Astro 501: Radiative Processes
Lecture 16
Feb 20, 2013

Announcements:

- **Problem Set 5** due 5pm Friday

Last time: thermal bremsstrahlung

Q: spectral shape of emission?

Q: spectral shape of absorption coefficient? implications?

Today: Special Relativity and Radiation

Q: when is special relativity astrophysically important?

Special Relativity for the Impatient

Spacetime

see S. Carroll, *Spacetime and Geometry*; R. Geroch, *General Relativity from A to B*

evolving view of space, time, and motion:
Aristotle → Galileo → Einstein

Key basic concept: **event**
occurrence localized in space and time
e.g., firecracker, finger snap
idealized → no spatial extent, no duration in time

a goal (*the* goal?) of physics:
describe relationships among events

Q: consider collection of all possible events–what's included?

Spacetime Coordinates

Each event specifies a unique point in space and time
collection of all possible events = **spacetime**

lay down coordinate system: 3 space coords, one time
4-dimensional, but as yet time & space unrelated

e.g., time t, Cartesian x, y, z: event $\rightarrow (t, x, y, z)$
physics asks (and answers) what is the relationship
between two events, e.g., (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2)

Note: more on spacetime in Director's Cut Extras to today's notes

Galilean Relativity

consider *two laboratories*
(same apparatus, funding, required courses, vending machines)
move at constant velocity with respect to each other

Galileo:
no experiment done in either lab (without looking outside)
can answer the question "which lab is moving"
$\rightarrow$ *no absolute motion*, only relative velocity

Newton: laws of mechanics invariant
for observers moving at const $\vec{v}$
"*inertial observers*"

Implications for spacetime
no absolute motion $\rightarrow$ *no absolute space*
(but still no reason to abandon absolute time)

Galilean Frames

each inertial obs has own personal frame:
obs ("Angelina") at rest in own frame: (x, y, z) same for all t
but to another obs ("Brad") in relative motion $\vec{v} = v\hat{x}$
B sees A's frame as time-dependent:

$$x_{\text{A as seen by B}} = x' \;=\; x - vt \tag{1}$$

but still absolute time: $t' = t$
Newton's laws (and Newtonian Gravity) hold in both frames
can show: $d^2\vec{x}/dt^2 = \vec{F}(\vec{x}) \Rightarrow d^2\vec{x}'/dt'^2 = \vec{F}(\vec{x}')$
"Galilean invariance"

Geometrically:
different inertial frames $\rightarrow$ transformation of spacetime
slide the "space slices" at each time
(picture "shear," or beveling a deck of cards)

Trouble for Galileo

Maxwell: equations govern light
very successful, but:

- predicts unique (constant) light speed c–relative to whom?
- Maxwell eqs **not** Galilean invariant

Lorentz: Maxwell eqs invariant when

$$t' = \gamma(t - vx/c^2) \tag{2}$$

$$x' = \gamma(x - vt) \tag{3}$$

$$y' = y \tag{4}$$

$$z' = z \tag{5}$$

with Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$

Einstein:
Lorentz transformation not just a trick
but correct relationship between inertial frames!
$\Rightarrow$ this is the way the world is

Einstein: Special Relativity

consider two *nearby events*
(t, x, y, z) and $(t + \Delta t, x + \Delta x, y + \Delta y, z + \Delta z)$

different inertial obs *disagree* about Δt *and* $\Delta \vec{x}$
but all *agree* on the value of the **interval**

$$\Delta s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \quad (6)$$

$$= (c\Delta t)^2 - (\Delta \ell)^2 \quad (7)$$

Note: interval can have $\Delta s^2 > 0, < 0, = 0$

quantities agreed upon by all observers: *Lorentz invariants*

Light pulse: $\Delta \ell = c\Delta t$
$\rightarrow \Delta s_{\text{light}} = 0$
$\rightarrow$ light moves at c in all frames!

Motion and time:

Consider two events, at rest in one frame:

$\Delta\vec{x}_{\text{rest}} = 0$ in rest frame, so

$\Delta s = c\Delta t_{\text{rest}}$: $c\times$ elapsed time in rest frame

In another inertial frame, relative speed v:

events separated in space by $\Delta x' = v\Delta t'$

$$\Delta s = \sqrt{c^2 \Delta t'^2 - \Delta x'^2} = \sqrt{c^2 - v^2}\Delta t' = \frac{1}{\gamma} c \Delta t' \tag{8}$$

since Δs same: infer $\Delta t' = \gamma \Delta t_{\text{rest}} > \Delta t_{\text{rest}}$

$\Rightarrow$ moving clocks appear to run slow

(special) relativistic **time dilation**

$\Rightarrow$ no absolute time (and no absolute space)

H. Minkowski:

"Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Spacetime and Relativity

Pre-Relativity: space and time separate and independent
but *rotations* mix *space* coords, e.g.,

$$x' = x \cos\theta - y \sin\theta \quad ; \quad y' = y \cos\theta + x \sin\theta \tag{9}$$

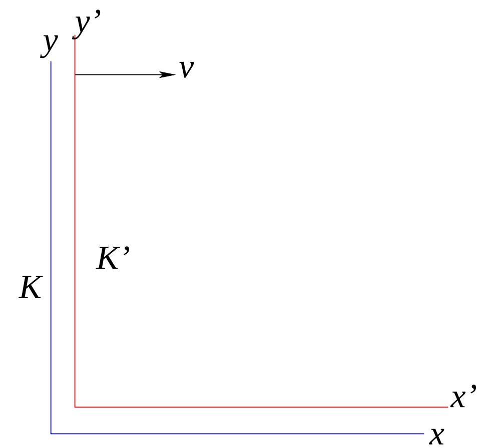
without changing underlying vector (rotation of coords only)
transform rule holds not only for $\vec{x}$
but all other physical directed quantities: e.g., $\vec{v}, \vec{a}, \vec{p}, \vec{g}, \vec{E}$
all transform under rotations following same rule, e.g.,

$$E'_x = E_x \cos\theta - E_y \sin\theta \quad ; \quad E'_y = E_y y \cos\theta + E_x \sin\theta \tag{10}$$

Lesson: express & guarantee underlying rotational invariance
by writing physical law in vector form
e.g., $\vec{F} = m\vec{a}$ gives same physics for any coord rotation

Lorentz Transformations

consider two coordinate systems K, K'
moving with *relative* speed $\vec{v} = v\hat{x}$



$$\begin{aligned} t' &= \gamma(t - vx/c^2) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned}$$

- **boost** from one frame to another
- truly mix space and time $\rightarrow$ *spacetime*
- look like rotations, but 4-dimensional

$\rightarrow$ should express laws in terms of 4-D vectors:
"4-vectors," t, x components transform via Lorentz

Length Contraction

consider a *standard ruler*: measure length at factory

- ruler is at rest wrt observer
- measure both ends *at same time* $\delta t = t_2 - t_1 = 0$
- ends are are $x_1 = 0$, $x_2 = L \to$ length $L = \delta x = x_2 - x_1$

observer flying by a speed $\vec{v} = v\hat{x}$, makes measurement

$$\delta x' = \gamma(\delta x - v\delta t) = \gamma(L - v\delta t/c^2) \tag{11}$$

$$\delta t' = \gamma(\delta t - v\delta x/c^2) = \gamma(\delta t - vL/c^2) \tag{12}$$

but length measurement is done at same time

$\to \delta t' = 0 \ \to \delta t = vL/c^2$ *Q: implications?*

and thus length found is $L' = \delta x' = \gamma(1 - v^2/c^2)L$

$\Rightarrow L' = L/\gamma$ **length contraction**

Q: what if the observer were moving in $\hat{y}$?

Addition of Velocities

consider an object moving wrt to frame K'
as seen by K': in time interval dt, moves distance dx $\rightarrow$ has velocity $u' = dx'/dt'$

What is speed in frame K (speed v wrt K)?

$$
\begin{aligned}
dt &= \gamma(dt' + v\,dx'/c^2) && (13)\\
dx &= \gamma(dx' + v\,dt') && (14)\\
dy = dy & \qquad dz = dz' && (15)
\end{aligned}
$$

and thus

$$
\begin{aligned}
u_x &= dx/dt = \frac{u'_x + v}{1 + u'_x v/c^2} && (16)\\
u_y &= dy/dt = \frac{u'_y}{\gamma(1 + u'_x v/c^2)} && (17)\\
u_z &= dz/dt = \frac{u'_z}{\gamma(1 + u'_x v/c^2)} && (18)
\end{aligned}
$$

If object is moving with arbitrary velocity $\vec{u}'$ in K' decompose $\vec{u}' = \vec{u}'_{\|} + \vec{u}'_{\perp}$ where $\|$ is along $K - K'$ motion:

$$u_{|} = \frac{u'_{\|} + v}{1 + u_{\|} v/c^2} \tag{19}$$

$$u_{\perp} = \frac{u'_{\perp}}{\gamma(1 + u_{\|} v/c^2)} \tag{20}$$

boost causes **change in velocity direction**

$$\tan\theta = \frac{u_{\perp}}{u_{\|}} = \frac{u' \sin\theta'}{\gamma(u' \cos\theta' + v)} \tag{21}$$

and consider the case where $u' = c$

$$\tan\theta = \frac{\sin\theta'}{\gamma(\cos\theta' + \beta)} \tag{22}$$

$$\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta} \tag{23}$$

angular shift is the **abberation of light**

a light signal emitted in K' at angle θ
is seen in K at angle

$$\cos\theta = \frac{\cos\theta' + v/c}{1 + v/c\ \cos\theta'} \tag{24}$$

Q: what if $\theta' = 0$*?* π*?*
Q: how can we understand this physically?

Q: what if $\theta' = \pi/2$*?*
Q: how can we understand this physically?

consider photons emitted *isotropically* in K'
with v/c not small
Q: what is angular pattern in K*? implications?*

Relativistic Beaming

for light emitted in K' at $\theta' = \pi/2$
observed angle after boosting is

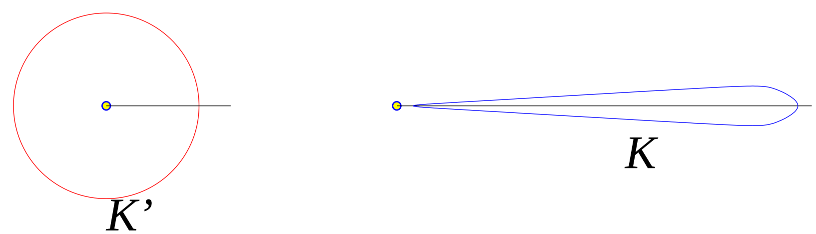
$$\tan\theta = \frac{1}{\gamma v/c} \tag{25}$$

and thus

$$\sin\theta = \frac{1}{\gamma} \tag{26}$$

if emitted K' is highly relativistic,
then $\gamma \gg 1$, and

$$\theta \rightarrow \frac{1}{\gamma}$$



i.e., a small forward angle!
a highly relativistic emitter gives a **beamed radiation pattern**
strongly concentrated ahead of emitter direction

Director's Cut Extras

Pre-Relativity: Aristotle

x, y, z Cartesian (Euclidean geometry)
spatial distance ℓ between events is:

$$\ell^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \tag{27}$$

and is independent of time
elapsed time between events is: $t_2 - t_1$
and is independent of space
"absolute space" and "absolute time"

Is a particle at rest? $\Leftrightarrow$ do (x, y, z) change?
$\rightarrow$ "absolute rest, absolute motion"

Diagram: Aristotelian spacetime
unique "stacking" of "time slices"

Life According to Aristotle

Note: even in absolute space
event location indep of coordinate description
e.g., two observers choose coordinates different by a rotation:
(x, y) and $(x', y') = (x\cos\theta - y\sin\theta, y\cos\theta + x\sin\theta)$
preserves distance from origin: $x^2 + y^2 = (x')^2 + (y')^2$

objects (observers) at rest:
same x, y, z always, t ticks forward
geometrically, a line in spacetime: **"world line"**
if at rest: world line vertical
constant speed: $x = vt$: diagonal line

light: moves at "speed of light" c
$\rightarrow$ well-defined, since motion absolute
in spacetime: light pulse at origin $(t, x, y, z) = (0, 0, 0, 0)$
moves so that distance $\ell = \sqrt{x^2 + y^2 + z^2} = ct$
geometrically: **light cone**