Astro 501: Radiative Processes
Lecture 35
April 22, 2013

Announcements:

- **Problem Set 11** last one! due Monday April 29

Last time: equivalent width and curve of growth

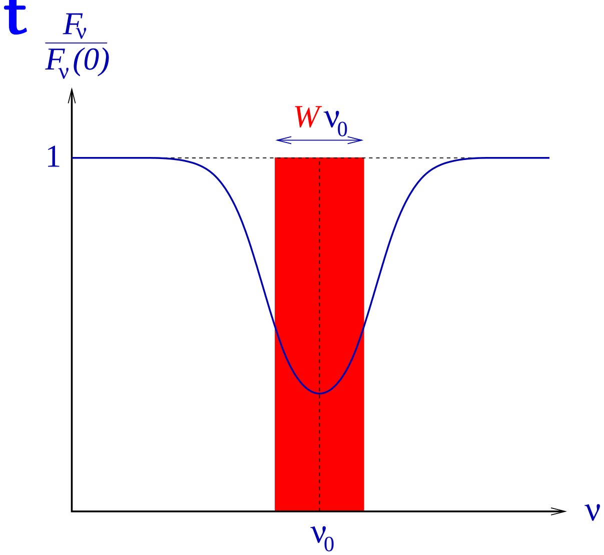
Q: what's equivalent width? why useful? equivalent to what?

Q: what's the curve of growth? why useful? what's growing?

Q: regimes in the curve of growth?

useful to define a dimensionless **equivalent width**

$$W = \int_{\Delta\nu_{\rm line}} \frac{F_\nu(0) - F_\nu}{F_\nu(0)} \, \frac{d\nu}{\nu_0}$$



so $W\nu_0$ equivalent to

width of 100% absorbed line

i.e., *saturated* line with "rectangular" profile

and W is width as fraction of ν_0

in terms of optical depth, equivalent width is

$$W = \int_{\Delta\nu_{\rm line}} \left[1 - \frac{F_\nu}{F_\nu(0)}\right] \, \frac{d\nu}{\nu_0} = \int_{\Delta\nu_{\rm line}} \left(1 - e^{-\tau_\nu}\right) \, \frac{d\nu}{\nu_0} \qquad (1)$$

and thus $W = W(N_\ell)$ via $\tau_\nu = \sigma_\nu N_\ell$

dependence of W vs N_ℓ: **curve of growth**

Absorption Line Spectroscopy: Doublets

what if absorbing level ℓ has transitions to
two "neighboring" excited states u_1 and u_2?
i.e., $\ell \to u_1$ and $\ell \to u_2$ both allowed

Q: how will this imprint on the spectrum?

Q: if both lines optically thin, what is equiv width ratio W_2/W_1*?*
Q: what is notable about this?

Q: how will W_2/W_1 *ratio change with* N_ℓ*?*
Q: what is notable about this?

Doublet Line Ratios

if the states u_1 and u_2 are "neighboring" levels
then spectra will show two neighboring lines: a **doublet**

in the optically thin limit, $W \to (\pi e^2/m_e c^2)\ N_\ell\ f_{\ell u}\ \lambda_{\ell u}$
so if both lines are *optically thin*, then

$$\frac{W_2}{W_1} \approx \frac{f_{\ell u_2}\lambda_{\ell u_2}}{f_{\ell u_1}\lambda_{\ell u_1}} \tag{2}$$

- ratio set entirely by atomic line properties
- transition with *larger* $f_{\ell u_2}\lambda_{\ell u_2}$ has stronger line

sketch curve of growth for both lines

as N_ℓ increases: optical depth increases
stronger line first enters *flat part of curve of growth*
eqivalent width set by thermal width: $W = 2b/c\sqrt{\ln\tau_0/\ln 2}$
$\rightarrow$ W_2/W_1 *drops*, until weaker line also in flat part:

$$\frac{W_2}{W_1} \approx \sqrt{1 + \frac{\ln(f_{\ell u_2}\lambda_{\ell u_2}/f_{\ell u_1}\lambda_{\ell u_1})}{\ln\tau_{\ell u_1}/\ln 2}} \;\rightarrow\; 1 \tag{3}$$

$\rightarrow$ limit is *independent of atomic properties*

as absorber column N_ℓ increases further
the stronger line first enters the damping wing regime
line ratio increases again, approaching

$$\frac{W_2}{W_1} \approx \frac{\lambda_{\ell u_2}}{\lambda_{\ell u_1}}\sqrt{\frac{f_{\ell u_2}\Gamma_{\ell u_2}}{f_{\ell u_1}\Gamma_{\ell u_1}}} \tag{4}$$

$\rightarrow$ set by atomic properties and *total damping width*

Q: why is all of this useful?

thus *doublet ratio is diagnostic of curve-of-growth regime*

Strategy:

compare observed W_2/W_1 value to
optically thin limiting value $f_{\ell u_2}\lambda_{\ell u_2}/f_{\ell u_1}\lambda_{\ell u_1}$

- agreement verifies both lines optically thin
- $W_2/W_1 \approx 1$ is warning: flat part of curve of growth
- $W_2/W_1 \to$ damping wing ratio gives $\Gamma_{\ell u_2}/\Gamma_{\ell u_1}$
  and thus collisional broadening & hence pressure/density

Atomic Hydrogen: the Lyman Series and Lyman Limit

neutral hydrogen in the ground state has transitions to all excited states; lines form the **Lyman series**

- Lyman-α: $n = 1 \rightarrow 2$; $\lambda_{\mathrm{Ly}\alpha} = 1215.67$ Å
  Q: EM regime? implications?
- Lyβ: $n = 1 \rightarrow 3$; Lyγ: $n = 1 \rightarrow 4$, etc

Lyman series line $n = 1 \rightarrow n_f$ has

$$\lambda_{n_f} = \left(1 - \frac{1}{n_f^2}\right) \frac{hc}{B_{\mathrm{H}}} \tag{5}$$

for $n_f \gg 1$, line *pileup* of lines
at **Lyman limit** $\lambda_{n_f \rightarrow \infty} = hc/B_{\mathrm{H}} = 911.75$ Å

but note: real spectra do not show infinite lines near Ly limit
Q: Why not? What will we really see?

The Lyman Limit

there are infinitely many transitions near the Lyman limit
but each line has a *finite width*
typically due to thermal broadening

lines overlap and *blend* when

$$\text{line width} \lesssim \text{line spacing} \tag{6}$$

$$\frac{v_{\rm FWHM}}{c}\lambda_n \lesssim \lambda_n - \lambda_{n-1} \approx \frac{\partial \lambda_n}{\partial n} \tag{7}$$

occurs when $1/n^3 \lesssim v_{\rm FWHM}/c$, and thus
quantum number $n \gtrsim 67\ (2\ {\rm km\,s^{-1}}/{\rm v_{FWHM}})^{1/3}$, and wavelength

$$\lambda \approx 911.75 + 0.2 \left(\frac{v_{\rm FWHM}}{2\ {\rm km/s}}\right)^{2/3}\ \text{Å} \tag{8}$$

∞ `www: Lyman limit QSO absorption line system`

Q: implications for atoms having $\lambda(n = 1 \to 2) < 912$ Å?

Atomic Resonance Lines

for neutral atoms, *permitted* lines due to
transitions from the *ground state* are called **resonance lines**

we saw: neutral H atoms absorb all photons with $\lambda < 912$ Å
in *bound-bound transitions* to $n_f \gg 1$ for λ near limit
or for smaller λ, in *bound-free transitions* that ionize H

but H is the most abundant element in the Universe!

thus for atoms with all resonance lines $\lambda < 912$ Å
local hydrogen absorbs photons that can drive these transition
$\rightarrow$ *cannot observe these elements in sightlines with H I!*
Q: are any atoms excluded this way? if so, which?

Good news:

the only atoms excluded this way are He and Ne

noble gasses, first excited states at very high energies

all other atoms have resonance lines $\lambda > 912$ Å!

Bad news:

for most atoms and ions, resonance lines have $\lambda < 3000$ Å

Q: why is this bad?

Q: how to get around the badness?

“Metal” Lines

quaint astro-lingo: **“metal”** any element $\neq$ H, He
including, e.g., famous “metals” C, N, O

most metal atoms and ions have resonance lines $\lambda < 3000$ Å
this is in the UV, blocked by Earth’s atmosphere
`www: atmosphere transmittance`
for such species, can only see absorption lines:

- by going to space
- when looking at high-redshift objects
  `www: metal lines in QSO spectrum`

but nature has not been totally unkind
a few atoms and ions have resonance lines with $\lambda > 3000$ Å
examples: Na I D doublet $\lambda = 5891.6, 5897.6$ Å,
Ca II doublet at $3934.8, 3969.6$ Å
`www: solar lines` *Q: why is the Na D line so weak?*

Atomic Hydrogen: the 21 cm Line

in *hydrogen*, both e and p have spin $S = 1/2$ (fermions!)
coupled via *spin-spin* or *hyperfine* interaction
with Hamiltonian $H_{\text{spin}-\text{spin}} = H_{\text{hf}} \; \vec{s}_e \cdot \vec{s}_p$

hydrogen ground state has two possible *spin configurations*

- proton and electron spins *parallel*: $\uparrow_e \uparrow_p$
  excited state: $S_u = 1$, $g_u = 3$
- spins *antiparallel*: $\downarrow_e \uparrow_p$
  ground state: $S_\ell = 0$, $g_\ell = 1$

transition $u \rightarrow \ell$ is an electron *spin flip*, with

$$A_{u\ell} = 2.8843 \times 10^{-15} \text{ s}^{-1} = (11.0 \text{ Myr})^{-1}$$
$$\Delta E = E_u - E_\ell = 5.86 \times 10^{-6} \text{ eV} = k_{\text{B}} \, (0.06816 \text{ K})$$
$$\nu_{u\ell} = 1420.4 \text{ MHZ} \qquad \lambda_{u\ell} = 21.106 \text{ cm}$$

implications of A value? ΔE? λ?

Einstein coefficient:

$A_{u\ell} = (11.0 \text{ Myr})^{-1}$: *very slow rate*

- spontaneous emission only occurs after a ~ 11 Myr
  if the atoms has been *undisturbed*: no collisions!
- spontaneous emission never observed in the laboratory!
  but can measure transition via stimulated emission!
- but can occur in low-density astrophysical environment
- excited state lifetime $A^{-1} \ll$ age of Universe
  $\rightarrow$ need some collisions to replenish excited state

EM regime:

$\nu = 1420.4$ MHZ and $\lambda = 21.106$ cm: "21 cm radiation" in radio

Thermal Properties:

$\Delta E/k_{\text{B}} = 0.06816$ K small splitting

$\rightarrow$ easy to thermally populate excited state

Q: recall that today, $T_{\text{cmb}} = 2.725$ K; implications?

Spin Temperature

the CMB has $T_{\text{CMB}} \gg \Delta E/k \rightarrow$ can populate upper level!

if states in thermal equilibrium at *excitation* or *spin temperature* with $T_{\text{ex}} \equiv T_{\text{spin}} \gg \Delta E/k$, then

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu_{u\ell}/kT_{\text{spin}}} \approx \frac{g_u}{g_\ell} = 3 \tag{9}$$

a nearly fixed ratio *independent of temperature*, so that

$$n_u \approx \frac{3}{4} n(\text{H I}) \; , \quad n_\ell \approx \frac{1}{4} n(\text{H I}) \tag{10}$$

thus: 21-cm emissivity also independent of spin temperature

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell} \, \phi_\nu \approx \frac{3}{16\pi} A_{u\ell} \, h\nu_{u\ell} \, n(\text{H I}) \, \phi_\nu \tag{11}$$

Q: absorption coefficient?

21-cm Absorption Coefficient

as usual, absorption coefficient has true and stimulated terms:

$$\alpha_\nu = n_\ell \sigma_{\ell u} - n_u \sigma_{\ell u} \quad (12)$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - \frac{n_u}{n_\ell} \frac{g_\ell}{g_u} \right] \quad (13)$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - e^{-h\nu_{u\ell}/kT_{\rm spin}} \right] \quad (14)$$

but in practice we always have $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1$, so
stimulated emission correction is very important!

using $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1 - h\nu_{u\ell}/kT_{\rm spin}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} A_{u\ell} \ \frac{hc\lambda_{u\ell}}{kT_{\rm spin}} n({\rm H\ I}) \ \phi_\nu \quad (15)$$

and thus $\alpha_\nu \propto 1/T_{\rm spin}$

Q: what determines ϕ_ν in practice?