Astro 501: Radiative Processes
Lecture 25
March 15, 2013

Announcements:

- **Problem Set 8** due today
- **Problem Set 7** due after break

Last time: Sunyaev-Zeldovich theory

Q: implications of Planck form of CMB?

Q: what is the SZ effect? integrated intensity?

Q: effect at low frequencies?

CMB has Planck (blackbody) form $I_\nu = B_\nu(T_0)$
recall: a blackbody spectrum arises from

- a *thermal emitter* having source function $S_\nu = B_\nu$
- that is also *optically thick*

thus we conclude: sometime in the past

- cosmic matter and radiation were *in thermal equilibrium*
- and the Universe was *opaque*

but the fact that the CMB is a *background*
to low-z objects $\rightarrow$ late-time U. is *transparent* to CMB

thus the CMB implies that the Universe is *evolving*
and in the past was much *denser*
so that equilibrium could be established

SZ: Low Frequencies

at low frequencies $h\nu \ll kT_e$, we have

$$\frac{\Delta n(\nu)}{n_0(\nu)} = \frac{\Delta I_\nu}{I_\nu^0} \approx -2y \tag{1}$$

- *frequency-independent fractional decrease* in intensity
- proportional to Compton y

physically reasonable? yes!
these wimpy photons are promoted to higher frequencies

Note: low-frequencies are Rayleigh-Jeans regime
using antenna temperature $T_\nu = c^2 I_\nu / 2hk\nu^2$, we have

$$\frac{\Delta T_\nu}{T_\nu} \approx -2y \tag{2}$$

constant fractional decrease in antenna temperature

Q: what about the high-frequency limit $h\nu \gg kT_e \sim m_e c^2 \beta^2$?

At *high frequencies* $h\nu \gg kT_e \sim \beta^2$, *Compton shift dominates*

roughly expect upscattered frequency

$$\nu \approx \left(1 + \frac{h\nu_0}{m_e c^2}\right) \nu_0 \tag{3}$$

so that $\Delta\nu/\nu_0 \sim h\nu_0/m_e c^2$

same song, second verse:
scattering kernel, again in simpleminded approximation

$$K(\nu, \nu_0) = (1 - \tau)\ \delta(\nu_0 - \nu) + \tau\ \delta\left[\nu_0 - (1 - h\nu/m_e c^2)\nu\right]$$

thus we have

$$\frac{\Delta n(\nu)}{n(\nu)} = \frac{\Delta I_\nu}{I_\nu} = -\tau\ \frac{h\nu}{m_e c^2}\frac{1}{n(\nu)}\ \nu \partial_\nu n(\nu) \tag{4}$$

$$= -y \frac{h\nu}{kT_{\rm rad}} \left(2 - \frac{h\nu/kT_{\rm rad}\ e^{h\nu/kT_{\rm rad}}}{e^{h\nu/kT_{\rm rad}} - 1}\right) \approx y \left(\frac{h\nu}{kT_{\rm rad}}\right)^2 \tag{5}$$

Q: implications?

SZ Effect: High Frequencies

for $h\nu \gg kT_{\mathrm{rad}}$, a careful calculation finds

$$\frac{\Delta I_\nu}{I_\nu} \approx y \left(\frac{h\nu}{kT_{\mathrm{rad}}}\right)^2 \quad (6)$$

now *fractional shift is frequency dependent*

To summarize, we have found the limiting SZ shifts
for $x = h\nu/kT_{\mathrm{rad}}$, we have

$$\frac{\Delta I_\nu}{I_\nu} \to \begin{cases} -2y & x \ll 1 \\ yx & x \gg 1 \end{cases} \quad (7)$$

Q: implication?

SZ Effect: Null

We see that SZ *decreases low-frequency intensity*
and *increases high-frequency intensity*
$\rightarrow$ there must be a transition that crosses zero!
$\rightarrow$ frequency ν_{null} must exist at which $\Delta I_\nu / I_\nu = 0$: **SZ null**

PS7 (modulo typos): Kompaneets SZ approximation

$$\frac{\Delta I_\nu}{I_\nu} = -y \; g(x) = -y \frac{x e^x}{e^x - 1} \left[4 - x \frac{e^x + 1}{e^x - 1} \right] \tag{8}$$

with $x = h\nu / kT_{\text{rad}}$; perturbation vanishes at $x_{\text{null}} = 3.83$

Q: SZ spectral dependence?
Q: how to exploit this observationally?

`www: SZ spectrum plotted`
Q: effect of cosmic redshifting on SZ: integrated intensity? spectrum?

SZ and Cosmic Redshifting

redshifting: a fundamental consequence the expanding universe
cosmologist learn at their mother's knee that
lengths stretch with to the *cosmic scale factor* $a(t)$:

$$\vec{r}(t) = a(t)\ \vec{r}_0 \tag{9}$$

usual convention: today (time t_0) distance is $\vec{r}_0$, so $a(t_0) = 1$

wavelengths are lengths – it's right there in the name!
so wavelengths stretch as $\lambda(t) \propto a(t)$

- photon emitted with λ_{em} at t_{em} is observed at $t_{\text{obs}} = t_0$ with

$$\lambda_{\text{obs}} = \frac{a(t_{\text{obs}})}{a(t_{\text{em}})} \lambda_{\text{em}} = \frac{1}{a(t_{\text{em}})}\ \lambda_{\text{em}} \tag{10}$$

- thus the *redshift* is related to scale factor via

$$z \equiv \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} - 1 = \frac{1}{a(t_{\text{em}})} - 1 \tag{11}$$

Q: redshifting effect on ν? T? SZ $\Delta I_\nu / I_\nu$? $\Delta I / I$?

Cosmic Redshifting

wavelength: $\lambda_{\rm obs} = \lambda_{\rm em}/a_{\rm em} = (1+z)\lambda_{\rm em}$

frequency: $\nu = c/\lambda$ and so $\nu_{\rm obs} = a_{\rm em}\ \nu_{\rm em} = \nu_{\rm em}/(1+z)$

temperature: Wein says $T/\nu_{\rm max} = const$
so $T_{\rm obs} \propto \nu_{\rm max,obs} \propto a_{\rm em}$ and so $T_{\rm obs} = T_{\rm em}/(1+z)$

But recall SZ fractional intensity change $\Delta I_\nu / I_\nu^0 = -y g(x)$
$\rightarrow$ only depends on dimensionless ratio x evaluated at emission

$$x = \frac{h\nu_{\rm em}}{kT_{\rm rad,em}} = \frac{h\nu_{\rm obs}/(1+z)}{kT_{\rm rad,obs}/(1+z)} = \frac{h\nu_{\rm obs}}{kT_{\rm rad,obs}} \qquad (12)$$

SZ fractional specific intensity change is *redshift independent!*
∞ $\rightarrow$ same goes for integrated intensity change $\Delta I/I = 4y$
Q: is this reasonable?

`www:  SZ clusters at different redshift`

this also means:

- shape of spectral distortion is redshift independent
- max, min, and null in $\Delta I_\nu / I_\nu$ always the same!
- robust signature of and test of (thermal) SZ effect!

SZ clusters usually resolved

Q: what information is in a cluster's SZ pattern on sky?

Q: what information is in total SZ flux across sky?

Q: SZ applications for cluster astrophysics?

Thermal SZ Effect as a Probe of Galaxy Cluster

in each line of sight
SZ measures Comptonization parameter in a cluster:

$$y = \sigma_{\rm T} \int \frac{n_e \, kT_e}{m_e c^2} ds = \frac{\sigma_{\rm T}}{m_e c^2} \int P_e \, ds \approx \frac{\sigma_{\rm T} \, kT_e}{m_e c^2} \int n_e \, ds \tag{13}$$

direct measurement of *projected pressure* in column
and if T_e known, a measure of electron column density

SZ flux measures

$$\int \cos\theta \; y \, d\Omega \approx \int y \, d\Omega = \frac{\int y \, dA}{D_{\rm A}^2} \tag{14}$$

where $D_{\rm A}(z)$ is the (angular diameter) distance

$$\int y \, dA \approx \frac{\sigma_{\rm T} \, kT_e}{m_e c^2} \int n_e \, ds \, dA \propto M_{\rm gas} \tag{15}$$

$\rightarrow$ SZ flux gives *intracluster cluster gas mass!* *Q: cosmo apps?*

SZ Effect: Cosmological Applications

- *SZ identifies all clusters without redshift bias!*
  $\rightarrow$ SZ can be used to discover high-z clusters

- SZ + X-ray gives cluster size, gas mass, T_e
  if cluster physics well-understood (Ricker, Vijayaraghavan)
  $\rightarrow$ *cluster mass*

- cluster number density ("abundance") and mass vs z
  i.e., cluster *mass function* a sensitive probe of cosmology

today: clusters are the *largest bound objects*; in early U: rare
number and mass vs time sensitive to *cosmic acceleration*
that competes with *structure growth via gravitational instability*
$\Rightarrow$ clusters probe this competition

Q: so how to find clusters, measure redshifts?

note that SZ redshift independence also means
SZ does not give cluster redshift

Dark Energy Survey key project:
optical images, redshifts of clusters
compare with SZ survey by South Pole Telescope

`www: SPT survey image`

SZ Effect: More Cosmological Applications

even for clusters not clearly imaged in SZ
SZ effect from all clusters still imprinted on CMB
affects $\Delta T_{\rm cmb}$ perturbation pattern on sky

typical angular size of cluster SZ:
for large cluster $\theta_{cluster} \sim R_{\rm cluster}/d_{\rm H} \sim 3$ Mpc/4 Gpc ~ 3 arcmin
i.e., SZ affects small angular scales
in C_ℓ multipole space this corresponds to $\ell \sim 200/\theta_{\rm deg} \sim 4000$

SZ statistical imprint on CMB anisotropies:
exquisitely sensitive measure of *cosmic structure*
for experts: angular power spectrum $C_\ell^{\rm SZ} \propto \sigma_8^7$!

To date: SZ contribution to power spectrum not seen! *Planck*?

Kinetic SZ Effect and Cosmology

Thus far: implicitly assumed that cluster is *at rest*
relative to CMB frame ("fundamental observers")

but if cluster moving along line of sight with velocity $v_{\rm los}$
bulk motion adds *uniform Doppler shift*
to usual thermal SZ effect
→ kinematic or **kinetic SZ effect** ("kSZ")

at lowe frequencies (Rayleigh-Jeans), kSZ has

$$\frac{\Delta I_\nu}{I_\nu} \approx -\tau \, \frac{v_{\rm los}}{c} \qquad (16)$$

what causes cluster motion = bulk flows?
→ large-scale density perturbations

Q: but how do we tell between kSZ and thermal SZ?

Other SZ Applications

SZ is sensitive to *any* population of high-energy electrons

should be a SZ contribution from relativistic electrons
in intracluster medium, arising from
• cosmic-rays escaped from galaxies, or from
• "structure formation cosmic rays" accelerated by cosmological shocks
nonthermal SZ

also recall that AGN jets lead to electron acceleration
in principle can measure jet electrons in SZ

Have a good break!