Astro 501: Radiative Processes
Lecture 23
March 11, 2013

Announcements:

- **Problem Set 7** due Friday

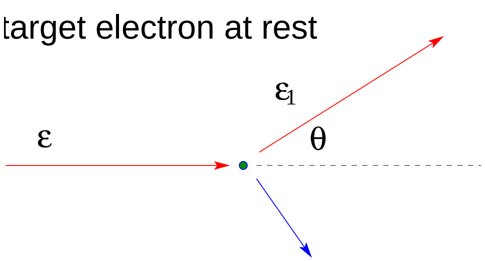
Last time: Compton scattering

Q: which is?

Q: differences with Thomson scattering?

Q: when are Compton/Thomson differences small? large?

Compton: treat light as massless particle



for photon incident on electron *at rest*
conservation of energy and momentum implies

$$\epsilon_1 = \frac{\epsilon}{1 + (\epsilon/m_e c^2)(1 - \cos\theta)} \tag{1}$$

scattered photon energy is lower, and direction different

New Dance Craze: Inverse Compton Style

Step 0: plant your feet = consider *lab/observer frame*:
- relativistic electrons with $E = \gamma m_e c^2$
- isotropic photon distribution, energies ϵ

Step 1: jump (boost) to *electron rest frame*
Ask ourselves: what does the electron "see"?
Q: incident photon angular distribution? typical energy ϵ'?

for simplicity: let $\gamma\epsilon \ll m_e c^2$
$\rightarrow$ Thompson approximation good in e frame K'
Q: then what is angular distribution of scattered photons in K'?
Q: scattered photon energy lab frame, roughly?

Step 2: jum (boost) again, return to *lab frame*
Q: what is angular distribution of scattered photons?
Q: scattered photon energy in e rest frame, roughly?

Inverse Compton and Beaming

Recall: a photon distribution isotropic in frame K
is *beamed* into angle $\theta \sim 1/\gamma$ in highly boosted frame K'

so in *electron rest frame* K'
most lab-frame photons "seen" in head-on beam
with energy $\epsilon' \sim \gamma\epsilon$

if rest-frame energies in Thompson regime:
- scattered photon directions $\propto d\sigma/d\Omega \propto 1 + \cos^2\theta$
  $\rightarrow$ isotropic + quadrupole piece
- scattered energy $\epsilon'_1 \sim \epsilon' \sim \gamma\epsilon$

back in lab frame
- boost $\rightarrow$ scattered photons beamed forward
- scattered photon energy *boosted* to $\epsilon_1 \sim \gamma\epsilon'_1 \sim \gamma^2\epsilon$

Q: implications for blazar spectra?

Inverse Compton Power for Single-Electron Scattering

Consider a relativistic electron (γ, β)
incident on an isotropic distribution of ambient photons

Order of magnitude estimate of *power* into inverse Compton

- if typical ambient photon energy is ϵ
  then typical *upscattered energy* is $\epsilon_1 \sim \gamma^2 \epsilon$
- if ambient photon number density is $n_{\rm ph}$
  then *scattering rate per electron* is $\Gamma = n_{\rm ph} \sigma_{\rm T} c$ *Q: why?*

thus expect power = rate of energy into inverse Compton

$$\frac{dE_{1,\rm upscatter}}{dt} \sim \Gamma \ \epsilon_1 \sim \gamma^2 \epsilon n_{\rm ph} \sigma_{\rm T} c \sim \gamma^2 \sigma_{\rm T} c u_{\rm ph} \qquad (2)$$

where $u_{\rm ph} = \langle \epsilon \rangle \, n_{\rm ph}$ is the ambient photon
energy density in the lab (observer) frame

Q: but what about scattering "removal" of incident photons?

some photons "removed" from ambient distribution by upscattering
removal rate is scattering rate per electron: $\Gamma = n_{\rm ph} \sigma_{\rm T} c$
and thus rate of energy "removal" per electron is

$$\frac{dE_{1,\rm init}}{dt} = -\Gamma \langle \epsilon \rangle = -\sigma_{\rm T}\ c\ \langle \epsilon \rangle n_{\rm ph} = -\sigma_{\rm T}\ c\ u_{\rm ph} \tag{3}$$

because $\langle \epsilon \rangle \equiv u_{\rm ph}/n_{\rm ph}$

Note that

$$\frac{dE_{1,\rm upscatter}}{dt} \simeq \gamma^2 \left| \frac{dE_{1,\rm init}}{dt} \right| \tag{4}$$

$\rightarrow$ for $\gamma \gg 1$, large net energy gain!

net inverse Compton power per electron, when done carefully:

$$P_{\rm Compt} = \frac{4}{3} \sigma_{\rm T}\ c\ \gamma^2\ \beta^2\ u_{ph} \tag{5}$$

Q: note any family resemblances?

Synchrotron vs Compton Power

We found the single-electron inverse Compton power to be

$$P_{\mathsf{Compt}} = \frac{4}{3}\sigma_{\mathsf{T}}\ c\ \gamma^2\ \beta^2\ u_{ph} \tag{6}$$

but recall *synchrotron power*

$$P_{\mathsf{synch}} = \frac{4}{3}\sigma_{\mathsf{T}}\ c\ \gamma^2\ \beta^2\ u_B \tag{7}$$

formally identical! and note that

$$\frac{P_{\mathsf{synch}}}{P_{\mathsf{Compt}}} = \frac{u_B}{u_{\mathsf{ph}}} \tag{8}$$

for *any* electron velocity as long as $\gamma\epsilon \ll m_e c^2$

- we turn next to spectra: good time to ask
  Q: what is conserved in Compton scattering? implications?

Inverse Compton Spectra: Monoenergetic Case

in Compton scattering, the *number of photons is conserved*
i.e., ambient photons given new energies, momenta
but neither created nor destroyed

thus: the photon *number* emission coefficient $\mathcal{J}(\epsilon_\infty)$
must have $4\pi \int \mathcal{J}(\epsilon_\infty) \ \lceil \epsilon_\infty$ = number of scatterings per unit volume
and $4\pi \int (\epsilon_1 - \epsilon) \ \mathcal{J}(\epsilon_\infty) \ \lceil \epsilon_\infty$ = net Compton power

detailed derivation appears in RL: answer is

$$j(\epsilon_1; \epsilon, \gamma) = \frac{3}{4} N(\gamma) \ \sigma_{\mathrm{T}} \ c \ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \ x \ f(x) \tag{9}$$

with $N(\gamma) = dN/d\gamma$ the electron flux at γ
and $du_{\mathrm{ph}}/d\epsilon$ the ambient photon energy density at ϵ
and $f(x) = 2x \ln x + 1 + x - 2x^2$, with $x = \epsilon_1/(4\gamma^2 \epsilon)$

Inverse Compton Scattering: Power-Law Electrons

as usual, assume power-law electron spectrum $N(\gamma) = C\ \gamma^{-p}$

still for a single ambient photon energy
integrate emission coefficient over all electron energies

$$j(\epsilon_1; \epsilon) = \int j(\epsilon_1; \epsilon, \gamma) d\gamma \tag{10}$$

$$= \frac{3}{4}\sigma_{\mathrm{T}}\ c\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \int\ x\ f(x)\ N(\gamma) d\gamma \tag{11}$$

with $x = \epsilon_1/(4\gamma^2\epsilon)$
and where $x\ f(x)$ is peaked, with max at $x = 0.611$

Q: notice a family resemblance?

Q: strategies for doing integral?

Q: anticipated result?

for both IC and synchrotron: spectrum is integral of form

$$j(\epsilon_1;\epsilon) \propto \int G\left(\frac{\epsilon_1}{\gamma^2\epsilon_0}\right)\ \gamma^{-p}\ d\gamma \tag{12}$$

strategy is to change variables to $x = \epsilon_1/(\gamma^2\epsilon_0)$

result factorizes into product of
- dimensionless integral, times
- power law $j \propto (\epsilon_1/\epsilon)^{-(p-1)/2}$

so once again:
peaked emission spectrum for single-energy electron
smoothed to power-law emission spectrum, index $s = (p-1)/2$
for power-law electron energy distribution

full result in RL, guts are (up to numerical factors)

$$4\pi\ j(\epsilon_1;\epsilon) \sim \sigma_{\rm T}\ c\ C\ \epsilon_1^{-(p-1)/2}\ \epsilon^{(p-1)/2}\ \frac{du_{\rm ph}}{d\epsilon}(\epsilon) \tag{13}$$

Q: interesting choice of ambient photon distribution?

emission coefficient is

$$4\pi\ j(\epsilon_1;\epsilon) \sim \sigma_{\mathsf{T}}\ c\ C\ \epsilon_1^{-(p-1)/2}\ \epsilon^{(p-1)/2}\ \frac{du_{\mathsf{ph}}}{d\epsilon}(\epsilon) \tag{14}$$

depends on background photon distribution via $du_{\mathsf{ph}}/d\epsilon$

for a thermal (Planck) photon distribution:

- $du/d\epsilon \sim T^4/T \sim T^3$, and
- $\epsilon^{(p-1)/2} \sim T^{(p-1)/2}$ and so

expect temperature scaling $j \sim T^{3+(p-1)/2} = T^{(p+5)/2}$

in fact:

$$4\pi\ j(\epsilon_1) = 4\pi \int j(\epsilon_1;\epsilon) \sim \frac{\sigma_{\mathsf{T}}\ C}{h^3 c^2}(kT)^{(p+5)/2}\ \epsilon_1^{-(p-1)/2} \tag{15}$$

Awesome Examples

`www: Fermi sky movie: mystery object`

Q: what strikes you?

Q: how does the mystery object radiate > 100 MeV *photons?*

`www: WMAP Haze`

Q: what strikes you?

haze spectrum: $\propto \nu^{-0.5}$, flatter than usual synchrotron

Q: what electron index would this imply?

Q: if electrons continue to high E*, what should we see?*

`www: Fermi search for that feature`