vec{v} > 0$ and $\vec{v} < 0$
macro: corresponds to bulk gas speed $\vec{u}_{\rm gas} = 0$

since particles are moving, speeds $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle > 0$
$\rightarrow$ avg KE of one gas particle is nonzero
Q: what does this mean for bulk, macroscopic gas?

particle motion $\rightarrow$ particle kinetic energy
proportional to bulk temperature

$$\langle KE \rangle_{\text{per particle}} = \frac{1}{2}\mu\langle v^2 \rangle = \frac{3}{2}kT \tag{11}$$

example of general rule of thumb:
in thermal system, typical particle energy $\boxed{E_{\text{particle}} \sim kT}$
with $k = 1.38 \times 10^{-23}$ Joules/Kelvin: Boltzmann's constant

for thermal gas: average particle speed ("root mean square") is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{\mu}} \tag{12}$$

where μ = mass of 1 gas particle: "molecular weight"
if a gas particles has

$$\mathcal{A} = \text{tot \# of } n, p = \text{sum atomic weights} \tag{13}$$

then $\boxed{\mu = \mathcal{A} m_p}$, with m_p = proton mass $= 1.67 \times 10^{-27}$ kg also: peak speed (most probable)

$$v_p = \sqrt{2kT/\mu} < v_{\rm rms}$$

Q: how can this be less than average?

Note:

- $v_{\rm rms} \propto \sqrt{T}$: hotter $\to$ faster on avg
  T measures avg particle energy & speed
- $v_{\rm rms} \propto 1/\sqrt{\mu}$: more massive particles $\to$ slower on avg

Example: air is mostly N_2 and O_2 molecules

Q: in this room, which faster: ?

but: if peel orange,
smell does not propagate at $v_{\rm rms} \sim 500$ m/s:
$\to$ don't smell it within 10 ms!
Q: why not?

Director's Cut Extras

More on Equilibrium Temperatures

we argued that a planet's temperature T is set by
and **equilibrium** in which the energy flow into the planet
is exactly balanced by the energy flow out

Here we look at this in more detail

The energy flow onto a planet is set by the flux of sunlight

$$\left(\frac{dE}{dt}\right)_{\text{in}} = W_{\text{abs}} = (1-A)F_{\text{sunlight}}S_{\text{shadow}} = \pi R^2(1-A)\left(\frac{R_\odot}{d}\right)^2 \sigma T_\odot^4 \tag{14}$$

which is essentially constant and most importantly is
independent of the planet's temperature

The energy flow out of a fast rotator is

$$\left(\frac{dE}{dt}\right)_{\text{out}} = W_{\text{emit}} = S_{\text{emit}}F_{\text{bb}} = 4\pi R^2\sigma T^4 \tag{15}$$

by energy conservation, the a planet's heat energy content *changes* due to the energy flows in and out, i.e.

$$\left(\frac{dE}{dt}\right)_{\text{heat}} = -\left(\frac{dE}{dt}\right)_{\text{out}} + \left(\frac{dE}{dt}\right)_{\text{in}} \quad (16)$$

and since $E_{\text{heat}} \propto T$, we have

$$\frac{dT}{dt} \propto W_{\text{abs}} - W_{\text{emit}} \quad (17)$$

$$= -4\pi R^2 \sigma T^4 + \pi R^2 (1-A) \left(\frac{R_\odot}{d}\right)^2 \sigma T_\odot^4 \quad (18)$$

we can rewrite this last expression as

$$\frac{dT}{dt} \propto -4\pi R^2 \sigma \left[T^4 - \frac{1-A}{4} \left(\frac{R_\odot}{d}\right)^2 \sigma T_\odot^4\right] \quad (19)$$

$$= -4\pi R^2 \sigma \left(T^4 - T_{\text{eq}}^4\right) \quad (20)$$

where T_{eq} is the fast-rotator equilibrium temperature found above

Let's look at the three possibilities for the temperature change

$$\frac{dT}{dt} \propto -4\pi R^2 \sigma \left(T^4 - T_{\mathrm{eq}}^4\right) \tag{21}$$

if $T = T_{\mathrm{eq}}$ planet has exactly equilibrium temperature
then $dT/dt = 0$: temperature does not change with time

if $T > T_{\mathrm{eq}}$ planet hotter than equilibrium temperate
then $dT/dt < 0$: temperature *de*creases: the planet cools
cooling continues until $T = T_{\mathrm{eq}}$

if $T < T_{\mathrm{eq}}$ planet cooler than equilibrium temperature
then $dT/dt > 0$: temperature *in*creases: the planet warms
warming continues until $T = T_{\mathrm{eq}}$

So: no matter what temperature a planet starts with

- it will always be driven to the equilibrium temperature $T = T_{\mathrm{eq}}$
- and then temperature remains constant

We have already guessed this, but now we have proven it