Astro 501: Radiative Processes
Lecture 36
April 24, 2013

Announcements:

- **Problem Set 11** last one! due next Monday April 29

Last time: atomic hydrogen lines

- Lyman limit *Q: what's that? Why does it arise?*
- 21 cm line *Q: Why does it arise? Why is it special?*

hyperfine transition $u \to \ell$ is an electron *spin flip*, with

$$A_{u\ell} = 2.8843 \times 10^{-15}\ \mathrm{s}^{-1} = (11.0\ \mathrm{Myr})^{-1} \tag{1}$$

Amendment to last time: *spontaneous emission* not measured in lab
but *stimulated emission* exquisitely well-measured in resonating cavities: hydrogen masers

$$\nu_{u\ell} = 1420.405751768(1)\ \mathrm{MHZ} \qquad \lambda_{u\ell} = 21.10611405413\ \mathrm{cm}$$

$\Delta E/k_{\mathrm{B}} = 0.06816\ \mathrm{K} \ll T_{\mathrm{cmb},0} \to$ CMB can populate upper level!

ratio $n_u/n_\ell \approx 3$ nearly fixed*independent of temperature*, so that

$$n_u \approx \frac{3}{4} n(\mathrm{H\ I})\ , \quad n_\ell \approx \frac{1}{4} n(\mathrm{H\ I}) \tag{2}$$

thus: 21-cm emissivity also independent of spin temperature

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell}\ \phi_\nu \approx \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ n(\mathrm{H\ I})\ \phi_\nu \tag{3}$$

Q: absorption coefficient?

21-cm Absorption Coefficient

as usual, absorption coefficient has true and stimulated terms:

$$\alpha_\nu = n_\ell \sigma_{\ell u} - n_u \sigma_{\ell u} \tag{4}$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - \frac{n_u}{n_\ell} \frac{g_\ell}{g_u} \right] \tag{5}$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - e^{-h\nu_{u\ell}/kT_{\mathrm{spin}}} \right] \tag{6}$$

but in practice we always have $e^{-h\nu_{u\ell}/kT_{\mathrm{spin}}} \approx 1$, so
stimulated emission correction is very important!

using $e^{-h\nu_{u\ell}/kT_{\mathrm{spin}}} \approx 1 - h\nu_{u\ell}/kT_{\mathrm{spin}}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} A_{u\ell} \ \frac{hc\lambda_{u\ell}}{kT_{\mathrm{spin}}} n(\mathrm{H\ I}) \ \phi_\nu \tag{7}$$

and thus $\alpha_\nu \propto 1/T_{\mathrm{spin}}$

Q: what determines ϕ_ν in practice?

since $A = \Gamma$ is very small, 21-cm line intrinsically very narrow
$\rightarrow$ width entirely determined by *velocity dispersion* of the emitting hydrogen

for a random, Gaussian velocity distribution

$$\phi_\nu = \frac{1}{\sqrt{2\pi}} \frac{c}{\nu_{u\ell}} \frac{1}{\sigma_v} e^{-u^2/2\sigma_v^2} \tag{8}$$

with $u = c(\nu_{u\ell} - \nu)/\nu_{u\ell}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} \frac{1}{\sqrt{2\pi}} \frac{A_{u\ell}\lambda_{u\ell}^2}{\sigma_v} \frac{hc}{kT_{\mathrm{spin}}} n(\mathrm{H\ I})\ e^{-u^2/2\sigma_v^2} \tag{9}$$

and optical depth

$$\tau_\nu = 2.190\ \left(\frac{N(\mathrm{H\ I})}{10^{21}\ cm^{-2}}\right) \left(\frac{100\ \mathrm{K}}{T_{\mathrm{spin}}}\right) \left(\frac{1\ \mathrm{km/s}}{\sigma_v}\right)\ e^{-u^2/2\sigma_v^2} \tag{10}$$

Q: implications?

21 cm Emission: Optically Thin Case

21 cm optical depth:

$$\tau_\nu = 2.190 \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}} \right) \left(\frac{100\text{ K}}{T_{\text{spin}}} \right) \left(\frac{1\text{ km/s}}{\sigma_v} \right) e^{-u^2/2\sigma_v^2} \quad (11)$$

real interstellar lines of sight can have $N(\text{H I}) > 10^{21}\ cm^{-2}$
$\rightarrow$ *self-absorption can be important!*

But in the optically thin limit, for $N(\text{H I}) \lesssim 10^{20}\ \text{cm}^{-2}$
then absorption is small and

$$I_\nu \approx I_\nu(0) + \int j_\nu\ ds = I_\nu(0) + \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I})\ \phi_\nu \quad (12)$$

with $N(\text{H I}) = \int n_{\text{H I}}\ ds$

if $I_\nu(0)$ is known *Q: how?*, then

$$\int [I_\nu - I_\nu(0)]\ d\nu = \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I}) \quad (13)$$

in terms of antenna temperature, integrating in velocity space

$$\int [T_{\mathsf{A}} - T_{\mathsf{A}}(0)]\ du = \int \frac{c^2}{2k\nu^2} [T_{\mathsf{A}} - T_{\mathsf{A}}(0)]\ c\,\frac{d\nu}{\nu} = \frac{3}{16\pi} A_{u\ell} \frac{hc\lambda_{u\ell}^2}{k}\, N(\mathrm{H\ I})$$

measures *hydrogen column* $N(\mathrm{H\ I})$ independent of spin temperature!

integrating over solid angles gives flux density

$$F_{\mathsf{obs}} = \int F_\nu\ d\nu = \int I_\nu \cos\theta\ d\Omega\ d\nu \approx \int I_\nu\ d\Omega\ d\nu \qquad (14)$$

and thus the integrated flux

$$F_{\mathsf{obs}} \propto \int N(\mathrm{H\ I})\ d\Omega = \frac{\int n_{\mathrm{H\ I}}\ ds\ dA}{D_L^2} \propto \frac{M_{\mathrm{H\ I}}}{D_L^2} \qquad (15)$$

measures the *total hydrogen mass* $M_{\mathrm{H\ I}}$
if we know the (luminosity) distance D_L

useful for H I clouds in our own Galaxy,
and measuring H I content of external galaxies

consider cold, diffuse atomic H in a galaxy that has bulk internal motions with speeds $v_{\text{bulk}} > \sigma_v$

Q: how would this arise?

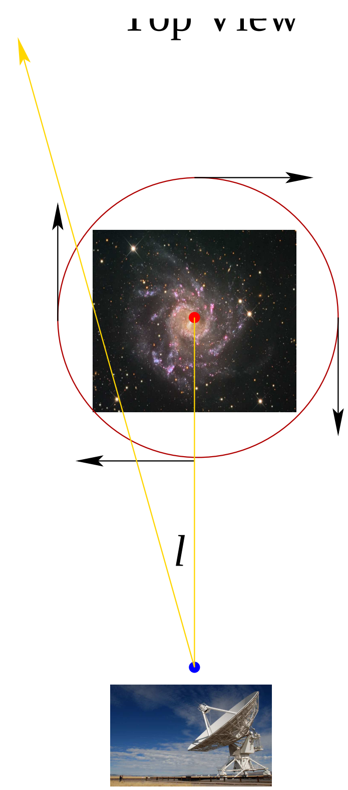
Q: what spectral pattern would uniform rotation give?

Q: what is a more realistic expectation?

Awesome Example: Galaxies in 21 cm

spiral galaxies observed in 21 cm emission, ellipticals are not
$\rightarrow$ spirals are gas rich, ellipticals gas poor `www: THINGS survey`

spiral galaxies also rotate:
bulk line-of-sight motion imprinted on 21 cm
via Doppler shift at different sightlines



spectrum depends on *rotation curve* $V(R)$

- uniform rotation: $V = \omega_0 R \propto R$
  small V near center, only large at edge
  $\rightarrow$21 cm peak near galaxy systemic speed $V = 0$
- "flat" curve: $V(R) \rightarrow V_0$, a constant
  small V only near center, large elsewhere
  ∞ $\rightarrow$21 cm peak at $V = \pm V_0$

`www: observed 21 cm spectrum`

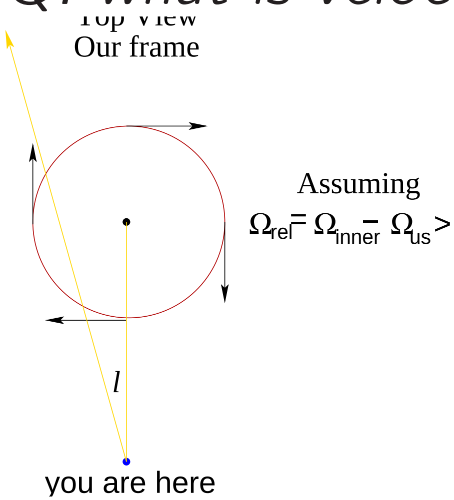
Awesome Example: the 21 cm Milky Way

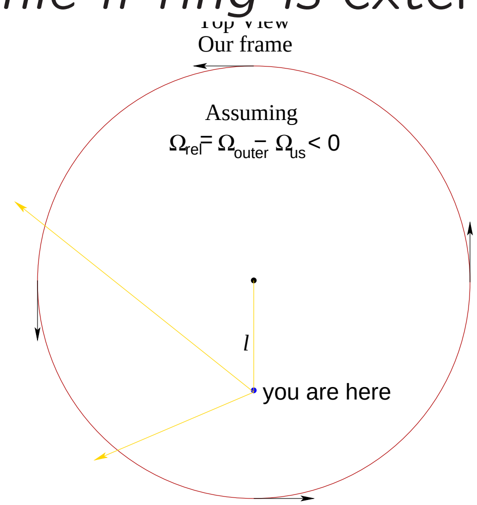
the Galactic plane is well-mapped in 21 cm

Q: what do we expect for the intensity map?

Q: what do we expect for the velocity map?

Hint: imagine single *rings* of rotating gas
Q: what is velocity profile if ring is interior *to us?*
Q: what is velocity profile if ring is exterior *to us?*





`www: observed MW velocity profile`

Awesome Example: Cosmic 21 cm Radiation

CMB today, redshift $z = 0$, has $T_{\rm cmb}(0) = 2.725\ {\rm K} \gg {\rm T}_{\rm ex,21\,cm}$
but what happens over cosmic time?

fun & fundamental cosmological result:
(relativistic) *momentum redshifts*: $p \propto 1/a(t)$, which means

$$p(z) = (1 + z)\ p(0) \tag{16}$$

where $p(0)$ is observed momentum today ($z = 0$)

why? photon or de Broglie wavelength λ is a *length*, so

$$\lambda(t) = a(t)\ \lambda_{\rm emit} = \frac{\lambda_0}{(1 + z)} \tag{17}$$

and quantum relation $p = h/\lambda$ implies $p \propto (1 + z)$

Q: implications for gas vs radiation after recombination?

Thermal History of Cosmic Gas and Radiation

until recombination (CMB formation) $z \geq z_{\rm rec} \sim 1000$
(mostly) hydrogen gas is ionized, tightly coupled to CMB
via Thomson scattering: $T_{\rm cmb} = T_{\rm gas}$

after recombination, before gas decoupling $z_{\rm dec} \sim 150 \lesssim z \leq z_{\rm rec}$

- most gas in the Universe is *neutral*
  but a small "residual" fraction $x_e \sim 10^{-5}$ of e^- remain ionized

- Thompson scattering off residual free e^- ($x_e \sim 10^{-5}$)
  still couples gas to CMB $\rightarrow$ $T_{\rm cmb} = T_{\rm gas}$ maintained
- until about $z_{\rm dec} \sim 150$, when Thomson scattering ineffective,
gas *decoupled*

Q: after decoupling, net effect of 21 cm transition?

radiation transfer along each sightline:

$$I_\nu = I_\nu^{\mathrm{cmb}} \ e^{-\tau_\nu} + I_\nu^{\mathrm{gas}} \ (1 - e^{-\tau_\nu}) \tag{18}$$

with τ_ν optical depth to CMB
in terms of *brightness or antenna temperature* $T_B = (c^2/2k\nu^2) I_\nu$

$$T_b = T_{\mathrm{cmb}} \ e^{-\tau_\nu} + T_{\mathrm{gas}}(1 - e^{-\tau_\nu}) \tag{19}$$

when $T_{\mathrm{gas}} = T_{\mathrm{cmb}}$ (really, $T_{\mathrm{spin}} = T_{\mathrm{cmb}}$)
gas is in equilibrium with CMB: emission = absorption
$\rightarrow T_b = T_{\mathrm{cmb}}$: no net effect from CMB passage through gas

after gas decoupling, before reionization $z_{\mathrm{reion}} \sim 10 \lesssim z \leq z_{\mathrm{dec}}$
separate thermal evolution: $T_{\mathrm{cmb}} \sim E_{\mathrm{peak}} \propto p_{\mathrm{peak}} \propto (1+z)$
but matter has $T_{\mathrm{gas}} \sim p^2/2m \propto p^2 \propto (1+z)^2$
$\rightarrow$ *gas cools* (thermal motions "redshift") *faster than the CMB!*

Q: net effect of 21 cm transitions in this epoch?

21 cm Radiation in the Dark Ages

before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\rm gas} < T_{\rm cmb}$

$$\delta T_b \equiv T_b - T_{\rm cmb} = (T_{\rm gas} - T_{\rm cmb})_z \, (1 - e^{-\tau_\nu})_z \tag{20}$$

we have $\delta T_b < 0$: gas seen in 21 cm *absorption*

Q: what cosmic matter will be seen this way?

Q: what will its structure be in 3-D?

Q: how will this structure be encoded in δT_b?