Astro 501: Radiative Processes
Lecture 26
March 27, 2013

Announcements:

- **Problem Set 8** extended to 5pm Monday

Last time: Sunyaev-Zeldovich finale
just in time for `www: Planck cosmology data release`

Today: plasma effects
Q: what's a plasma?
Q: why do we care?
`www: the sky in H`α

Plasma Effects

Plasmas

roughly speaking a **plasma** is a

- *globally neutral*
- *partially or completely ionized gas*

more quantitatively:

ionization $\rightarrow$ (at least some) particles have $E_{\mathrm{thermal}} > E_{\mathrm{binding}}$

"a little ionization goes a long way"

- electrons and ions in plasma are not *free*
  but have Coulomb interactions with each other
  and can interact with static and propagating EM fields
- gas does *not* need to be fully ionized to show plasma effects

Plasma Frequency

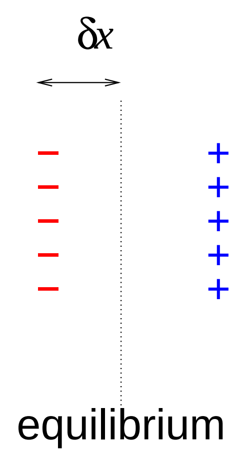
on average (*globally*) the plasma is *neutral*:

$$\langle n_e \rangle = \sum Z_i \langle n_i \rangle \tag{1}$$

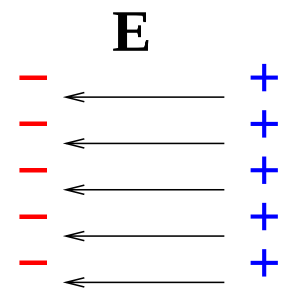
with n_e the electron density
and n_i the density of ion species i of atomic number Z_i

but *locally* the unbound charges can move
fluctuations can create small separation between e and ions

consider idealized picture:
"walls" of electrons and ions
both displaced from equilibrium
Q: effect of charge distribution?
Q: response of particles?



Charge separation → capacitor
electric field $\vec{E}$ between "walls"



find electric field given electron density n_e
equilibrium separation δx, and wall area A:
Gauss box around electrons: $EA = 4\pi Q_{\text{enc}} = 4\pi e n_e A \ \delta x$
$\rightarrow E = -4\pi e n_e \ \delta x$: note area A drops out!

electron equation of motion

$$m_e \ddot{\delta x} = -eE = -4\pi \ e^2 n_e \ \delta x \quad (2)$$

$$\ddot{\delta x} = -\frac{4\pi \ e^2 n_e}{m_e} \ \delta x \quad (3)$$

Q: and so? fundamental scales?

electric field due to plasma charge separation
restores charges back to equilibrium position
$\to$ a stable equilibrium!

charges *oscillate*

$$\delta\ddot{x} = -\omega_{\rm p}^2 \ \delta x \tag{4}$$

with **plasma frequency** $\omega_{\rm p}^2 = 4\pi \ e^2 n_e / m_e$ and so

$$\nu_{\rm p} = \frac{\omega_{\rm p}}{2\pi} = \sqrt{\frac{4\pi \ e^2 n_e}{m_e}} = 8.97 \ \text{kHz} \ \left(\frac{n_e}{1 \ cm^{-3}}\right)^{1/2} \tag{5}$$

sets fundamental plasma timescale $\tau = 1/\omega_{\rm p}$
e thermal speed is $v_T \sim \sqrt{kT/m_e}$
$\to$ fundamental lengthscale: *Debye length* $\lambda_{\rm D} = v_T \tau = \sqrt{kT/4\pi e^2 n}$
$\to$ plasma like behavior on timescales $\gg \tau$, on lengthscales $\gg \lambda_{\rm D}$

Electromagnetic Waves in a Plasma

Till now: assumed EM propagation in *vacuum*
but astrophysically, almost always in plasma!

must revisit Maxwell equations, now allowing for
- electron charge density $\rho_q = -e n_e$
- current density (charge flux!) $\vec{j} = \rho_q \vec{v}_e = -e n_e \vec{v}_e$

look for wavelike solutions: all quantities $\propto e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$\begin{array}{cc} i\vec{k}\cdot\vec{E} = -4\pi e n_e & i\vec{k}\cdot\vec{B} = 0 \\ i\vec{k}\times\vec{E} = i\frac{\omega}{c}\vec{B} & i\vec{k}\times\vec{B} = -4\pi e n_e \frac{\vec{v}}{c} - i\frac{\omega}{c}\vec{E} \end{array} \tag{6}$$

electron velocity governed by $m_e \dot{\vec{v}} = -e(\vec{E} + \vec{v}/c \times \vec{B}) \approx -e\vec{E}$, so

$$\vec{v} = \frac{e\vec{E}}{i\omega m_e} \tag{7}$$

note that e *velocity* $\propto$ *electric field*

$$\vec{v} = \frac{e\vec{E}}{i\omega m_e} \tag{8}$$

and thus $\vec{j} = \sigma\vec{E}$ with *conductivity*

$$\sigma = \frac{ie^2 n_e}{\omega m_e} \tag{9}$$

continuity equation: $i\omega e n_e = i e n_e \vec{k} \cdot \vec{v} = \sigma \vec{k} \cdot \vec{E}$

using this, can rewrite Maxwell as

$$\begin{array}{cc} i\left(1 - \frac{4\pi\sigma}{i\omega}\right)\vec{k}\cdot\vec{E} = 0 & i\vec{k}\cdot\vec{B} = 0 \\ i\vec{k}\times\vec{E} = i\frac{\omega}{c}\vec{B} & i\vec{k}\times\vec{B} = -i\left(1 - \frac{4\pi\sigma}{i\omega}\right)\frac{\omega}{c}\vec{E} \end{array} \tag{10}$$

∞ a miracle! *Q: why?*

have recast Maxwell in plasmas into "source-free" form
so still have:

- wavelike solutions
- $\vec{k}$, $\vec{E}$, $\vec{B}$ mutually orthogonal

but now have new **dispersion relation**

$$c^2 k^2 = \epsilon \; \omega^2 \tag{11}$$

with the **dielectric constant**

$$\epsilon = 1 - \frac{4\pi\sigma}{i\omega} = 1 - \frac{4\pi e^2 n_e}{\omega^2 m_e} \tag{12}$$

and thus we have

$$\omega^2 = \omega_{\rm p}^2 + c^2 k^2 \tag{13}$$

Q: implications for EM propagation in plasmas?

Plasma Dispersion Relation

vacuum relation $\omega = ck$ replaced by

$$\omega^2 = \omega_{\rm p}^2 + c^2 k^2 \tag{14}$$

where $\omega_{\rm p}^2 = 4\pi e^2 n_e / m_e$

if $\omega < \omega_{\rm p}$, then $k^2 < 0$!
$\rightarrow$ wavenumber imaginary!

$$k = \frac{i}{c}\sqrt{\omega_{\rm p}^2 - \omega^2} \tag{15}$$

wave amplitude *damped* as e^{-kr}
$\rightarrow$ low frequency waves do not propagate! "cutoff" in spectrum
e.g., Earth ionosphere damps waves with $\nu < 1$ MHz

characteristic *damping scale* $2\pi c/\omega_{\rm p}$

Group and Phase Velocity

in the other limit $\omega > \omega_{\rm p}$
waves do propagate without damping

waves move according to $e^{i\phi}$, with phase
$\phi = \vec{k} \cdot \vec{x} - \omega t = k\hat{n} \cdot (\vec{x} - \omega/k \; t \; \hat{n})$
$\rightarrow$ wavefronts propagate with *phase velocity*

$$v_\phi = \frac{\omega}{k} = \frac{c}{n_r} \tag{16}$$

where the *index of refraction* is

$$n_r \equiv \sqrt{\epsilon} = \sqrt{1 - \left(\frac{\omega_{\rm p}^2}{\omega^2}\right)} \tag{17}$$

but *signals* move with *group velocity* (PS8)

$$v_{\rm g} \equiv \frac{\partial \omega}{\partial k} = c \sqrt{1 - \left(\frac{\omega_{\rm p}^2}{\omega^2}\right)} \tag{18}$$

Group Velocity Awesome Example: Pulsars

Pulsars: spinning, magnetized neutron stars
pulsed emission with period = spin period
pulsed $\rightarrow$ narrow in time $\rightarrow$ broadband in frequency

www: pulsar signals in audio
www: pulsar sky distribution

pulsar signals propagate through interstellar medium–a plasma!
every small band of frequencies propagates with different $v_{\rm g}(\omega)$
$\rightarrow$ pulses *dispersed*, arrive with spread time

if pulsar distance is d
then arrival time at Earth at each frequency ω is

$$t_{\rm pulsar}(\omega) = \int_0^d \frac{ds}{v_{\rm g}(\omega)} \tag{19}$$

Q: how should arrive time depend on ω?

pulsar at d has arrival time

$$t_{\text{pulsar}}(\omega) = \int_0^d \frac{ds}{v_{\text{g}}(\omega)} \tag{20}$$

frequency dependence set by

$$\frac{1}{v_{\text{g}}} = \frac{1}{c}\left(1 - \frac{\omega_{\text{p}}^2}{\omega^2}\right)^{-1/2} \approx \frac{1}{c}\left(1 + \frac{\omega_{\text{p}}^2}{2\omega^2}\right) \tag{21}$$

where we used $\omega \gg \omega_{\text{p}} \sim$ kHz, and so

$$t_{\text{pulsar}}(\omega) \approx \frac{d}{c} + \frac{1}{2c\omega^2}\int_0^d \omega_{\text{p}}^2 \, ds = \frac{d}{c} + \frac{1}{2c\omega^2}\ \mathcal{D} \tag{22}$$

Q: implications? how can we be sure dispersion is real?

pulsar time delay

$$t_{\text{pulsar}}(\omega) \approx \frac{d}{c} + \frac{1}{2c\omega^2} \int_0^d \omega_{\text{p}}^2 \, ds = \frac{d}{c} + \frac{1}{2c\omega^2} \, \mathcal{D} \qquad (23)$$

- depends on frequency: $\delta t \propto \nu^{-2} \propto \lambda_{\text{obs}}^2$
- free electron column: **dispersion measure** $\mathcal{D} = \int_0^d n_e \, ds$

to test whether dispersion is real:
should obey correct frequency dependence
$\rightarrow$ this isolates dispersion measure

- if have estimate of electron density n_e

$\rightarrow$ get *distance* to pulsar!

- if have idea of pulsar distance
  can use pulsar ensemble to *map free electron density* n_e!
  $\rightarrow$ reveals Galactic spiral arm pattern!
  `www: Taylor & Cordes 1993`

Q: applications for Sgr A?*

Sgr A*: our very own neighborhood black hole
a laboratory for study of General Relativity

so far: black hole properties studied via orbits
of closely approaching stars
available closest approach distances still $\gg GM/c^2$
$\rightarrow$ GR effects too small to detect

the great hope: find a *pulsar* around Sgr A*
not crazy! many supernova remnants near Galactic center!

- good news: hyperaccurate pulsar timing $\rightarrow$ GR probe
- bad news: surrounding free e "screen" will disperse signal
  limit the strength of GR probe...