Astro 501: Radiative Processes
Lecture 18
Feb 25, 2013

Announcements:

- good news: *no problem set this week!*
- bad news: **Midterm in class Friday**
  info online

Last time: more Special Relativity

Q: relationship between relativistic energy and momentum?

Q: relativistic particle energy and momentum dependence on $\vec{v}$?

Q: electric & magnetic field Lorentz transformation properties?

Q: electric field of a relativistic point charge with large v?

Q: relativistic power–nice feature?

4-momentum of particle with mass m:

$$P = (E/c, \vec{p}) = (mc\gamma, m\gamma\vec{v}) \tag{1}$$

where the invariant norm is

$$E^2 - (cp)^2 = (mc^2)^2 \tag{2}$$

and where relativistic energy is

$$E = mc^2\gamma \xrightarrow{\text{nonrel}} mc^2 + \frac{1}{2}mv^2 + \cdots \tag{3}$$

and relativistic momentum is

$$\vec{p} = m\gamma\vec{v} \tag{4}$$

note also that

$$\frac{\vec{v}}{c} = \frac{c\vec{p}}{E} \tag{5}$$

Lorentz transformation: if boost with velocity $\vec{v} = c\vec{\beta}$

$$E'_{\parallel} = E_{\parallel} \qquad B'_{\parallel} = B_{\parallel} \tag{6}$$

$$E'_{\perp} = \gamma(E_{\perp} + \vec{\beta} \times \vec{B}) \tag{7}$$

$$B'_{\perp} = \gamma(B_{\perp} - \vec{\beta} \times \vec{E}) \tag{8}$$

Relativistic version of force law:

$$ma^{\mu} = \frac{dP_{\mu}}{d\tau} = qU_{\nu}F^{\nu}_{\mu} \tag{9}$$

zero component: energy conservation

$$\frac{dE}{dt} = q\vec{v} \cdot \vec{E} \tag{10}$$

space components: Lorentz force

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v} \times \vec{B} \tag{11}$$

$$P = \frac{dW}{dt} = \frac{dW'}{dt'} = P' \tag{12}$$

total power emitted is Lorentz invariant!

Power Emitted by a Relativistic Charge

Larmor: $P' = 2q^2/3c^3\ |\vec{a}'|^2$
want to re-express using 4-acceleration

can show: in instantaneous rest frame, $a^{0\prime} = 0$
and thus $|\vec{a}'|^2 = a \cdot a$
Lorentz-invariant Larmor expression for total radiated power

$$P = \frac{2}{3}\frac{q^2}{c^3} a \cdot a \tag{13}$$

manifestly invariant, can evaluate in any frame

$$P = \frac{2}{3}\frac{q^2}{c^3} a \cdot a \tag{14}$$

in instantaneous rest frame, 4-acceleration transforms as

$$a'_{\|} = \gamma^3 a_{\|} \tag{15}$$

$$a'_{\perp} = \gamma^2 a_{\perp} \tag{16}$$

$$\tag{17}$$

and so power emitted is

$$P = \frac{2}{3}\frac{q^2}{c^3} a' \cdot a' = \frac{2}{3}\frac{q^2}{c^3}(a'^2_{\perp} + a'^2_{\|}) \tag{18}$$

$$= \frac{2}{3}\frac{q^2}{c^3}\,\gamma^4\,(a^2_{\perp} + \gamma^2 a^2_{\|}) \tag{19}$$

note large boost for relativistic particles ($P \propto \gamma^4$ or γ^6)

Cosmic Rays

The Mystery of the Ionizing Radiation

Early history: $\sim 1900 - 1912$
pioneers of radioactivity studies found that some elements (isotopes)
would emit α, β, or γ-rays
  powerful ionizing agents
  with different ranges in matter = "penetrating power"
Q: what are α, β, γ rays? why emitted?
Q: for, e.g., $\sim few$ MeV, which is most, least penetrating?

But soon realized that even *without* radioactive samples
  ionization gauges give nonzero signal!
$\Rightarrow$ "background" radiation

Q: possible sources?
Q: how would you design an experiment to
  discriminate among then using 1912 technology?

Victor Hess and the Discovery of Cosmic Rays

possible background ionization sources:

- terrestrial: from radioactive isotopes in Earth's crust
  e.g., uranium, thorium
- extraterrestrial: from Sun?

Victor Hess, 1912: take ionization detectors on hot-air balloon

- ionization signal first goes *down*,
  but by $h \sim 5$ km goes *up* to $\sim few\times$ sea level rate!

$\Rightarrow$ terrestrial ionization sources dominant at ground
...but extraterrestrial sources exist!

- survive passage thru atmosphere $\Rightarrow$ very penetrating: γ rays?

∞ *Q: how to tell is cosmic rays are solar or extrasolar?*

Hess repeated balloon experiment during solar eclipse:

- no reduction in signal

$\Rightarrow$ radiation does not come from Sun!

$\Rightarrow$ *"cosmic radiation"* = **cosmic rays**

`www: 1936 Nobel Prize in Physics`

Cosmic Rays: Vital Statistics

Cosmic rays: population of particles which are

- electrically charged
- energetic ($\gtrsim 1$ MeV)
- nonthermal *Q: meaning?*

Cosmic Ray Sources:

- solar activity: ~ 0.1 MeV to ~ 1 GeV, typically few MeV
  `www: Solar Flares`
- all others = bulk of cosmic rays: origin outside solar system

`www: real-time satellite data`

composition: mostly nuclei (fully stripped of e^-)

- nuclear (hadronic) component:
  90% are protons
  of remainder, 90% are α
  elements up to Se detected
  `www: proton flux`

- electron/positron (leptonic) component: mostly e^-, some e^+
  at fixed energy, electron flux $\mathcal{I}_E(e) \sim \mathcal{I}_E(p)/100$ of protons

angular distribution:
isotropic over most of energy range

cosmic rays are often annoyance to non-CR astronomers *Q: why?*

Observed Electron Component

Experimental techniques:

- balloons
- space missions
- ground-based (high-energy): atm Čerenkov, air shower arrays

flux at top of atmosphere depends on location *Q: why?*
  and on time

anti-correlation between CR flux at Earth and solar activity

$\Rightarrow$ solar "modulation" of CR

- excludes $\lesssim 100$ MeV particles
- reduces $\lesssim 1$ GeV flux

must correct for solar effects ("demodulate") to infer
  interstellar spectra

Cosmic Ray Spectrum

specific *number* intensity usually expressed in units of energy

$$\mathcal{I}_E = \frac{d\mathcal{N}}{dA\,dt\,d\Omega\,dE} \quad (20)$$

$$= v(E)\frac{d\mathcal{N}}{dV\,d\Omega\,dE} \quad (21)$$

or of Lorentz $\gamma = E_{\rm tot}/m_e c^2$

`www: CR electron spectrum`

Q: what is number spectrum $\mathcal{I}_E$*? energy spectrum* $\mathcal{I}_E$*?*

`www: CR proton spectrum`

cosmic ray spectrum clearly **nonthermal**
rather: a succession of *power laws*

- observed CR electrons: roughly

$$\mathcal{I}_E(e) \propto E^{-3} \tag{22}$$

and thus $I_E(e) = E/\mathcal{I}_E(e) \propto E^{-2}$

Q: will protons be the same? different?

- protons w/ $1\ \text{GeV} \lesssim E \lesssim 300\ \text{TeV}$:

$$I_E(p) \simeq 1.4\ \left(\frac{E}{\text{GeV}}\right)^{-s}\ \text{protons cm}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1}\text{GeV}^{-1} \tag{23}$$

  where spectral index (“slope”) $s \simeq 2.7$
- beyond “knee’ at $E_{\text{knee}} \sim 10^{15}$ eV
  power law index steepens to $s \sim 3$
- then beyond “ankle” at $E_{\text{anlke}} \simeq 10^{18}$ eV, flattens again

Q: Tevatron energy? LHC? implications?
historically: many particles first discovered via CRs

Q: in which regime are most CR particles? most CR energy?

What's typical?

cosmic-ray number flux
$\Phi(>E) = 4\pi \int I(E)\ dE = 4\pi \int E\ I(E)\ d\ln E$
per log energy interval, number distribution is
$d\Phi/d\ln E \sim E\ I(E) \sim E^{-(s-1)}$
$\rightarrow$ *number peaks at smallest (but still relativistic) energies*
typical proton: $E \sim 1$ GeV

cosmic-ray energy proton flux $F(>E) = 4\pi \int E\ I(E)\ dE$
per log energy interval, $dF/d\ln E \sim E^2\ I(E) \sim E^{-(s-2)}$
$\Rightarrow$ since $s > 2$, *energy also peaks at low energies*

ensemble of cosmic rays acts as *mildly relativistic gas*

spectrum poses questions:
- origin(s) of the power-law behavior?
- what leads to the different regimes?

Cosmic Ray Propagation

consider cosmic ray protons and electrons
moving through interstellar space

Q: what interactions will each have?

Q: what will be the effect on interstellar matter?

Q: how will this affect CR propagation ("radiative transfer")?

Q: how could we detect evidence for this?

cosmic rays are highly energetic and penetrating
fill the Galaxy (and other galaxies)

cosmic ray electrons and protons
can and do collide with and scatter off interstellar matter
...and interstellar radiation!

- heating source for interstellar matter
- propagation should include scattering effect

But more important:
Cosmic rays are *charged particles*
$\rightarrow$ couple to Galactic (and intergalactic!) magnetic fields

- cosmic ray trajectories are bent by fields
- *cosmic rays do not point back to their sources*
- *accelerated motion* means that *cosmic rays radiate*
- electrons much more strongly accelerated $\rightarrow$ e synchrotron radiation dominates

`www: radio continuum sky, edge-on spirals, SN remnants`