Astro 501: Radiative Processes
Lecture 24
March 13, 2013

Announcements:

- **Problem Set 7** due Friday

Last time: inverse Compton power and spectra

Q: family resemblance with synchrotron?

Q: applications?

Q: assumptions we made?

net inverse Compton power per electron, when done carefully:

$$P_{\mathrm{Compt}} = \frac{4}{3}\sigma_{\mathrm{T}}\ c\ \gamma^2\ \beta^2\ u_{\mathrm{ph}} \tag{1}$$

formally identical to synchrotron power, with

$$\frac{P_{\mathrm{synch}}}{P_{\mathrm{Compt}}} = \frac{u_B}{u_{\mathrm{ph}}} \tag{2}$$

for *any* electron velocity as long as $\gamma\epsilon \ll m_e c^2$

IC spectrum for *power-law electron energy distribution*

$$j(\epsilon_1;\epsilon) \sim \sigma_{\mathrm{T}}\ c\ C\ \epsilon_1^{-(p-1)/2}\ \epsilon^{(p-1)/2}\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \tag{3}$$

also close formal similarities with synchrotron

Q: what changes (and not) for non-relativistic electrons?

Inverse Compton: Non-Relativistic Electrons

if electrons are nonrelativistic
but still on average more energetic than the photons
we have $\beta = v/c \ll 1$
and $\gamma \approx 1 + \beta^2/2 + \cdots$, so that

$$P_{\mathrm{Compt}} = \frac{4}{3}\sigma_{\mathrm{T}}\ c\ \gamma^2\ \beta^2\ u_{\mathrm{ph}} \approx \frac{4}{3}\sigma_{\mathrm{T}}\ c\ \beta^2\ u_{\mathrm{ph}}\ +\ \vartheta(\beta^4) \qquad (4)$$

if electrons has a **thermal velocity distribution** at T_e
then velocities have Maxwell-Boltzmann distribution $e^{-v^2/2v_T^2} v^2\, dv$
with $v_T^2 = kT_e/m_e$, and so averaging, we get

$$\left\langle v^2 \right\rangle = 3v_T^2 = 3\frac{kT_e}{m_e} \qquad (5)$$

and thus

$$\left\langle P_{\mathrm{Compt}} \right\rangle = 4\sigma_{\mathrm{T}}\ c\ \frac{kT_e}{m_e c^2}\ u_{\mathrm{ph}} \qquad (6)$$

Sunyaev-Zel'dovich Effect

The Cosmic Microwave Background

Spectrum

best data: FIRAS instrument on
Cosmic Background Explorer (COBE)
Fixsen et al (1996):

- `www:` T_{antenna} `plot` – consistent with *purely thermal*
- present all-sky temperature

$$T_0 = 2.725 \pm 0.004 \text{ K} \tag{7}$$

thus, the CMB has, within our ability to measure
precisely the Planck spectral form

$$I_\nu = B_\nu(T_0) \tag{8}$$

Q: what does this imply?

CMB has Planck (blackbody) form $I_\nu = B_\nu(T_0)$

recall: a blackbody spectrum arises from

- a *thermal emitter* having source function $S_\nu = B_\nu$
- that is also *optically thick*

thus we conclude: sometime in the past

- cosmic matter and radiation were *in thermal equilibrium*
- and the Universe was *opaque*

but the *present* universe
must be *transparent* to the CMB

Q: why is this?

Q: what does this imply about epoch probed by CMB?

The CMB Implies a Dense Past

the fact that the CMB is a *background*
to low-z objects $\rightarrow$ late-time U. is *transparent* to CMB

thus the CMB implies that the Universe is *evolving*
and in the past was much *denser*
so that equilibrium could be established

thus: the CMB probes exactly the epoch
i.e., the last time U. was *opaque* to its thermal photons

CMB created by (and gives info about)
an epoch of cosmic transition: *opaque* $\rightarrow$ *transparent*

CMB as Cosmic "Baby Picture": Last Scattering Surface

but transparent/opaque transition is
controlled by photon *scattering*
e.g., CMB released at epoch of **"last scattering"** $z_{\rm ls}$
$\rightarrow$ CMB sky map is a picture of the U. then:
"surface of last scattering'

as long as density of scattering particles is nonzero
scattering rate > 0, mean free path and mean free time $\neq \infty$
naively would think scattering never stops!

Q: what's going on here?

it is true that as long as scatterers exist
some CMB photons will always be scattered

but: when mean free time $>$ age of universe
scattering ineffective, and a *typical* CMB photon
will *no longer be scattered*: CMB photons "released"
thereafter "free stream" across the Universe

in other words: CMB arises from cosmic "photosphere"
where cosmic *optical depth* against scattering becomes small

More later on this:
we will find this occurs at $z \sim 1000$, $t \sim 400,000$ yrs
a long ago $\rightarrow$ last scattering really far far away

The CMB Reprocessed: Hot Intracluster Gas

CMB is cosmic photosphere: "as far as the eye can see"
CMB created long ago, comes from far away

- all other observable cosmic objects are in *foreground*
- CMB passes through all of the observable universe

Sunyaev & Zel'dovich:
what happens when CMB passes through hot gas *Q: examples?*

consider gas of electrons at temperature $T_e \gg T_{\rm cmb}$
but where $kT_e \ll m_e c^2$ *Q: how good an approximation is this?*

Q: what's probability for scattering of CMB photon with ν?

CMB Scattering by Intracluster Gas

mean free path is that for Thompson scattering:
$\ell_\nu^{-1} = \alpha_\nu = n_e \sigma_{\mathsf{T}}$ independent of frequency
and thus optical depth is integral over cloud sightline

$$\tau_\nu = \int \alpha_\nu \ ds = \sigma_{\mathsf{T}} \int n_e \ ds \tag{9}$$

thus transmission probability is $e^{-\tau_\nu}$, and so
absorption probability is $1 - e^{-\tau_\nu}$

but for galaxy clusters: $\tau < 10^{-3} \ll 1$,
and so *absorption probability* is just τ
Q: implications?
Q: effect of scattering if electrons cold, scattering is elastic?
Q: what if electrons are hot?

if electrons are hot, they transfer energy to CMB photons change temperature pattern, in frequency-dependent way

What is net change in energy?
initial photon energy density is $u_0 = u_{\rm cmb} = 4\pi B(T_{\rm cmb})/c$
power transfer per electron is $P_{\rm Compt} = 4(kT_e/m_e c^2)\sigma_{\rm T} c\ u_0$, so

$$\frac{\partial u}{\partial t} = P_{\rm Compt}\ n_e = 4\frac{kT_e}{m_e c^2}\sigma_{\rm T} c\ u_0\ n_e \tag{10}$$

and thus net energy density change

$$\Delta u = 4\sigma_{\rm T}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \tag{11}$$

Q: implications?

CMB energy density change through cluster

$$\Delta u = 4\sigma_{\mathrm{T}}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \equiv 4y\ u_0 \tag{12}$$

- dimensionless **Compton-y parameter**

$$y \equiv \sigma_{\mathrm{T}} \int \frac{n_e\ kT_e}{m_e c^2} ds \simeq \tau \frac{kT_e}{m_e c^2} \simeq 3\tau\beta^2 \tag{13}$$

- note $n_e k T_e = P_e$ electron pressure
  $\rightarrow y$ set by line-of-sight pressure

fractional change in (integrated) energy density $\Delta u/u_0 = 4y$

- positive change $\rightarrow$ (small) net heating of CMB photons
- since $u \propto I$, this also means

$$\frac{\Delta I_{\mathrm{cmb}}}{I_{\mathrm{cmb}}} = 4y \tag{14}$$

cluster generated net CMB "hotspot"

Q: expected frequency dependence?

SZ Effect: Frequency Dependence

on average, we expect photons to gain energy
adding intensity at high ν, at the expense of low ν

but note that in isotropic electron population
• some scatterings will reduce energy
• while others will increase it

detailed derivation is involved:
• allow for ordinary and stimulated emission
• include effects of electron energy distribution
• allow for Compton shift in energy
• use Thomson (Klein-Nishina) angular distribution

full equation (Kompaneets and generalization)
describes *"diffusion" in energy (frequency) space*
but key aspect comes from basic Compton property *Q: namely?*

The SZ Scattering Kernel

recall: Compton scattering conserves photon number

thus useful to consider occupation number $f(\nu)$

- and number density $n(\nu) = n_\nu = dn/d\nu = 8\pi\nu^2/c^3 \ f(\nu)$
- where $I_\nu = c \ h\nu \ n_\nu/4\pi = 2 \ h\nu^3/c^2 \ f(\nu)$

conservation implies that effect of scattering

of incident photons $f_0(\nu) \stackrel{\text{cmb}}{=} (e^{h\nu/kT_{\text{cmb}}} - 1)^{-1}$

can be cast in the *Green's function* form

$$n(\nu) = \int K(\nu, \nu_0) \ n_0(\nu_0) \ d\nu_0 \tag{15}$$

Q: what does $K(\nu, \nu_0)$ represent physically?

Q: what does photon conservation require?

Q: what is K if we turn scattering off?

the *scattering kernel* is

$$n(\nu) = \int K(\nu, \nu_0)\ n_0(\nu_0)\ d\nu_0 \tag{16}$$

physically: gives the *probability* that
a photon observed at ν had frequency ν_0

photon conservation:
number of scattered photons $\int n(\nu)\ d\nu$
must be equal to initial number $\int n_0(\nu_0)\ d\nu$
requires $\int K(\nu, \nu_0)\ d\nu = 1$

if no scattering: must have $n(\nu) = n(\nu_0)$
and so $K(\nu, \nu_0) \to \delta(\nu - \nu_0)$
note: has right integral property

Q: main SZ frequency shift effect at low ν? high ν?

recall electron rest-frame Compton formula

$$\nu' = \frac{\nu_0'}{1 - (h\nu_0'/m_e c^2)(1-\cos\theta)} \approx \left[1 - \frac{h\nu_0'}{m_e c^2}(1-\cos\theta)\right]\nu' \quad (17)$$

at low frequencies $h\nu_0' \ll m_e c^2$:
Compton frequency shift tiny: $\nu' \approx \nu_0'$
but scattering off moving electrons gives *Doppler* shifts

Doppler: $\nu_0' = \gamma(1 - \beta\cos\theta)\nu_0$
initial electron distribution is isotropic, so at fixed γ

$$\left\langle \nu_0' \right\rangle = \gamma\,(1 - \beta\,\langle\cos\theta\rangle)\,\nu_0 = \gamma\nu_0 \approx \left(1 + \frac{v^2}{2c^2}\right)\nu_0 \quad (18)$$

- first order effect averages to zero
- but *second order* effect survives!
- boosting back to lab frame

$$\langle\nu\rangle \approx \gamma^2\nu_0 \approx \left(1 + \frac{v^2}{c^2}\right)\nu_0 \quad (19)$$

for low frequencies: $\nu \approx (1+\beta^2)\nu_0$

thus observed frequency ν arises from frequency $\nu_0 \approx (1-\beta^2)\nu$

simpleminded approximation:

$$\begin{aligned} K(\nu,\nu_0) &= (1-\tau)\ K_{\mathrm{unscattered}}(\nu,\nu_0) + \tau\ K_{\mathrm{scattered}}(\nu,\nu_0) \\ &= (1-\tau)\ \delta(\nu_0-\nu) + \tau\ \delta\left[\nu_0 - (1-\beta^2)\nu\right] \end{aligned}$$

thus we have

$$n(\nu) = \int K(\nu,\nu_0)\ n_0(\nu_0)\ d\nu_0 \qquad (20)$$

$$= (1-\tau)\ n_0(\nu) + \tau\ n_0\left[(1-\beta^2)\nu\right] \qquad (21)$$

Q: and so?

SZ: Low Frequencies

our low-frequency approximation gives

$$n(\nu) = (1-\tau)\ n_0(\nu) + \tau\ n_0\left[(1-\beta^2)\nu\right] \tag{22}$$

and so the change at low frequency ν is

$$\Delta n(\nu) = n(\nu) - n_0(\nu) = -\tau\ \left\{n_0(\nu) - n_0\left[(1-\beta^2)\nu\right]\right\} \tag{23}$$

but $\beta^2 \ll 1$, so expand

$$\Delta n(\nu) \approx -\tau\ \Delta\nu\ \partial_\nu n_0(\nu) = -\tau\beta^2\nu\ \partial_\nu n_0(\nu) \tag{24}$$

using the Planck form for n_0, and with $\tau\beta^2 = 2y$, we have

$$\Delta n(\nu) = -2y\ n_0(\nu)\ \left(2 - \frac{h\nu/kT_e}{e^{h\nu/kT_e} - 1}\right) \approx -2y\ n_0(\nu) \tag{25}$$

where the last expression uses $h\nu/kT_e \ll 1$
Q: implications?

at low frequencies $h\nu \ll kT_e$, we hve

$$\frac{\Delta n(\nu)}{n_0(\nu)} = \frac{\Delta I_\nu}{I_\nu^0} \approx -2y \tag{26}$$

- *frequency-independent fractional decrease* in intensity
- proportional to Compton y

physically reasonable? yes!
these wimpy photons are promoted to higher frequencies

Q: what about the high-frequency limit $h\nu \gg kT_e \sim m_e c^2 \beta^2$*?*