Astro 501: Radiative Processes
Lecture 38
April 29, 2013

Announcements:

- **Problem Set 11** last one! *extended* to Wed. May 1
- **Final Exam**
  will consist of 24 hour, take-home problem set
  to be done without collaboration
  due at the end of scheduled exam time
- Please fill out ICES survey! Time is running out!

Last time: collisional excitation

- cross section estimate for atom-atom collisions?
- critical density
  Q: what is it mathematically? physically?
  Q: why is it important? useful?

rate of inelastic collisions $a + c$ per volume is

$$\frac{d\mathcal{N}_{\text{collisions}}}{dV\ \ dt} \equiv \dot{n}_{ac\rightarrow a'c'} = \langle \sigma_{ac} v \rangle\ \ n_a\ n_c \tag{1}$$

collision rate *per* a is

$$\Gamma_{ac\rightarrow a'c'} = \frac{\dot{n}_{ac\rightarrow a'c'}}{n_a} = \langle \sigma_{ac} v \rangle\ \ n_c \tag{2}$$

mission and by collisions are equal when

$$\langle \sigma_{10} v \rangle\, n_c = (1 + f_\nu) A_{10} \tag{3}$$

this defines a **critical density**

$$n_{c,\text{crit}} = \frac{(1 + f_\nu) A_{10}}{\langle \sigma_{10} v \rangle} \tag{4}$$

Electron-Atom Collisions

consider inelastic collisions of *atoms* with
thermal *electrons* at T

Q: geometric cross section of electron?

Q: quantum mechanical lengthscale for non-relativistic e*?*

Q: collision cross section, reaction rate estimate for e *at* T*?*

electrons are quantum particles
with de Broglie wavelength $\lambda_{\mathrm{deB}} = h/p_e = h/m_e v$
so thermal electrons have a *thermal de Broglie wavelength*

$$\lambda_{\mathrm{deB},e} \sim \frac{h}{m_e v_T} = \frac{h}{\sqrt{m_e\, kT}} = 52\ \text{Å} \ \left(\frac{1000\ \mathrm{K}}{T}\right)^{1/2} \tag{5}$$

so for T of interest, $\lambda_{\mathrm{deB},e} \gg r_{\mathrm{atom}} \sim a_0$

so to order-of-magnitude, atom-electron cross section is

$$\sigma_{ae} \sim \pi \lambda_{\mathrm{deB},e}^2 = \pi \frac{h^2}{m_e\, kT} \tag{6}$$

and thermal collision rate coefficient is

$$\langle \sigma_{ae} v \rangle \sim v_T\ \sigma_{ae} \sim \frac{h^2}{m_e \sqrt{m_e kT}} \propto \frac{1}{\sqrt{T}} \tag{7}$$

to order of magnitude,

$$\langle \sigma_{ae} v \rangle \sim v_T \; \sigma_{ae} \sim \frac{h^2}{m_e \sqrt{m_e k T}} \propto \frac{1}{\sqrt{T}} \tag{8}$$

useful to define dimensionless **collision strength** $\Omega_{u\ell}$
for electron-atom transition $u \rightarrow \ell$:

$$\langle \sigma_{ae} v \rangle \equiv \frac{h^2}{(2\pi m_e)^{3/2} (kT)^{1/2}} \frac{\Omega_{u\ell}}{g_u} \tag{9}$$

in principle, $\Omega_{u\ell}(T)$ depends on T
but in practice, nearly constant with T,
and values are in range $\Omega_{u\ell} \sim 1 - 10$

Nebular Diagnostics

Nebular Diagnostics

consider a *diffuse nebula*: low-density gas
generally irradiated by stellar and/or stellar objects

Q: expected optical spectrum?

`www: example spectra`

Q: how to use spectra to measure T? density?

Nebular Temperature Diagnostic

diffuse nebulae: usually optically thin in visible band
continuum radiation is not blackbody
and reprocesses stellar radiation with $T \sim 3000 - 30,000$ K
spectra dominated by *emission lines*
$\rightarrow$ need to use these to determine T, density

temperature diagnostics: *pairs of lines* that are

- energetically accessible: $E_{u\ell} \lesssim kT$
- widely spaced: $\Delta E \sim kT$

consider an idealized *3-level atom*

- ground state $n = 1$, excited states $n = 2, 3$
- excited states populated by *electron collisions* at volume rate $\dot{n}_{13} = \langle \sigma_{e1\rightarrow 3} v \rangle \; n_1 \; n_e \propto \Omega_{13} e^{-E_{13}/kT} n_1 n_e$
- probability for $3 \rightarrow 1$ transition: $A_{31}/(A_{31} + A_{32})$ *Q: why?*

if electron density $n_e \ll n_{e,\text{crit}}$

then de-excitation occurs via spontaneous emission

and integrated emissivity from the $3 \to 1$ transition is

$$j_{31} = E_{31}\dot{n}_{13}\frac{A_{31}}{A_{31}+A_{32}} = E_{31}\langle\sigma_{31}v\rangle\frac{A_{31}}{A_{31}+A_{32}}n_1 n_e \qquad (10)$$

and from the $3 \to 1$ transition is

$$j_{21} = E_{21}\left(\langle\sigma_{12}v\rangle + \langle\sigma_{13}v\rangle\frac{A_{32}}{A_{31}+A_{32}}\right)n_1 n_e \qquad (11)$$

thus the emissivity ratio and hence line ratio is

$$\frac{j_{31}}{j_{21}} = \frac{A_{31}E_{31}}{A_{32}E_{32}}\frac{(A_{31}+A_{32})\langle\sigma_{31}v\rangle}{(A_{31}+A_{32})\langle\sigma_{21}v\rangle + A_{31}\langle\sigma_{31}v\rangle} \qquad (12)$$

$$= \frac{A_{31}E_{31}}{A_{32}E_{32}}\frac{(A_{31}+A_{32})\Omega_{31}e^{-E_{32}/kT}}{(A_{31}+A_{32})\Omega_{21} + A_{31}\Omega_{31}e^{-E_{32}/kT}} \qquad (13)$$

excellent! Q: Why?

3-level atom line ratio

$$\frac{j_{31}}{j_{21}} = \frac{A_{31}E_{31}}{A_{32}E_{32}} \frac{(A_{31}+A_{32})\Omega_{31}e^{-E_{32}/kT}}{(A_{31}+A_{32})\Omega_{21}+A_{31}\Omega_{31}e^{-E_{32}/kT}} \tag{14}$$

depends only on T and atomic properties

so: for appropriate systems

- measure line ratio
- look up the atomic properties
- use observed ratio to solve for T!

Interstellar Dust

Strange Things are Afoot at the Circle K

E. E. Barnard (1907, 1910)

noted "vacancy" on the sky, now called "*dark clouds*"

`www: Barnard's images; modern images of dark clouds`

"It almost seems to me that we are here brought face to face with a phenomenon that may not be explained with our present ideas of the general make-up of the heavens." –Barnard 1907

R. J. Trumpler (1930)

compared distance measures to open star clusters

- *luminosity distance* $d_L = \sqrt{L/4\pi F}$
- *angular diameter distance* $d_A = R/\theta$

  Q: but how did he know luminosity L? physical size R?

`www: Trumpler data`

$\Rightarrow$ found that for distant clusters, $d_L > d_A$

also found stellar *colors* increasingly *red* with larger distance

Q: possible explanations? implications?

Cosmic Dust: Evidence

Trumpler 1930: found increasing ratio $d_L/d_A > 1$ with distance, with

$$\frac{d_L}{d_A} \propto \frac{1}{R}\sqrt{\frac{L}{F}} \tag{15}$$

observed d_L/d_A increase requires distant clusters are either:

- progressively more luminous – but why?
- progressively smaller – but why?
- anomalously dimmer, i.e., flux F increasingly *attenuated*

increased reddening with distance $\rightarrow$ not a geometric effect
$\rightarrow$ space filled with medium that *absorbs* and *reddens* light
$\Rightarrow$ **interstellar dust**

Interstellar Extinction

consider an object of *known flux density* F_λ^0
Q: candidates?

due to dust absorption, *observed flux* density is $F_\lambda < F_\lambda^0$
quantify this via **extinction** A_λ

$$\frac{F_\lambda}{F_\lambda^0} = 10^{-(2/5)A_\lambda} \tag{16}$$

compare optical depth against dust absorption:
$F_\lambda/F_\lambda^0 = e^{-\tau_\lambda}$, so

$$A_\lambda = \frac{5}{2}\log_{10} e^{\tau_\lambda} = 2.5\ \log_{10}(e)\ \tau_\lambda = 1.086\ \tau_\lambda \text{ mag} \tag{17}$$

extinction measures optical depth

Q: what does reddening imply about A_λ?

Reddening

observed reddening implies A_λ stronger for shorter λ
$\rightarrow$ increases with $1/\lambda$

for source of known F^0_λ, can measure this
`www: extinction curve as a function of wavelength`
observed trend: "*reddening law*"
- general rise in A_λ vs $1/\lambda$
- broad peak near $\lambda \sim 2200$ AA $= 0.2\mu$ m

Q: implications of peak? of reddening at very short λ?

in photometric bands, define *redding*: for: B and V

$$E(B-V) \equiv A_B - A_V \tag{18}$$