Astro 501: Radiative Processes
Lecture 32
April 10, 2013

Announcements:

- **Problem Set 10** due 5pm next Friday April 19
- **no class meeting Monday or Wednesday**
  time off for good behavior

Last time: the physics and astrophysics of line shapes

Q: why not a delta function? what about energy conservation?

Q: sources of broadening?

Q: lineshapes in astrophysical applications?

Linewidths

naïvely: in transition $u \to \ell$, *energy conservation* requires
$h\nu = E_u - E_\ell \equiv h_{u\ell}$, so $\phi_{\mathrm{naive}}(\nu) = \delta(\nu - \nu_{u\ell})$: *zero width!*

But real observed linewidths are nonzero, for several reasons

- *intrinsic width*
quantum effect, due to nonzero transition rate $\Gamma = 1/\tau$
and energy-time uncertainty principle $\Delta E \; \Delta t \gtrsim \hbar/2$

- *thermal broadening*
thermal motion of absorbers $\to$ Doppler shifts

- *collisional broadening*
absorber collisions add to transition probability

Collisional Linewidth

if particle densities are high, atomic collisions are rapid
and can drive transitions $u \leftrightarrow \ell$

thus there is a nonzero collision rate $\Gamma_{\rm coll}$ per atom
where $\Gamma_{\rm coll} = n \ \sigma_{\rm coll} v$

heuristically: this decreases excited state lifetimes
and thus adds to energy uncertainty

so total transition rate includes both $\Gamma_{\rm int}$ and $\Gamma_{\rm coll}$:
$\rightarrow$ collisions add damping, which depends on
photospheric density and temperature via $\Gamma_{\rm coll}$

thus collisional broadening measures density and temperature
thus also know as "pressure broadening"

Q: effect of collisions on lineshape?

recall: atomic transition $u \to \ell$ has

$$\sigma_{u\ell}(\nu) = \pi e^2/m_e c \; f_{u\ell} \; \phi_{u\ell}(\nu) = B_{\text{classical}} \; f_{u\ell} \; \phi(\nu) \tag{1}$$

without collisions, intrinsic profile shape that is *Lorentzian*

$$\phi_{u\ell}^{\text{intrinsic}}(\nu) = \frac{4\Gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \Gamma_{u\ell}^2}$$

full width at half-maximum: $(\Delta\nu)_{\text{FWHM}} = \Gamma_{u\ell}/2\pi$
set by intrinsic level de-excitation rate $\Gamma_{u\ell}$

With collisions: $\Gamma_{\text{coll}} = n \; \sigma_{\text{coll}} \; v$
still a Lorentzian profile, but with effective transition rate to

$$\frac{\Gamma_{u\ell}}{2} = \frac{\Gamma_{u\ell}^{\text{intrinsic}}}{2} + \Gamma_{\text{coll}} \tag{2}$$

4 `www: solar Hα line`

Awesome Example: Classifying Stars

Q: how can spectra determine stellar (photosphere) T?

`www: spectra of main sequence (dwarf) stars`

Q: many lines are strongest in middle of sequency–why?

`www: white dwarf spectrum`

`www: O star spectrum`

Q: similar temperatures, why different?

Q: at fixed T, how can spectrum distinguish main sequence vs giant stars?

Q: which of the above requires distance to star?

Q: what stellar properties do require distance?

Awesome Example: Classifying Stars

to a good approximation, stellar spectra are:

- blackbody = Planck form, at photospheric T
- with lines (often many!) due to photospheric absorption

Star Type: *OBAFGKMLT*

a sequence in *temperature*; Sun is G5

"early types" hotter than Sun: *OBAF*

"late types" solar and cooler: *GKMLT*

main sequence spectra: lines very temperature sensitive

Balmer H lines: weak→strong→weak for types O→A→M

- O stars $T > 30,000$ K: most H is ionized
- A stars $T \sim 10,000$ K: most H neutral, but $n = 2$ populated
- M stars $T \sim 4000$ K: H neutral, tiny $n = 2$ population

Stellar Luminosity Class: I, II, III, IV, V

determined by shapes of strong lines at fixed spectral type
i.e., at (nearly) fixed temperature
V: line wings broader than intrinsic damping width
I: no additional broadening

physically: damping wings sensitive to *pressure broadening*
i.e., by collision rate $\Gamma_{\rm coll} = n\sigma_{\rm coll}v$
at fixed T, this corresponds to different *density* and pressure
but hydrostatic equilibrium: $\nabla P = \rho\vec{g} = G\rho M/R^2$
linewidth set by pressure $\to$ set by stellar *radius* R

Class I: supergiant
Class II: bright giants
Class III: normal (“red” giants)
Class IV: subgiants
Class V: main sequence (non-giants = “dwarfs”); Sun is G5V

Absorption Lines: Probing the Depths

so far: focused on absorption line *shape*
but important information also in line *depth*
below the continuum level

Q: what is needed to measure line depth?
Q: in high-resolution spectra, what sets line depth at each ν?
Q: as absorber density increases, effect on line?

absorption cross section (line oscillator strength) generally known
`www: online databases`
Q: given this, what quantitative information does line depth give?

Absorption Lines: Radiation Transfer

consider a (spatially) unresolved source, with angular area $\Delta\Omega$
if no material in foreground, observed flux $F_\nu(0) \approx I_\nu(0)\ \Delta\Omega$

with intervening absorbers of density n at T, observed flux is

$$F_\nu = e^{-\tau_\nu}\ F_\nu(0) + (1 - e^{-\tau_\nu}) S_\nu(T)\ \Delta\Omega \tag{3}$$

but usually for bright sources, $S_\nu(T)\ \Delta\Omega \ll F_\nu(0)$
and we have $F_\nu \approx e^{-\tau_\nu}\ F_\nu(0)$

near $\nu_{u\ell}$ for absorber transition $\ell \to u$, optical depth is

$$\tau_\nu = \sigma_\nu\ N_\ell\ \left(1 - \frac{g_u N_u}{g_\ell N_\ell}\right) \tag{4}$$

where $N_i \equiv \int n_i\ ds$ is absorber *column density* for level i

the last factor accounts for stimulated emission
but usually $g_u N_u \ll g_\ell N_\ell$ *Q: why?*, so that $\tau_\nu \approx \sigma_\nu\ N_\ell$

So *if we assume we know the spectral shape* $F_\nu(0)$
of the background source across the line profile
then the observed deviation from this continuum
i.e., line *profile* $F_\nu/F_\nu(0) = e^{-\tau_\nu}$
directly measures optical depth $\tau_\nu \approx \sigma_{\ell u} N_\ell$

but the absorption cross section is

$$\sigma_{\ell u}(\nu) = \pi e^2/m_e c \; f_{\ell u} \; \phi_{\ell u}(\nu) \tag{5}$$

oscillator strength $f_{\ell u}$ usually known (i.e., measured in lab)
so at high resolution:

- line profile*depth* $\rightarrow$ absorber *column density* N_ℓ
- line profile *shape* $\rightarrow$ absorber profile function $\phi_{\ell u}(\nu)$ which encodes, e.g., temperature via core width $b = \sqrt{2kT/m}$, and collisional broadening via wing with Γ

Depth of Line Center

if the absorbers have a Gaussian velocity distribution
then the optical depth profile is $\tau_\nu = \tau_0 \; e^{-v^2/b^2}$
with the Doppler velocity $v = (\nu_0 - \nu)/\nu_0 \; c$, and thus
τ_ν is also Gaussian in ν

the optical depth a the line center is

$$\tau_0 = \sqrt{\pi} \left(\frac{e^2}{m_e c} \right) \; \frac{N_\ell f_{\ell u} \lambda_{\ell u}}{b} \; \left[1 - \frac{g_u N_u}{g_\ell N_\ell} \right] \tag{6}$$

ignoring the stimulated emission term $[\cdots]$, for H Lyman α

$$\tau_0 = 0.7580 \; \left(\frac{N_\ell}{10^{13} \text{ cm}^{-2}} \right) \; \left(\frac{f_{\ell u}}{0.4164} \right) \left(\frac{\lambda_{\ell u}}{1215.7 \text{ Å}} \right) \; \left(\frac{10 \text{ km/s}}{b} \right)$$

so if we can measure τ_0, we get column N_ℓ

Q: in low-resolution spectra, what information is lost?

Q: what information remains?