Astronomy 501 Spring 2013
Problem Set #2

Due in class: Friday, Feb. 1

Total points: 7+1

1. [**3 points**] *The Eddington Limit.* Rybicki & Lightman problem 1.4; each part is worth 1 point.
   Note: you'll want to look at the Director's Cut Extras from Lecture 4

2. [**1 point**] *Kirchhoff's Law.* Rybicki & Lightman problem 1.9; each part is worth 0.5 points.

3. *A circumstellar disk.*[1] Consider a perfectly flat, infinitely thin disk around a star of luminosity L and radius $R_\star$. Let the disk absorb all starlight that falls on it, and re-radiate as a perfect blackbody. Ignore any energy transport through the disk, i.e., let each annulus exchange heat with its neighbors.

   This represents a simple but illustrative model of disks that are found around, e.g., young stars and around supermassive black holes (AGN).

   (a) [**1 point**] Find the disk temperature profile $T(r)$ as a function of the radius r from the star. You may take $r \gg R_\star$. The most important part of the problem is to find how the temperature scales with radius.

   Hint: to simplify the problem you may approximate the disk illumination at a distance r to come from a point source a height $R_\star$ above the disk plane. Ask yourself what is the *flux* onto the disk at radius r.

   If you are familiar with the expression for planetary temperatures, compare the $T(r)$ scalings. If they are different, why? If not, why not?

   (b) [**1 bonus point**] Now treat the star not as a point source but as a half-disk. You may still take $r \gg R_\star$. You should find your answer only changes by a numerical factor.

4. *The CMB and Reionization.*[2] The cosmic microwave background has a specific intensity which we may write as

$$I_\nu = I_\nu^{\rm iso} + \delta I_\nu \tag{1}$$

   i.e., an isotropic signal $I_\nu^{\rm iso}$, to which is added an *anisotropic* perturbation δI_ν that is small: $|\delta I_\nu| \ll I_\nu$. The CMB arises at great distances (i.e., high redshifts $z \sim 1100$) during cosmic recombination. This is when the universe cools such that it goes from an ionized and opaque plasma of mostly free protons and electrons, to a neutral and transparent gas of mostly neutral hydrogen atoms. Thus the sightline to the CMB passes through almost the entire observable universe.

   Subsequent to the formation of the CMB, the neutral universe was largely ionized again; this "reionization" was likely due to ultraviolet radiation from the first stars,

[1] Based on a problem assigned by Eugene Chiang.

[2] Based on a problem assigned by Wayne Hu.

and/or from the first quasars. The CMB passes through the reionized universe, which effectively presents a cloud of free electrons that act as a "screen" that scatters the CMB photons coherently and isotropically. We will study the effect of this scattering screen on the initial, unscattered CMB signal, which we will call $I_\nu(0)$.

(a) **[0.5 points]** Show that for the initial CMB signal, without loss of generality we can always write the perturbation $\delta I_\nu(0)$ such that we have $J_\nu(0) = I_\nu^{\rm iso}(0)$ and $\int \delta I_\nu(0)\ d\Omega = 0$.

(b) **[0.5 points]** Write down the equation of radiative transfer for the case of no absorption, and no sources other than scattering that is isotropic and coherent. Express the equation in terms of optical depth τ, which turns out to be frequency independent.

Ignore for the moment the perturbation term, and then write the transfer equation for the zeroth order approximate $I_\nu \approx I_\nu^{\rm iso}$. Solve the equation to show how the zeroth order term depends on the reionization optical depth τ.

Interpret your result physically. The WMAP-measured CMB optical depth is about $\tau \simeq 0.09$; how large an effect will this have on the CMB?

(c) **[0.5 points]** Now include the first order correction δI_ν in the transfer equation. Also use the solution from part (b) for the zeroth order term. Solve the equation to show how the first order term depends on the reionization optical depth tau.

Interpret your result physically. What effect does reionization have on CMB temperature anisotropies?

(d) **[0.5 points]** Working in the Rayleigh-Jeans regime of the CMB spectrum, rewrite the CMB specific intensity field I_ν in terms of the antenna temperature field $T = T_{\rm iso} + \delta T$, where $T_{\rm iso}$ is isotropic. Show that, given our assumptions, $|\delta T| \ll T_{\rm iso}$.

Go on to find how the observed $T_{\rm iso}$ and δT are related to the initial, unscreened $T_{\rm iso}(0)$ and $\delta T(0)$.

Interpret your result physically.