Astro 501: Radiative Processes
Lecture 27
March 29, 2013

Announcements:

- **Problem Set 8** extended to 5pm Monday

Last time: plasma effects

Q: plasma characteristic scales?

Q: EM wave propagation in plasma vs vacuum?

plasma characteristic scales:

- plasma frequency $\omega_{\mathrm{p}}^2 = 4\pi e^2 n_e / m_e$
- Debye length $\lambda_{\mathrm{D}} = v_T/\omega_{\mathrm{p}} = \sqrt{kT/4\pi e^2 n_e}$
  plasma behavior of $\lambda_{\mathrm{D}} \ll$ collision mean free path

dispersion relation in plasma:

$$\omega^2 = \omega_{\mathrm{p}}^2 + (ck)^2 \quad (1)$$

also note: in plasma, changes to electrostatic potential of a charge q at distance r:

$$\phi(r) = \frac{q}{r} e^{-\sqrt{2}r/\lambda_{\mathrm{D}}} = \frac{q_{\mathrm{sc}}(r)}{r} \quad (2)$$

"screening" effect of unbound electrons and ions in plasma damps interaction of a charge at distances $r \gtrsim \lambda_{\mathrm{D}}$
timescale for onset of screening is $\tau \sim \lambda_{\mathrm{D}}/v_T \sim \omega_{\mathrm{p}}^{-1}$

EM Propagation Along A Magnetic Field

the interstellar medium (ISM) contains not only plasma
but also *magnetic fields*

thus we are obliged to understand
EM propagation in a magnetized plasma

consider idealized case:
a fixed, uniform external field $\vec{B}_0$
in a nonrelativistic plasma: $v_T \ll c \rightarrow kT \ll m_e c^2 \rightarrow T \ll 10^{10}$ K

Q: effect on plasma electrons?
Q: effect on EM waves propagating $\perp \vec{B}_0$*?*
Q: effect on EM waves propagating along $\vec{B}_0$*?*

in a fixed uniform external field $\vec{B}_0$
(non-relativistic) electrons move in Larmor orbits
and new frequency/timescale introduced: Larmor/gyro-frequency

$$\omega_B = \frac{eB_0}{m_e c} = 17 \text{ Hz } \left(\frac{B_0}{1 \ \mu\text{Gauss}} \right) \tag{3}$$

magnetic field introduces a special direction and thus anisotropy
which affects EM propagation $\rightarrow$ dielectric constant anisotropic

that is:

- electrons orbit around field lines
- for *waves* $\parallel$ *field*: $\hat{k} = \hat{B}_0$
  e motion due to $\vec{E}_{\text{wave}}$ in Larmor orbit plane
  $\rightarrow$ expect B_0 to change wave propagation
- for *waves* $\perp$ *field*: $\hat{k} \cdot \hat{B}_0 = 0$
  e motion due to $\vec{E}_{\text{wave}}$ is orthogonal to orbit
  $\rightarrow$ expect no/less change in EM propagation

Electron Motion in a Magnetized Plasma

if $B_0 \gg B_{\rm wave}$, then e equation of motion

$$m_e \dot{\vec{v}} \approx -e\vec{E} - e\frac{\vec{v}}{c} \times \vec{B}_0 \tag{4}$$

assume a propagating, sinusoidal, *circularly polarized* EM wave:

$$E(t) = E \ e^{i\omega t} \left(\hat{\epsilon}_1 \mp \hat{\epsilon}_2\right) \tag{5}$$

where $\mp \ \leftrightarrow$ right/left circular polarization

also assume propagation is *along the field*

$$\vec{B}_0 = B_0 \ \hat{\epsilon}_3 \tag{6}$$

solutions with $v(t) \propto e^{i\omega t}$ have

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_B)} \vec{E}(t) \tag{7}$$

electron velocity has

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_B)}\vec{E}(t) \tag{8}$$

so still have Ohm's law current density $\vec{j} = -en_e\vec{v} = \sigma\vec{E}$
but now with $\sigma = ie^2 n_e / m_e(\omega \pm \omega_B)$

and so now the dielectric constant is

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega \pm \omega_B)} \tag{9}$$

- right(+) and left(-) circular waves travel with different speeds
- speed difference sense is $v_{\mathrm{R}} > v_{\mathrm{L}}$

Q: effect of sending circularly polarized radiation thru a plasma?

Q: effect of sending linearly polarized radiation thru a plasma?

Faraday Rotation

for EM waves *along* magnetic field, dielectric constant is

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega \pm \omega_B)} \tag{10}$$

if incident radiation is *circularly* polarized (either R or L)
then will encounter different dispersion than unmagnetized case
but still remain circularly polarized

if incident radiation is *linearly* polarized
then it has equal superposition of R and L components
$\rightarrow$ these components dispersed differently $\rightarrow$ tblue*changes phase*
$\rightarrow$ *polarization rotated due to magnetic field*
$\Rightarrow$ **Faraday rotation**

Faraday Rotation

after wave propagates distance $\vec{d}$, phase is $\vec{k} \cdot \vec{d}$
but if k nonuniform in space, then

$$\phi_{\mathrm{R,L}} = \int_0^d k_{\mathrm{R,L}} \, ds \tag{11}$$

with $ck_{\mathrm{R,L}} = \omega \sqrt{\epsilon_{\mathrm{R,L}}}$

if $\omega \gg \omega_{\mathrm{p}}$ and $\omega \gg \omega_B$ then

$$k_{\mathrm{R,L}} \approx \frac{\omega}{c} \left[1 - \frac{\omega_{\mathrm{p}}^2}{2\omega^2} \left(1 \mp \frac{\omega_B}{\omega} \right) \right] \tag{12}$$

and thus *polarization plane rotates* through angle

∞

$$\Delta\theta = \frac{\Delta\phi}{2} = \frac{1}{2} \int_0^d (k_{\mathrm{R}} - k_{\mathrm{L}}) ds = \frac{1}{2} \int_0^d \frac{\omega_{\mathrm{p}}^2 \omega_B \, ds}{c\omega^2} \tag{13}$$

Faraday rotation of linear polarization angle is therefore

$$\Delta\theta = \frac{2\pi e^3}{m_e^2 c^2 \omega^2} \int_0^d n_e \; B_{\parallel} \; ds \tag{14}$$

Q: how can we be sure Faraday rotation really has occurred?

Q: what does Faraday rotation directly tell us? with other information?

Q: what if field changes along line of sight?

Astrophysics of Faraday Rotation

effect occurs when *linearly polarized* radiation
passes through a *magnetized plasma*

But we don't know initial polarization angle!
true, but $\Delta\theta \propto \nu^{-2} \propto \lambda^2$
$\rightarrow$ use this dependence to confirm effect

if Faraday rotation observed:

- immediately know $B_{\parallel} \neq 0$: existence of interstellar magnetism
- if know n_e and d, then *measure* $B_{\parallel}$
- if field direction changes, then $B > B_{\parallel}$:
  Faraday gives *lower limit* to true field strength

Q: what astrophysical situation needed to observe this? examples?

to observe Faraday rotation, need both

- polarized background source and
- foreground plasma

typical example:

- AGN have (partially) linearly polarized emission and are cosmological $\rightarrow$ isotropically distributed on sky
- if you are lucky, one is behind your source!

Awesome Example I: our Galaxy

find rotation for many AGN across the sky

plot *rotation measure* $\Delta\theta = \mathrm{RM}\ \lambda^2$

$$\mathrm{RM} = \frac{1}{2\pi} \frac{e^3}{m_e^2 c^4} \int n_e\ B_{\parallel}\ ds \tag{15}$$

`www: results` *Q: implications?*

Results:

- Faraday rotation detected! *the Galaxy is magnetized!*
- largest signal in plane $\rightarrow$ fields associated with ISM
- typical strength $B_{\text{ism}} \sim few\ \mu$Gauss

Awesome Example II: supernova remnants

recall: supernovae are mighty particle accelerators

the engines of cosmic-ray acceleration

$\rightarrow$ supernova remnants are very bright in synchrotron
from electrons accelerated in the remnant
and this radiation is polarized

$\rightarrow$ so can measure Faraday rotation in the remnant
using its own synchrotron!

Awesome Example III: ask David

Radiation and Bound States

Radiation and Bound States

In the course up until now, focused largely on
continuum radiation
i.e., processes that emit/absorb across a wide range of ν
and (mostly) involve *unbound* electrons

- blackbodies
- bremsstrahlung
- synchrotron
- Thompson/Compton

But what about *lines*!?
these arise from transitions involving electrons in *bound states*
$\rightarrow$ atoms and molecules

To discuss these, begin with refresher on atoms and molecules

Atomic Structure

Atomic Structure: Order of Magnitude

Atoms and molecules inherently *quantum* systems

Cowgirl/cowboy view of hydrogen:
quantum bound state of electron around proton, with energy

$$E = \frac{p^2}{2m_e} - \frac{e^2}{r} \tag{16}$$

but Heisenberg: $rp \geq \hbar/2$

Wild West: cowgirl/cowboy approximation $p \sim \hbar/r$

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{r} \tag{17}$$

Q: so how to find ground state?

Heisenberg-ized sketch of hydrogen energy

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{r} \qquad (18)$$

ground state: E is *minimum*

$$\partial_r E = -\frac{\hbar^2}{m_e r^3} + \frac{e^2}{r^2} = 0 \qquad (19)$$

gives electron radius $r_{\text{min}} \equiv a_0$:

$$a_0 = \frac{\hbar^2}{e^2 m_e} = 0.05 \text{ nm} \qquad (20)$$

and electron energy $E(r_{\text{min}}) \equiv E_1$:

$$E_1 = -\frac{e^4 m_e}{2\hbar^2} = -\frac{1}{2}\alpha^2 m_e c^2 = 13.7 \text{ eV} \qquad (21)$$

where $\alpha = e^2/\hbar c \approx 1/137$

Q: how do these compare with results of honest calculation?

Hydrogen Atom: Honest Results

a full quantum mechanical calculation for hydrogen involves

- specifying the Hamiltonian (energy)
- include Coulomb, but should also include e and p spins
- solve for wavefunctions

for hydrogen-like species: single electron, nuclear charge Z

- ground state energy $E_1 = -Z^2 e^4 m_e / 2\hbar^2$
- ground state mean radius $\langle r_1 \rangle = a_0 / Z$
- excited states, ignoring spin effects: $E_n = E_1/n^2$, $\langle r_n \rangle = n^2 r_1$
- wavefunction: 3-D system $\rightarrow$ nees 3 quantum numbers

  principal quantum number $n = 1, 2, \ldots$

  orbital angular momentum $\ell = 0, 1, \ldots, n-1$

  $\hat{L}^2 \psi = \ell(\ell+1)\hbar^2 \psi$

  z-projection of $\hat{L}$ $m = -\ell, \ldots, +\ell$