Astro 210
Lecture 33
April 15, 2011

Announcements

- HW 9 due
- HW 10, due in 1 week: computer-based, pick one of two
  for the theory-inclined: simulate a star
  for the observation-inclined: cosmology data analysis
- also due in 1 week: OBAFGKM(LT) mnemonic contest
  win 10 bonus points, and maybe also glamourous prizes
- Hour Exam 2 back today (most did well!)

Last time:

- star luminosity $L \propto M^4$
  on HR: main sequence is a *sequence of mass*
- star lifespan $\tau \propto M^{-3}$
  on HR: main sequence also a sequence of *lifespan*

The Sun: Main Sequence Phase

solar evolution on main sequence: 4p$\rightarrow$ ^{4}He
$\rightarrow$ over time: H "fuel" $\rightarrow$ He "ash"
e.g., today, Sun's core $<$ 50% H!

so average particle mass μ *increases*: fewer but heavier

consequences:

- pressure $P = nkT = \frac{\rho}{\mu}kT$: larger $\mu \rightarrow$ pressure drop
- but Sun interior must still support Sun's weight
  $\Rightarrow$ pressure must stay *same*
- to maintain P, core contracts & heats
  $\rightarrow$ larger μ drives T up to compensate: core *hotter*

- fewer particles $\rightarrow$ fewer scatterers
  $\rightarrow$ light can escape more easily, faster
  $\rightarrow$ luminosity goes *up*!
  main sequence brightening

iClicker Poll: A Helium-Core Sun

What happens when *all* core H converted to He?

A the Sun's core expands

B the Sun's core contracts

C the Sun begins to burn helium

D the Sun ignites unburnt hydrogen outside core

$1M_\odot$ Star: Old Age

after core H exhausted

- core cools $\rightarrow$ loses pressure support
  core can't maintain hydrostatic equilibrium
- core contracts!
- H material overlying core aslo contracts, heats
  new fuel, can begin to burn!
  $\rightarrow$ H burning in "shell" around core

$\rightarrow L \uparrow$

- outer layers ("envelope") of star expands
  $\rightarrow$ cools: $T \downarrow$

red giant

`www: HR diagram`

The Dense Core

core $\rightarrow$ high density ρ
contraction slowed by Pauli exclusion principle
$\rightarrow$ quantum law: can't put $2e$'s in same state

at high densities:
quantum "degeneracy" pressure resists compression
like in ordinary solids

in high-density gas/solid:
pressure $P_{\text{degen}} = K\rho^{5/3}$
depends only on ρ, not T ($\neq$ ideal gas!)

structure: degenerate core, H-burning shell, envelope

core heats $\rightarrow$ He fusion ignites

normal gas: $T\uparrow$, $P\uparrow$ $\rightarrow$ expand $\rightarrow$ cool

degen. gas: $T\uparrow$, P const: no exp, cool:

$\rightarrow$ reaction speedup $\rightarrow$ explosion!

helium flash (few min)

but note: occurs deep in star $\rightarrow$ hidden by envelope!

after flash: core He burning

$^{4}\text{He} + ^{4}\text{He} + ^{4}\text{He} \rightarrow ^{12}\text{C} + \gamma$

ash $\rightarrow$ fuel!

similar to H-burning (main seq) but hotter, faster burn

most red giants in this phase

HR Diagram: Comparing Burning Phases

Note: in fair sample of stars:
main sequence makes up about 90% of the population
red giants make up most of the remaining 10%

`www: HR diagram`

Q: what does this tell us?
hint–imagine snapshot of fair sample of people
for example, attendance at White Sox/Cubs

HR Diagram and Stellar Life Stages

Main Sequence

- $\approx 90\%$ of stars
- hydrogen burning: $4p \rightarrow {}^4\text{He}$

Red Giants

- $\approx 10\%$ of stars
- helium burning: $3\,{}^4\text{He} \rightarrow {}^{12}\text{C}$

if stars born at roughly constant rate
most stars will be seen in longest life phase
$\Rightarrow$ main sequence phase longest, most of star life
red giant phase $\approx 1/10$ as long

Q: what happens when core He exhausted?

$1M_\odot$ Star: Death Throes

ultimately, core runs out of ^{4}He
now 2 shells: H- and He- burning
unstable! $\rightarrow$ thermal pulses
(every 10^3 yrs, for a few yrs)
expel mass in "superwind"
hot ejected gas $\rightarrow$ "planetary nebula"

`www: HST planetary nebulae`

hot core exposed! $\rightarrow$ cools rapidly
star core is exposed as bare "cinder"
supported by degeneracy pressure (electrons)
- very hot, but
- very compact $\rightarrow$ small

$\Rightarrow$ becomes white dwarf

White Dwarfs

"stellar corpse" – leftover after $1M_\odot$ star death

and for other low-mass stars too; see below

nearby example: Sirius B

`www: X-ray image`

- $M = 0.96M_\odot$
- $R = 0.0084R_\odot = 0.8R_{\text{Earth}}$!
- $\rho = (M/R^3)\rho_\odot \approx 2 \times 10^6 \rho_\odot = 2 \times 10^9 \text{ kg/m}^3$!

  $\Rightarrow$ 1 cm^3 contains 2 tons!

  compact! ultradense!

White Dwarf Structure

white dwarf *not* an ideal gas

supported by degenerate electrons $\rightarrow$ ultradense solid

equation of state:

$$P = K\rho^{\gamma} \begin{cases} \gamma = 5/3 & \text{“low” density} \\ \gamma = 4/3 & \text{“high” density} \rho \gg 10^9 \text{ kg/m}^3 \end{cases} \tag{1}$$

hydrostatic equilib gives $R^2 P \sim GM^2/R^2$

$\Rightarrow$ use this to eliminate P, relate M and R

Lower density white dwarfs: $\gamma = 5/3$

$GM^2/R^4 \sim KM^{5/3}/R^5$

$\Rightarrow R \propto M^{-1/3}$

Mass increases $\rightarrow$ radius *decreases!*

High density white dwarfs: $\gamma = 4/3$

for pressure to balance gravity: $GM^2/R^4 \sim KM^{4/3}/R^4$

$\Rightarrow M \sim (K/G)^{3/2}$!

mass is indep of R, ρ! numerically:

$$\boxed{M = M_{\text{Chandra}} = 1.4 M_{\odot}}$$

"Chandrasekhar limit!"

Q: what if white dwarf has $M < M_{\text{Chandra}}$?

Q: what if white dwarf has $M > M_{\text{Chandra}}$?

if high-density WD has $M < M_{\text{Chandra}}$
then pressure (more than) enough to balance gravity
$\rightarrow$ WD is stable against collapse

but: if high-density WD has $M > M_{\text{Chandra}}$

then pressure *not enough* to balance gravity
$\rightarrow$ gravity force not balanced
$\rightarrow$ star unstable $\rightarrow$ collapses under its own weight!
$\rightarrow$ catastrophe!

conclusion: Chandrasekhar mass is
maximum mass of white dwarfs!

Confirmed! All observed white dwarfs have $M < M_{\text{Chandra}}$

Testing Stellar Evolution

recall: evolution depends on mass

thus far: looked in detail at $1 M_\odot$ evolution

now need to know: how do other stars evolve?

Beyond $1M_\odot$: Low-Mass Stars

since $\tau = 10^{10}$ yr$/m^3$
long lifetime if $m < 1M_\odot$
τ = 14 Gyr = age of universe for $m \sim 0.9M_\odot$
$\rightarrow$ if m lower, "live forever"

for $m \lesssim 0.08M_\odot$, core too cool to burn H
"brown dwarfs"
Q: what (if any) is heat source? how does star evolve?

Bottom line:
not much going on with low-mass stars
but (by number) most stars are low-mass
high-mass stars are rare...but spectacular...

Lives and Deaths of Stars

a star's life history, death controlled by it **mass**

$M < 0.9 M_\odot$

history like that of the Sun to date
burn H $\rightarrow$ He
lifetime $>$ age of universe: live "forever"
i.e., none have yet died

$0.9 M_\odot < M < 8 M_\odot$

history like that of the Sun
life: burn H $\rightarrow$ He ("main sequence" phase)
then "giant" phase burning He $\rightarrow$ C
death: eject $>$ 50% of mass as enriched gas–"planetary nebula"
leave behind compact object: white dwarf

$M > 8 M_\odot$

history begins like Sun, but then very different...