Astro 507

Lecture 11

Feb. 14, 2014

Announcements:

- Preflight 2 was due this morning
- Problem Set 2 out today, due in class next Friday

Last time: Robertson-Walker metric

Q: what is it?

Q: parameters? variables?

Q: what coordinate system?

Q: what does it mean physically?

Friedmann-Lemaître-Robertson-Walker Metric

Robertson & Walker:
maximal symmetry imposes metric form

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2/R^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right)$$

where cosmic geometry encoded via κ:

$$\kappa = \begin{cases} +1 & \text{pos curv: “spherical”} \\ 0 & \text{flat: “Euclidean”} \\ -1 & \text{neg curv: “hyperbolic”} \end{cases} \tag{1}$$

gives **interval** for neighboring events

Consider event pairs (t, r, θ, ϕ) and $(t + \delta t, r, \theta, \phi)$

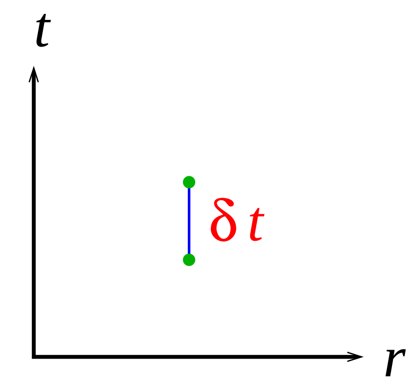
- *Q: what is* ds^2*?*
- *Q: what does* ds^2 *tell us physically?*

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2/R^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right)$$

$$= \text{(apparent elapsed time)}^2 - \text{(apparent distance)}^2$$

event separation $(dt, dr, d\theta, d\phi) = (\delta t, 0, 0, 0)$

- spatial coords unchanged:
  events at rest w.r.t. FO frame
- FO's apparent elapsed time is



$$ds = \delta t$$

lesson: dt is FO clock rate = **cosmic time**

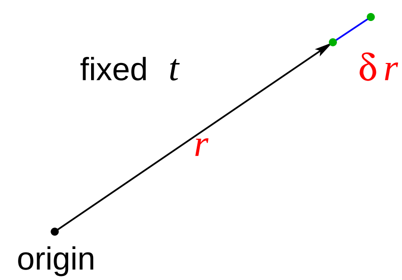
now consider pair: (t, r, θ, ϕ) and $(t, r + \delta r, \theta, \phi)$

Q: what is ds^2? physical significance?

$$ds^2 \;=\; dt^2 - a(t)^2\left(\frac{dr^2}{1-\kappa r^2/R^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right)$$
$$= \;(\text{apparent elapsed time})^2 - (\text{apparent distance})^2$$

event separation $(dt, dr, d\theta, d\phi) = (0, \delta r, 0, 0)$

- time coords unchanged:
  events simultaneous in FO frame
  $\Rightarrow ds^2$ gives $-(\text{apparent distance})^2 = -d\ell^2$
- separation is radial only
  $\Rightarrow$ FO finds physical radial distance is



$$d\ell = d\ell_r = a(t)\;\frac{\delta r}{\sqrt{1-\kappa r^2/R^2}}$$

Q: lessons?

for event separation $(dt, dr, d\theta, d\phi) = (0, \delta r, 0, 0)$
physical radial distance is

$$d\ell = d\ell_r = a(t)\ \frac{\delta r}{\sqrt{1 - \kappa r^2/R^2}} \tag{2}$$

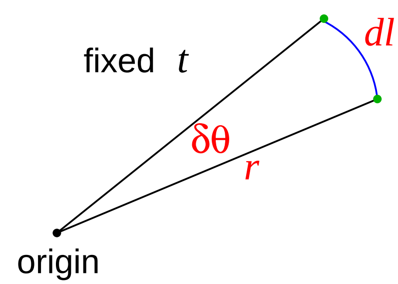
lessons:

- radial distances sensitive to curvature R
  not directly measured by r unless $\kappa = 0$
- radial distances evolve as $a(t)$ – of course!
- cosmoving radial distance is $d\ell_{r,\mathrm{com}} = \delta r/\sqrt{1 - \kappa r^2/R^2}$

now consider pair (t, r, θ, ϕ) and $(t, r, \theta + \delta\theta, \phi)$
Q: what is ds^2? physical significance?

$$
\begin{aligned}
ds^2 &= dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2/R^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \right) \\
&= (\text{apparent elapsed time})^2 - (\text{apparent distance})^2
\end{aligned}
$$

event separation $(dt, dr, d\theta, d\phi) = (0, 0, \delta\theta, 0)$



- time coords unchanged: events give FO distance
- separation is angular only
  $\Rightarrow$ FO finds distance = *arc length*

$$d\ell = d\ell_\theta = a(t)\ r\ \delta\theta \qquad (3)$$

- arc lengths depend on radial coord r
  $\neq$ physical radial distance unless $\kappa = 0$
- arc lengths evolve as $a(t)$ – of course!
- comoving angular distance is $d\ell_{\theta,\mathrm{com}} = r\ \delta\theta$
- similarly, $d\ell_\phi = a(t)\ r\ \sin(\theta)\ \delta\phi$

consider a region with

- $dt = dr = 0$, and
- $d\theta, d\phi \neq 0$

Q: physical significance?

Q: relevant quantity?

consider a region with

- $dt = 0$
- $dr, d\theta, d\phi \neq 0$

Q: physical significance?

Q: relevant quantity?

region with $dt = dr = 0$ and $d\theta, d\phi \neq 0$:

- fixed time coordinate: events give spatial separation
- fixed radial coordinate r: separation is angular only
- both angular coordinates vary: sweeps 2-D region on sphere
- *area* of region is

$$dA = d\ell_\theta \; d\ell_\phi = a(t)^2 \; r^2 \; \sin(\theta) \; d\theta \; d\phi = a(t)^2 \; r^2 \; d\Omega \qquad (4)$$

lesson:

- *solid angle* is usual $d\Omega = dA/a(t)^2 r^2 = \sin(\theta) \; d\theta \; d\phi$
- physical area of sphere at r is $A_{\rm sph} = 4\pi \; a(t)^2 \; r^2$

region with $dt = 0$ and $dr, d\theta, d\phi \neq 0$:

- sweep out 3-D *spatial volume* on sphere

∞

$$dV = d\ell_r \; d\ell_\theta \; d\ell_\phi = a(t)^3 \; \frac{r^2}{\sqrt{1 - \kappa r^2/R^2}} \; dr \; d\Omega \qquad (5)$$

Cosmic Time Dilation

⋆ for *any* pairs of photons

$$\frac{\delta t_{\rm obs}}{\delta t_{\rm em}} = \frac{a(t_{\rm obs})}{a(t_{\rm em})} = \frac{1+z_{\rm em}}{1+z_{\rm obs}}$$

and since $a(t_{\rm obs}) > a(t_{\rm em})$

$\rightarrow$ $\delta t_{\rm obs} > \delta t_{\rm em}$

$\rightarrow$ distant happenings appear in slow motion!

$\rightarrow$ time dilation!

Note: effect depends only on redshift, not on geometry

cosmic time dilation recently observed!

Q: how would effect show up?

Q: why non-trivial to observationally confirm?

`www: cosmic time dilation evidence`

Cosmological Redshifts Revisited

consider light with wavelength λ, frequency $\nu = c/\lambda$
FO emits wavecrests with period $\delta t_{\rm em} = 1/\nu = \lambda/c$

⋆ if photon pairs are wavecrests, then

$$\frac{\delta t_{\rm obs}}{\delta t_{\rm em}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}}$$

and thus

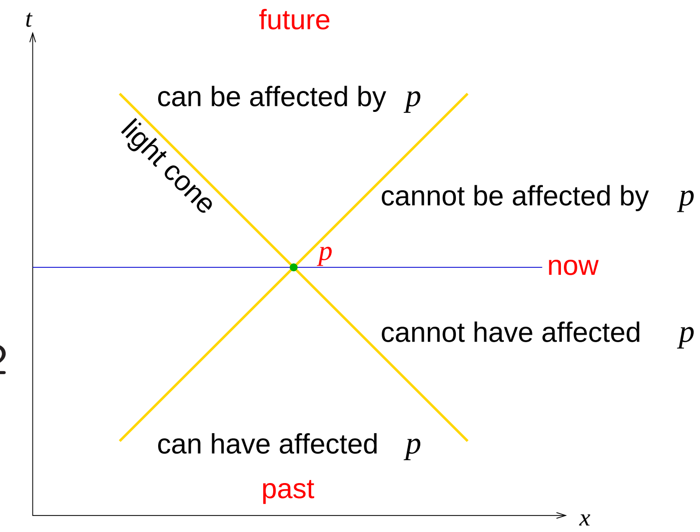
$$\frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \frac{\nu_{\rm em}}{\nu_{\rm obs}} = \frac{a(t_{\rm obs})}{a(t_{\rm em})} = \frac{1+z_{\rm em}}{1+z_{\rm obs}}$$

$\rightarrow$ $\lambda_{\rm obs} > \lambda_{\rm em}$ and $\nu_{\rm obs} < \nu_{\rm em}$
$\rightarrow$ cosmic redshifting!

Note: one-to-one relationships
redshift $z \leftrightarrow$ emission time $t_{\rm em} \leftrightarrow$ comov. dist. at emission $r_{\rm em}$
any/all of these denote a cosmic **epoch**

Cosmic Causality

Recall special relativity (Minkowski space)
$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$



light: $ds = 0$ → *cone* $dt^2 = dx^2 + dy^2 + dz^2$

Now RW metric: $ds^2 = dt^2 - a^2 d\ell^2_{\rm com}$
introduce new time variable η: **conformal time**
defined by $d\eta = dt/a(t)$ (see PS2)

$$ds^2 = a(\eta)^2 \left(d\eta^2 - d\ell^2_{\rm com} \right)$$

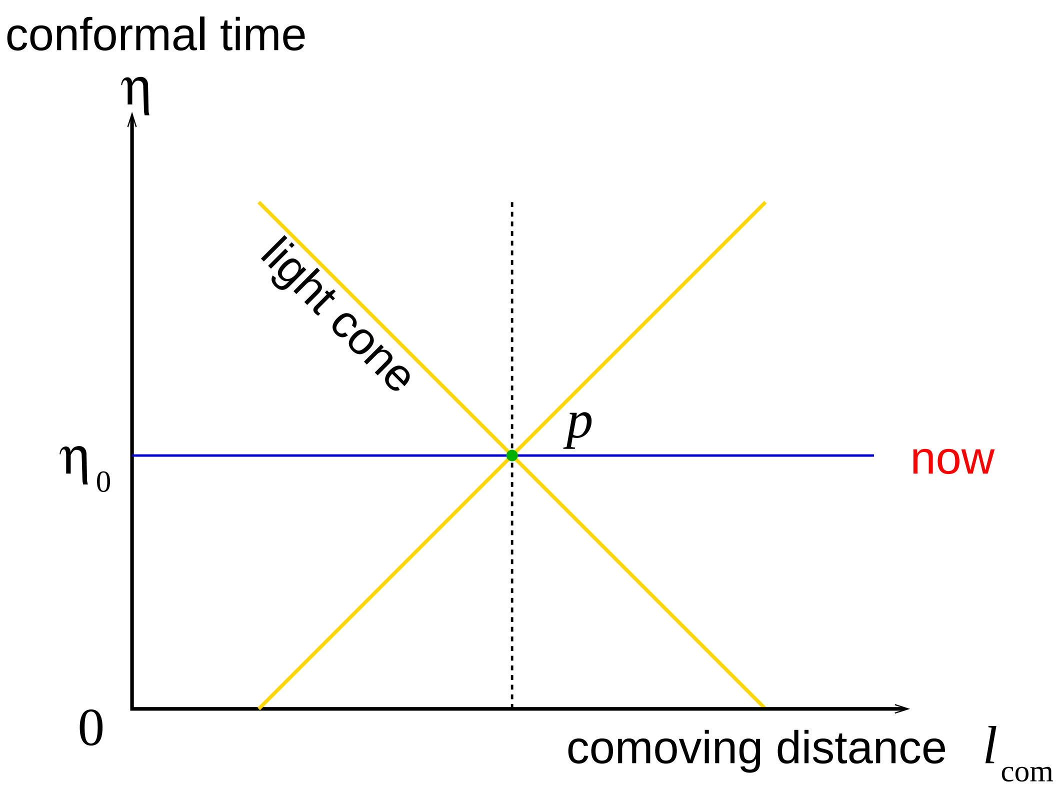
Q: implications?

$$ds^2 = a(\eta)^2 \left(d\eta^2 - d\ell^2_{\text{com}}\right) = a(\eta)^2 \;\times\; \text{(Minkowski structure)}$$

has same features as Minkowski space

$\Rightarrow$ *light cones still defined*

when use comoving lengths and conformal time



For a flat universe ($\kappa = 0$), it's even better:

$$ds^2 = a(\eta)^2 \left(d\eta^2 - dr_{\rm com}^2\right) = a(\eta)^2 \ \times \ \text{(exact Minkowski form)}$$

In either case $\rightarrow$ spacelike, timelike, lightlike divisions same
and in $(\eta, \ell_{\rm com})$ space:
light cone structure the same $\Rightarrow$ *causal structure the same*!

Namely:

- a spacetime point can only be influenced by events in past light cone
- a spactime point can only influence events in future light cone

So far: like Minkowski
New cosmic twist: finite cosmic age
Q: implications for causality?

Causality: Particle Horizon

past light cone at t defined by
photon propagation over cosmic history:

$$\int_{t_{\rm em}=0}^{t_{\rm obs}=t_0} \frac{d\tau}{a(\tau)} = \int_0^{r_{\rm em}} \frac{dr}{\sqrt{1-\kappa r^2/R^2}} \equiv d_{\rm hor,com}(t_0)$$

where $d_{\rm hor,com}$ is comoving distance
photon has traveled since big bang

if $d_{\rm hor,com} = \int_0^t d\tau/a(\tau)$ converges
then only a finite part of U has affected us
$\rightarrow$ $d_{\rm hor}$ defines *causal boundary*
$\rightarrow$ "**particle horizon**"

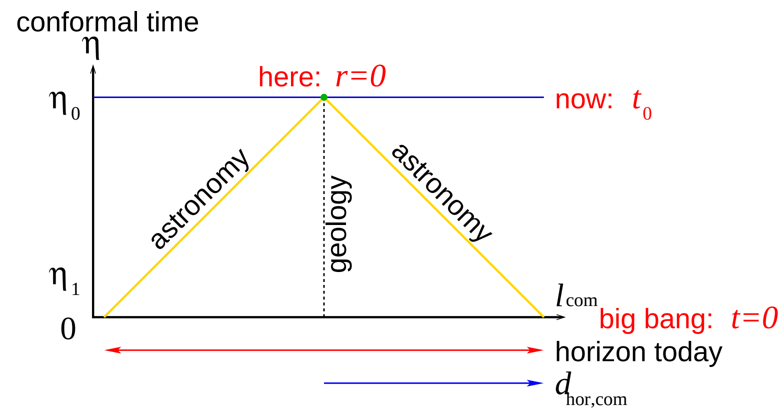
Q: physical implications of a particle horizon?
Q: role of finite age?
Q: sanity check–simple limiting case with obvious result?

Particle Horizons: Implications

our view of the Universe:

★ astronomical info comes from events along *past light cone*

★ *geological info* comes from *past world line*



if particle horizon finite (i.e., $\neq \infty$), then $d_{\text{horiz,com}}$:

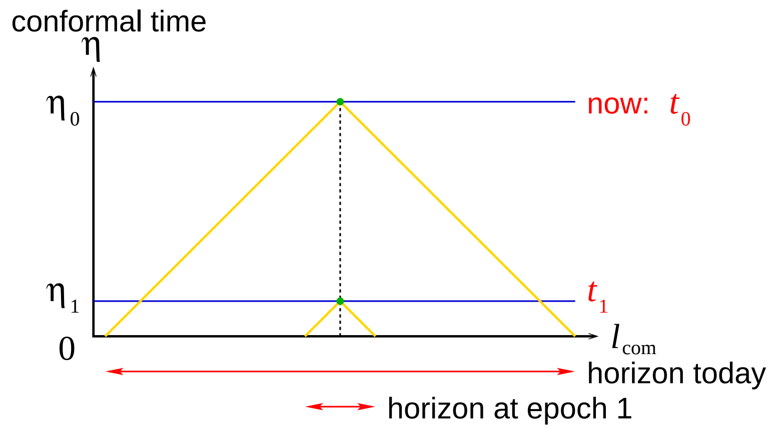
- gives comoving size of **observable universe**
- encloses region which can communicate over cosmic time $\rightarrow$ causally connected region
- sets "zone of influence" over which particles can "notice" and/or affect each each other and local physical processes can "organize" themselves

e.g., shouldn't see bound structures large than particle horizon!

So *is* d_{hor} finite?
depends on details of $a(t)$ evolution as $t \to 0$:
behavior near singularity crucial

will see in PS2:
for matter, radiation domination:

- d_{hor} finite
- and $d_{\text{hor}} \to 0$ for $t \to 0$



Q: implications for CMB?
Hint: observed $T_{\text{CMB}}(\theta, \phi)$ isotropic to 5th decimal place...

will see in coming weeks
▷ inflation (if real!) adds twist!