Astronomy 507   Spring 2014
Problem Set #7: The Final Frontier

This Problem Set takes the place of the final exam, and is *open book and open notes, and open web.* **You may not collaborate and your work must be entirely your own.**

Due on Compass, or to the instructor, on or before: Thursday, May 15, 4:30pm
Total points: 10+1

1. *Redshift Evolution in Real Time as a Probe of Cosmic Expansion History.* In class we showed that redshifts are related to the cosmic scale factor at photon emission and observation via

$$z = \frac{a(t_{\rm obs})}{a(t_{\rm em})} - 1 \tag{1}$$

where $a_{\rm obs} = 1$ for present-epoch observations of interest to us. One usually thinks of the redshift of an object at fixed comoving distance r as a fixed measure equivalent to r, and/or a fixed measure of the emission epoch. While this is true for most practical purposes, it is not strictly correct. Since the 1960's work of Alan Sandage[1] and UIUC's own George McVittie[2] , it has been known that the time *change* of redshifts pose a potentially powerful test of cosmology generally and of cosmic acceleration (and hence dark energy) particularly.

(a) **[1 point]** Starting with eq. (1), derive the McVittie equation for the observed evolution of redshift for an object

$$\frac{dz}{dt_{\rm obs}} = (1+z)\,H(t_{\rm obs}) - H(t_{\rm em}) = (1+z)\left[1 - \frac{1}{1+z}\frac{H(t_{\rm em})}{H_0}\right]\frac{1}{t_{H,0}} \tag{2}$$

where $H(t)$ is the expansion rate evaluated at (and observed at) time t.

(b) **[1 point]** Show that $dz/dt_{\rm obs} = 0$ for an "coasting" universe which has no acceleration, i.e., an expansion with $\ddot{a} = 0$. This implies that $dz/dt_{\rm obs}$ is a probe of cosmic acceleration/deceleration.

For a matter-only universe show that $dz/dt_{\rm obs} < 0$, while for a Λ-dominated universe show that $dz/dt_{\rm obs} > 0$. Interpret these results physically.

(c) **[1 point]** If we monitor the spectrum of an object at fixed comoving distance over a time δt, then the wavelength $\lambda_{\rm obs} = (1+z)\lambda_{\rm em}$ of spectral feature will drift by a fractional amount $\delta\lambda_{\rm obs}/\lambda_{\rm obs} = \lambda_{\rm em}/\lambda_{\rm obs}\, dz/dt_{\rm obs}\,\delta t = (1+z)^{-1} dz/dt_{\rm obs}\,\delta t$, equivalent to a Doppler velocity drift of $\delta v = c\delta z/(1+z)$.

Sensitive spectroscopic techniques have been developed to find planets via the small change in the Doppler shift of the parent star due to the gravitational

[1] Sandage, A. 1962, ApJ, 136, 319. This paper also calculates the time evolution of the *luminosity* of a source at fixed comoving distance. I leave it to the reader to see why this would be an even more difficult thing to measure than the time-change of redshift.

[2] McVittie, G. C. 1962 ApJ, 136, 334. Somewhat oddly, this is a separately-authored appendix to the Sandage (1962) paper, in which McVittie extends Sandage's analysis for general combinations of $\Omega_{\rm m}$ and Ω_Λ.

influence of the planet. Current methods can detect velocity changes down to about $\delta v_{\rm obs} \sim 1$ m/s over timescales as long as $\delta t \sim 10$ yr.

Using the above results, find the Doppler velocity drift of a $z = 3$ object over a timescale of $\delta t = 10$ yr, in a $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$ cosmology. Can this be observed with current techniques? What complications might make this measurement and its interpretation difficult? (*Hint*: real objects at $z = 3$ are not point sources, and do have internal motions.) How might some of these difficulties be overcome?

Such an observational campaign is sometimes known as the Sandage-Loeb[3] test, which has been known of for decades (thanks to Sandage and McVittie) but has received a revival of accelerated interest recently[4].

(d) **[1 bonus point]** Imagine it is (or becomes) possible to make reliable measurements of redshift drifts over a substantial redshift range, say $z = 0.5 - 3$. Explain how such measurements could be used to test cosmology in general, and dark energy models in particular.

2. *Baryon Acoustic Oscillations*

(a) **[1 point]** Explain qualitatively what BAO lengthscale is, and why it is a standard ruler.

(b) **[1 point]** Write an expression for the BAO comoving lengthscale $r_{\rm bao}$. *Hint:* it involves an integral of the cosmic soundspeed c_s. Briefly explain your choices for the bounds of integration.

(c) **[1 point]** Use your expression from part (b) to evaluate the BAO scale using the approximation $c_s \approx c/\sqrt{3}$, and express your answer in Mpc.

Compare this scale to that at which structures are nonlinear today, and comment on the significance of the mismatch.

(d) **[1 point]** Imagine we observe a large matter overdensity at redshift z. Along the same sightline, the BAO feature should appear as overdensities displaced in redshift by $\pm\Delta z$.

Find an expression for Δz; you may assume that $\Delta z \ll z$.

How can we test that the features detected this way are really due to BAO?

(e) **[1 point]** Briefly explain what cosmological information we gain if we measure Δz in part (d) for objects at many redshifts z. Be sure to indicate why it would be useful to do this measurement at multiple z.

3. *The Cosmic Star-Formation Rate.* Measuring and understanding the history of cosmic star formation is a major topic in cosmology today. As discussed in class, the cosmic star formation rate $\dot{\rho}_\star(z)$ is now fairly well-determined out to redshifts $z \sim 2$.

(a) **[1 point]** A famous evaluation of the cosmic star formation rate appears in Hopkins & Beacom (2006, ApJ 451, 142), linked in the final lecture webpage.

[3] Loeb, A. 1998, ApJL, 499, L111

[4] See, e.g., Lake, K. 1981, ApJ, 247, 17; Lake, K. 2007, PRD, 76, 063508; Balbi, A., & Quercellini, C. 2007, MNRAS, 382, 1623; Uzan, J.-P., Bernardeau, F., & Mellier, Y. 2008, PRD, 77, 021301;

Consult Hopkins & Beacom's Figure 1 and the surrounding discussion, and find the value $\dot{\rho}_\star(0)$ of the cosmic star-formation rate today, i.e., at $z = 0$. To see if this makes sense, consider the quantity $\bar{\psi} = \dot{\rho}_\star(0)/n_{\rm gal}$, where $n_{\rm gal}$ is a measure of the number density of galaxies at $z = 0$, for example as you found in Problem Set 1.

Explain why $\bar{\psi}$ should be a measure of the average star-formation rate of a typical galaxy today.

Then evaluate $\bar{\psi}$ using the Hopkins & Beacom value for $\dot{\rho}_\star(0)$, and $n_{\rm gal}$ you found in Problem Set 1 (or from class notes). Compare your result to the Milky Way star-formation rate $\psi_{\rm MW} \simeq 1\ M_\odot/{\rm yr}$ and comment.

(b) **[1 point]** Out to redshift $z \sim 1$, the cosmic star-formation rate grows roughly as $\dot{\rho}_\star \propto (1+z)^3$, so that $\dot{\rho}_\star(z) = (1+z)^3\ \dot{\rho}_\star(0)$. Assuming this dependence, integrate this rate over cosmic time, i.e., find $\rho_{\rm sf} = \int_{t(z=1)}^{t_0} \dot{\rho}_\star\, dt = \int_{a(z=1)}^{1} \dot{\rho}_\star\, da/(aH)$, using the expansion rate for a matter-dominated universe with $\Omega_{\rm m} = 0.3$. Also find $\Omega_{\rm sf}$. What physically should $\rho_{\rm sf}$ and $\Omega_{\rm sf}$ measure?

Compare your results with the baryon density parameter $\Omega_{\rm baryon}$, and the density parameter for stellar luminous matter $\Omega_{\rm lum}$ found in Problem Set 1 and in class notes. Comment on the results.