

Astro 501: Radiative Processes

Lecture 2

Jan 16, 2013

Announcements:

- Pick up syllabus

Last time:

★ Overview and Appetizer

Today: The great work begins!

★ Radiation Transfer

Program Notes: ASTR 501 Bugs/Features

- ▶ notes online—but come to class!
some people find it convenient to print 4 pages/sheet
- ▶ class \in diverse backgrounds: ask questions!
- ▶ Socratic questions
- ▶ typos/sign errors
Dirac story
please report errors in lectures
pretty please promptly report errors in problem sets;
if need be, errata posted and emailed

Radiation Transfer

Warmup: Electromagnetic Radiation

want to define terms and try to be clear about assumptions

What is light?

classically: electromagnetic waves

- move at speed c in vacuum
- monochromatic wavelength and frequency related by $\lambda\nu = c$
- visible band roughly $\lambda_{\text{vis}} \sim 400 - 700 \text{ nm}$

quantum mechanically: photons

- electromagnetic quanta: massless, spin-1 particles
- Planck: energy $E_\gamma = h\nu = hc/\lambda$
- but Einstein: $E^2 - (cp)^2 = (mc^2)^2$, and here $m_\gamma = 0$, so momentum $p_\gamma = E_\gamma/c = h/\lambda$

⊢

→ particle/wave duality

Q: when is each description appropriate?

CSI: Cosmos

Astronomy is (mostly) a “forensic” science

- usually: “can look but can’t touch”

Q: exceptions?

- usually: can’t repeat the experiment (big bang in lab?)

Astronomy is also expensive! And publicly funded!

to get taxpayers money’s worth, need to extract all possible info from the signals that arrive in our detectors

Q: very broadly, what is (are) the messenger(s)?

Q: what is (are) the signal(s)?

Astronomical Messengers

Solar System

can (sometimes!) bring matter into lab!

e.g.: terrestrial, lunar (and Martian?) rocks; meteorites

also solar wind and interplanetary dust return missions

- gives precise, detailed information on composition, age
- but lab samples unavailable even for most of Solar System

the rest of the Universe

must catch and decode particles arriving from afar

- *EM radiation* – the vast bulk of available signal
- but EM is not all! other “messengers” now accessible!

cosmic rays, neutrinos, gravity waves

○

...and perhaps dark matter?

The Electromagnetic Window to the Cosmos

in this course:

we will focus mostly on *EM radiative processes*

→ but much the technology we will build
also applies to other messengers

*Q: very broadly, what devices/methods exist to
detect EM signals?*

Q: very broadly, what do the detectors measure?

Detecting EM Radiation

historically:

- until 19th century, astro-detector = human eye
- photographic film revolutionized astronomy

today:

- CCDs measure *photons* via the photoelectric effect
span IR, optical, UV, X-ray
- bolometers and radiometers measure *energy*
in mm, radio

Q: what are astronomical (EM) observables?

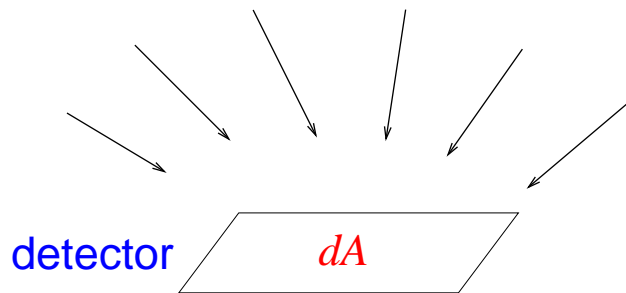
Electromagnetic Observables in Astronomy

In part drawn from http://background.uchicago.edu/%7Ewhu/Courses/ast305_10.html

- *flux*: energy or photon flow
really: measure *energy* accumulated over exposure time
- *spectrum*: flux distribution in
different energy (frequency, wavelength) bands
- *direction on sky*
- *solid angle* – *if* source is *resolved* on sky
i.e., angular size $>$ scope angular resolution
- *phase* information *if* measured (radio, optical)
- *polarization* (linear, circular) *if* measured
- *light curve* = time history of observables
if measurements span multiple epochs

Energy Flow

idealized detector of area dA
receives all incident radiation (all directions, all ν)
over exposure time dt



energy received in exposure $d\mathcal{E}$ depends on detector
because $d\mathcal{E} \propto dA dt$ *Q: why?*

- ⊞ thus energy received is detector-dependent via dA
Q: how to remove detector dependence?

Energy Flux

independent of detector, and
intrinsic to source and distance: **energy flux** (or just “flux”)

$$F = \frac{dE}{dA dt} = \frac{d\text{Power}}{d\text{Area}} \quad (1)$$

cgs units: $[F] = [\text{erg cm}^{-2} \text{ s}^{-1}]$

Note:

- usually, detector really measures energy received during exposure, i.e., time-integrated flux

fluence $f = d\mathcal{E}/dA = \int_{\delta t} F(t) dt$

derive $F_{\text{obs}} = f/\delta t = \text{time-avg flux during exposure}$

- if measure *photon counts* dN , sometimes report **photon** or **number flux** $\mathcal{F} = dN/dA dt$

cgs units: $[\Phi] = [\text{photons cm}^{-2} \text{ s}^{-1}]$

Inverse Square Law

consider spherical source of size R
emitting isotropically
with constant power L (“luminosity”)

at radius $r > R$ (outside of source)
area $A = 4\pi r^2$, and flux is

$$F = \frac{L}{4\pi r^2} \quad (2)$$

inverse square law

Q: what principle at work here?

Q what implicitly assumed?

if L known somehow (Q: how?): “standard candle”
then measure F and infer **luminosity distance**

$$d_L = \sqrt{\frac{L}{4\pi F}} \quad (3)$$

Inverse Square Law

Ultimately relies on *energy conservation*

→ energy emitted $d\mathcal{E}_{\text{emit}} = L dt_{\text{emit}}$ from source
is same as energy observed $d\mathcal{E}_{\text{obs}} = F A dt_{\text{obs}}$

Thus: inverse square derivation assumes

- no emission, absorption, or scattering outside of source
we will soon consider these in detail
- no relativistic effects (redshifting, time dilation)
- Euclidean geometry—i.e., no spatial curvature,
usually fine unless near strong gravity source

Note: inverse square suggests similarity with electrostatics
and invites use of Gauss' Law

for fun: think about why things aren't so simple for radiation

13

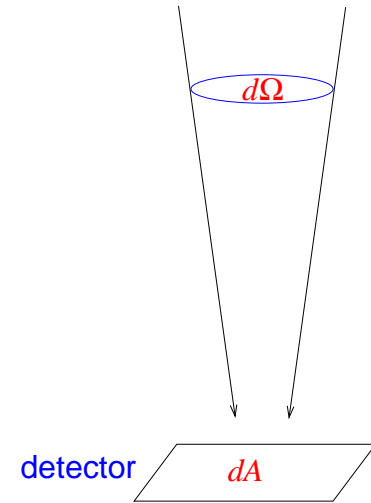
Q: what if we want to concentrate on one part of sky?

Q: how do we change measurement? what is new observable?

Intensity or Surface Brightness

Isolate small region (solid angle $d\Omega$) of sky by introducing a *collimator*

If source is extended over this region sky, energy flow received depends on collimator acceptance $d\Omega$: $d\mathcal{E} \propto dA dt d\Omega$



so define flux per unit “surface area” of sky:

intensity or **surface brightness** (or sometimes just “brightness”)

$$I = \frac{d\mathcal{E}}{dt dA d\Omega} \quad (4)$$

cgs units: $[I] = [\text{erg cm}^{-2} \text{s}^{-1} \text{sr}^{-1}]$, with sr = steradian

Q: what has been implicitly assumed?

have assumed light travels in straight lines: “rays”

- for infinitesimal solid angle $d\Omega$, collimator selects a small “bundle” or “pencil” (Chandrasekhar) of rays
- intensity I describes *one* individual ray (one direction) while flux describes *all* rays (all directions)

thus: implicitly adopted *geometric optics* approximation:

we have ignored diffraction effects

good as long as system scales $\gg \lambda$

So far, radiation direction is normal to detector surface

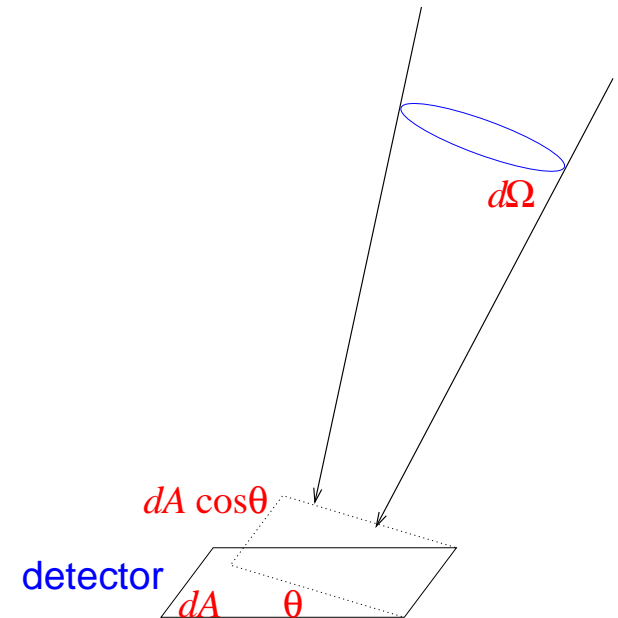
Q: what if it is at an angle?

Intensity for Arbitrary Incidence

if incidence is at angle θ to normal
radiation “sees” detector with
projected normal area $dA_{\perp} = \cos \theta dA$

where unit direction vectors give

$$\cos \theta = \hat{n}_{\text{rad}} \cdot \hat{n}_{\text{detector}}$$



and thus energy flow onto detector has geometric factor

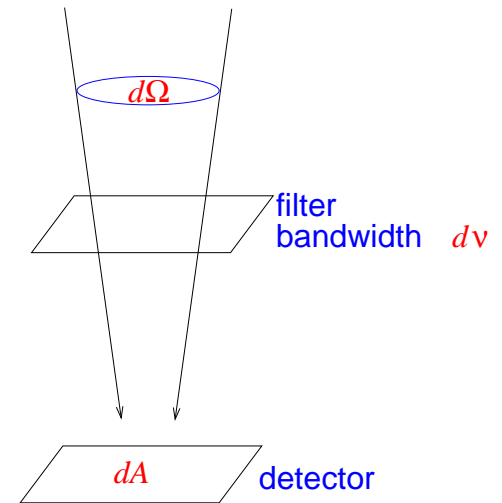
$$d\mathcal{E} = I \cos \theta dA dt d\Omega \quad (5)$$

Q: What if we are interested in the spectrum?

Specific Intensity

introduce a filter, or grating to disperse by λ
so detector receives small range of frequencies
in $(\nu, \nu + d\nu)$: **monochromatic** frequency ν
with **bandwidth** $d\nu$

energy received: $d\mathcal{E} \propto dA \, dt \, d\Omega \, d\nu$



define **specific intensity** or **spectral energy distribution (SED)**

$$I_\nu = \frac{d\mathcal{E}}{dt \, dA \, d\Omega \, d\nu} \quad (6)$$

cgs units: $[I_\nu] = [\text{erg cm}^{-2} \text{s}^{-1} \text{sr}^{-1}]$

- a less compact but more explicit notation is $dI/d\nu$
- can identify monochromatic flux by λ or photon energy E
and thus can also define $I_\lambda = dI/d\lambda$ and $I_E = dI/dE$

Net Flux

flux measures rays in all direction
passing through surface of normal (unit vector \hat{n}_{surf})

the contribution to the specific flux through area dA
from a ray with solid angle $d\Omega$,
coming from direction \hat{n}_{ray} , is:

$$dF_\nu = I_\nu \hat{n}_{\text{ray}} \cdot \hat{n}_{\text{surf}} d\Omega = I_\nu \cos \theta d\Omega \quad (7)$$

and thus the *total* or *net* specific flux
sums over all solid angles

$$F_\nu = \int I_\nu \cos \theta d\Omega \quad (8)$$

Note: in general, I_ν varies in different directions
e.g., in spherical coords, $I_\nu(\theta, \phi)$
→ an then integral for F_ν is non-trivial

Mean Intensity

the direction-averaged **mean or average intensity** is

$$J_\nu = \langle I_\nu \rangle \quad (9)$$

$$= \frac{\int I_\nu d\Omega}{\int d\Omega} \quad (10)$$

$$= \frac{1}{4\pi} \int I_\nu d\Omega \quad (11)$$

note that here, oppositely-directed rays do *not* cancel

(this is a *scalar* average = undirected)

unlike flux which has an associated direction (normal)

but important special case:

if I_ν is *same* in all directions: *isotropic*

- ¹⁹ Consider isotropic radiation incident on two-sided detector
Q: what's the net flux?

Isotropic Radiation

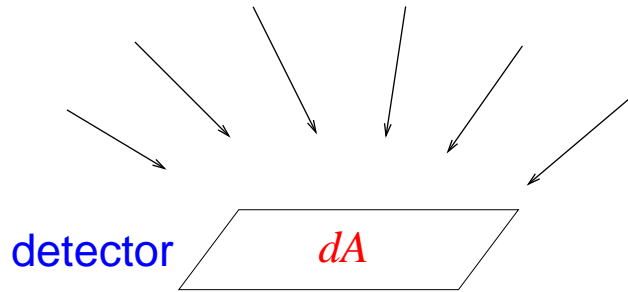
isotropy: $I_\nu(\theta, \phi) = I_\nu^{\text{iso}}$ indept of θ and ϕ
“looks the same in all directions”

on *two-sided detector*, net flux sums over *all* solid angles

$$\begin{aligned} F_{\nu, 2\text{-sided}}^{\text{iso}} &= I_\nu^{\text{iso}} \int_{4\pi} \cos \theta \, d\Omega = I_\nu^{\text{iso}} \int_0^{2\pi} d\phi \int_{-1}^1 \cos \theta \, d\cos \theta \\ &= 2\pi I_\nu^{\text{iso}} \int_{-1}^1 \mu \, d\mu = \pi I_\nu^{\text{iso}} [\mu^2]_{-1}^{+1} = 0 \end{aligned} \quad (12)$$

where in spherical coords $d\Omega = \sin \theta \, d\theta \, d\phi = d\cos \theta \, d\phi$
and where $\theta \in [0, \pi]$, $\cos \theta = \mu \in [-1, +1]$, and $\phi \in [0, 2\pi]$

20 Q: why physically is $F_{\nu, 2\text{-sided}} = 0$?



on *one-sided detector = plane* $\theta \in (0, \pi/2)$:

$$F_{\nu,1\text{-sided}}^{\text{iso}} = \pi I_{\nu}^{\text{iso}} \left[\mu^2 \right]_0^{+1} = \pi I_{\nu}^{\text{iso}} \quad (13)$$

a factor π larger than naive Gauss' law result

Q: *why?*