Astro 210
Lecture 40
May 2, 2011

Announcements

no HW for this week!

- Final Exam: **Monday May 9, 1:30–4:30 pm**

  info online
- **ICES** course evaluation available online

  please fill it out–I *do* read & use results

iClicker Poll: ICES

Vote your conscience!

Did you get one or more ICES notification emails for this course?

A Yes, and I promise to fill out the form if I have not already

B No, but I promise to go to https://ices.cte.uiuc.edu/ and fill out the form if I have not already

Last time: Began cosmology

- cosmic structure
  Q: cosmic "building blocks"?
  Q: distribution in space?

galaxies are cosmic “building blocks”
like stars are basic unit of galaxies

Cosmic Structure

on large scales, Universe is

1. **homogeneous**:
   galaxies fill space with (nearly) uniform density

2. **isotropic**:
   universe looks same in all directions

Motions of Galaxies

measure velocity respect to us
i.e., in galaxy spectra, look for shifts in lines

Results:
(almost) all galaxies show redshift
all galaxies move away from us!

Hubble (1929):
galaxy distance d and speed v related
`www: Hubble's original data; modern data`
$v \propto d$, or

$$\boxed{v = Hd} \quad (1)$$

Q: in simple terms, what does this say about galaxy motion?

Hubble's Law

Hubble sez: farther $\rightarrow$ faster

more specifically: Hubble Law

$$v = Hd \tag{2}$$

H: "Hubble constant" (sometimes written H_0)
Q: units of H?
value: $H_0 \approx 70$ km/(s · Mpc)

draw random (homog) dots
Q: add velocity vectors–what's the pattern?

- What does it mean?
  Q: logical possibilities?

The Meaning of Hubble's Law

observe: all galaxies move away from us!
galaxy motion *not* random, but rather highly organized
→ cries out for interpretation!

Egoist view: we are at center of Universe!

but:

- MW typical galaxy, not special
- center un-Copernican

Einstein view:
no center! ...and no edge! Universe is homogeneous!

the universe is expanding

highfalutin mathematics:
Hubble's law a very special mathematical form
in fact, the **only** form that has the following property:
photocopy universe

Hubble's law universal:
in expanding U, **all** galaxies
find $v = Hd$
$\rightarrow$ don't need to live in a special place to
find Hubble's law

Revolution Re-Re-Revisited

Copernican Revolution I (17th Century):
Earth is one typical planet among many
not center of solar system

Copernican Revolution II (earth 20th Century):
Sun is one typical star among many
not center of Milky Way Galaxy

Copernican Revolution III (1920's):
Milky Way is one typical galaxy among many
Universe much larger than previously thought

Copernican Revolution IV (1929):
we live in an expanding universe
which has no center at all!

Expanding Universe: Theory

recall General Relativity (Einstein Gravity):
★ space (& time) dynamic!
★ Universe is dramatic example of this

on cosmic lengthscales, space uniformly expands
stretches "like rubber sheet"
diagram: grid lines, galaxies at t_1, $t_2 > t_1$

Describe mathematically: write $d(t) = a(t)r$
$d(t)$: physical separation
r: fixed coords on rubber sheet
$a(t)$: measures expansion – "scale factor" or stretch-o-meter
put present age of Universe: $t = t_0$ today
then $a(t_0) = a_0 = 1$

Expansion Rate

Consider two observers (e.g., galaxies) at time t
separated by distance $d(t) = a(t)r$

what is velocity of one point as measured by the other?

$$\begin{aligned} v(t) &= \frac{\partial}{\partial t} d = \frac{\partial}{\partial t}(ar) = \dot{a}r & (3) \\ &= \frac{\dot{a}}{a}\, ar = \frac{\dot{a}}{a}\, d \equiv H\ d & (4) \end{aligned}$$

so expansion gives $v = Hd$, with $H = \dot{a}/a$

⋆ Hubble's law!

⋆ we see $H = H(t)$ measures expansion rate
and need not be constant! → "Hubble Parameter"
H_0 is present value, can (& does!) change with time

Expanding Universe: The Past

In the past, Galaxies closer together: $a(t) < 1$
U. was **denser**, also *hotter*

Universe began in very
hot
dense
state: $\Rightarrow$ big bang

expanded, cooled to present state

Where did the Big Bang Happen?
Q: Already know enough–where?

Where Was the Big Bang?

Universe is homogeneous & isotropic: no special points!
$\rightarrow$ big bang has no center $\rightarrow$ happened everywhere

Age of the Universe

Simple-minded estimate: imagine

- each galaxy initially launched with speed $v = v_{\rm init}$
- keeps constant speed over age of universe t_0

distance traveled: $d = vt_0$

but today $v = H_0 d$, so

$$d = (H_0 d)t_0 \quad \Rightarrow \quad H_0 t_0 = 1 \tag{5}$$

estimate "expansion age" $t_0 \approx 1/H_0 = 14$ billion years

Q: how does this compare to age of solar system?

Q: why is this estimate too simple?

Solar system age: 4.6 billion years
$\rightarrow$ Sun & Earth alive for last 1/3 of age of Universe

note: estimate too simple!
• galaxies feel each other's gravity, do not "coast"
• expansion rate changes with time
for the record, full answer is:

$$t_0 = \int dt = \int_{a=0}^{a=1} \frac{da}{\dot{a}} = \int_0^1 \frac{da}{aH} \tag{6}$$

using best estimate of expansion history H,
best estimate of expansion age is $t_0 = 13.7$ billion years
$\rightarrow$ close to our simple estimate!

Q: given that U expanding today
what are possible fates in future?

The Fate of the Universe

The story until ∼8 years ago
expansion and fate of universe is competition:
outward inertia of expansion vs *inward gravity*
...just like *pop fly* (ball hit upward)

currently: U expanding
like ball (pop fly) launched upward

future possibilities:

- gravity > inertia: *recollapse*
  like $v_{\rm esc} > v$–ball falls back
- gravity < intertia: *expand forever*
  like $v_{\rm esc} < v$–ball (rocket!?) leaves earth!
- gravity = intertia: *expand forever but $H \to 0$ at $t \to \infty$*
  like $v_{\rm esc} = v$–ball escapes but $v \to 0$ at $t \to \infty$

iClicker Poll: Cosmic Acceleration/Deceleration

How should the cosmic expansion *rate* change w/ time?

A *increase*: U. accelerates, $d^2a/dt^2 > 0$

B *decrease*: U. decelerates, $d^2a/dt^2 < 0$

C *constant*: U. coasts, $d^2a/dt^2 = 0$

Director's Cut Extras

How does the Universe expand?

- need to find $a(t) \to$ encodes cosmic expansion history
- How to find $a(t)$?
  similar to Newtonian physics we know and love:
  find law of motion that relates motion to gravity sources

consider two points (observers) in Universe at t

- (arbitrary) central point at $r = 0$
- a test mass m at distance $d(t) = a(t)\, r$

find energy, motion of test mass
$\to$ tells motion of any arbitrary point in universe
$\to$ cosmic dynamics!

what is energy, motion of test mass?

- sees spherical "enclosed" cosmic mass
  $M = \rho V_{\text{sphere}} = 4\pi/3\ d^3 \rho$
- has energy $E_{\text{tot}} = KE + PE = const$ (< 0 if bound)

$$
\begin{aligned}
\frac{1}{2} m v^2 - \frac{GMm}{d} &= const &(7)\\
m\left[\frac{1}{2}(Hd)^2 - \frac{4\pi G}{3}\rho d^2\right] &= const &(8)\\
H^2 - \frac{8\pi G}{3}\rho &= \frac{2const}{md^2} = -\frac{\kappa}{a^2} &(9)\\
\left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\,\rho - \frac{\kappa_2}{a} &(10)
\end{aligned}
$$

Friedmann equation for evolution of $a(t)$

Cosmodynamics

Friedmann is cosmic "equation of motion"
kinda a cosmic "$F = ma$" (really, cosmic energy equation)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{\kappa}{a^2} \tag{11}$$

Which means that the evolution of the universe
- expansion rate $H = \dot{a}/a$, and thus
- cosmic history $a(t)$, are controlled by

are controlled by the contents of the universe
- density ρ (can change with time)
- "energy" (really–curvature) constant $\kappa = const$

A Matter-Only Universe

Simplest situation (simplest universe!): matter only

- $\rho = \rho_{\text{matter}}$ only (no funny business!)
  then in sphere d, mass $M = 4\pi d^3 \rho/3 = const$
  so $\rho \propto a^{-3}$: as U expands, ρ decreases!
- total energy (curvature) zero: $\kappa = 0$

Evolution Solved

Assemble the pieces and turn the crank:

$(\dot{a}/a)^2 = 8\pi G/3\ \rho_0 a^{-3}$

$\dot{a}/a \propto a^{-3/2}$

$a^{1/2} da \propto dt$

integrate: $\int_0^a a^{1/2} da \propto \int_0^t dt$

$a^{3/2} \propto t$

$a \propto t^{2/3}$

put $t = t_0$ today

want $a(t_0) = 1$

So finally arrive at solution for matter-only Universe: $a(t) = (t/t_0)^{2/3}$

Q: fate of such a universe? i.e., what happens when $t \to \infty$*?*

What does it mean?
For matter-only universe, $a(t) = (t/t_0)^{2/3}$

fate: a always increases with time
$\rightarrow$ universe expands forever
as $t \rightarrow \infty$, then $a \rightarrow \infty$
universe becomes very low density!

what is t_0 = age of Universe?
since $H = \dot{a}/a = 2/3t$ always!
today: $t_0 = 2/3 \ 1/H_0 \simeq 10$ billion years

Q: is this reasonable?
compare: Earth 4.6 billion yrs
oldest stars: 10–12 billion yrs