Astro 507

Lecture 12

Feb. 17, 2014

Announcements:

- **Problem Set 2 due Friday**

  office hours: 3:10-4pm Thurs., or by appt

  for problem 1: see also extras in todays notes

Today: last day of boot camp!

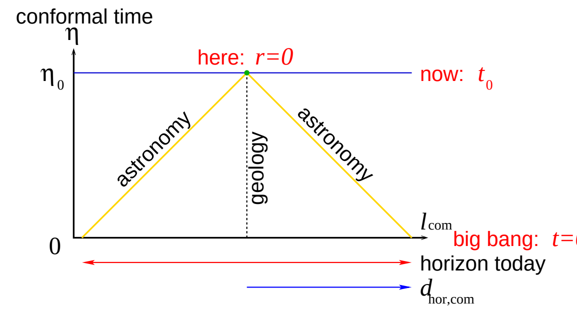
- cosmic distance measures

Last time: lifestyles in a Robertson-Walker universe

▷ cosmic causality

▷ particle horizon *Q: what's that? why important?*

Recap: Photon Propagation in FLRW



for a radial photon (i.e., coming to us)

$$d\ell_{\text{com}} = \frac{dr}{\sqrt{1 - \kappa r^2/R^2}} = \frac{dt}{a(t)} = d\eta$$

Why is η a "*conformal*" time?
conformal transformation = *angle-preserving*

$ds^2 = a(\eta)^2 \ (d\eta^2 - d\ell^2_{\text{com}}) = a(\eta)^2 \ \times$ (Minkowski form)

preserves Minkowski "angles" in spacetime

$\rightarrow$ lightcones keep straight slopes: $d\eta/d\ell_{\text{com}} = 1$ on cone

compare photon trajectory in (t, ℓ_{com}) plane:

at early times: light cone "slope" $dt/d\ell_{\text{com}} = a(t) \ll 1$

Q: what does this look like? why inconvenient?

`www: light cones:` (t, ℓ_{com}) `vs` $(\eta, \ell_{\text{com}})$`plane`

Cosmic Distance Measures

More examples of how spacetime properties impose relationships among observables

Warmup: Newtonian cosmology
another sanity check, limiting case
Q: validity range?

Consider Newtonian cosmo:

- given observed z, what is distance d_{Newt}?
- *Q: good for which z?*
- *Q: complications in full FLRW universe?*

"Newtonian Distance"

Newtonian cosmology:

- small speeds, weak gravity
  ignore curvature

Hubble's Law:

$$H_0 d_{\text{Newt}} \equiv v \simeq cz \tag{1}$$

applicability: $z \ll 1$

solve:

$$d_{\text{Newt}} = \frac{c}{H_0} z = d_H \; z$$

naïve distance is *linear in* z

Distances and Relativity

Basic but crucial distinction, important to remember:

In *Newtonian/pre-Relativity* physics: space is *absolute*

- "distance" has unique, well-defined meaning:
  $\Rightarrow$ Euclidean separation between points
- can think of as "intrinsic" to objects and points

In *Special and General Relativity*: space *not* absolute

- distance observer-dependent, not intrinsic to objects, events
- different well-defined measurements can lead to different results for distance

In FLRW universe, "distance" not unique: answer depends on

- *what you measure*
- *how you measure it*

Proper Distance

So far: have constructed *comoving* coordinates
which expand with Universe ("home" of FOs)

RW metric: encodes **proper distance**
i.e., *physical* separations as measured by metersticks/calipers:
▷ in RW frame i.e., by comoving observers=FOs
▷ *at one* fixed cosmic instant t

$$d\ell_{\mathrm{prop}}^2 = a(t)^2 d\ell_{\mathrm{com}}^2 = a(t)^2 \left(\frac{dr^2}{1-\kappa r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)$$

Can read off proper distances for small displacements
as measured by FOs at time t:

- $d\ell_r^{\mathrm{prop}} = a(t)\, d\ell_r^{\mathrm{com}} = a(t)\, dr/\sqrt{1-\kappa r^2/R^2}$
- $d\ell_\theta^{\mathrm{prop}} = a(t)\, d\ell_\theta^{\mathrm{com}} = a(t)\, r d\theta$
- $d\ell_\phi^{\mathrm{prop}} = a(t)\, d\ell_\phi^{\mathrm{com}} = a(t)\, r \sin\theta d\phi$

Q: how to find distance for finite displacements?

for finite displacements: integrate small ones

e.g., radial distance (at t) between $r = 0$ and r is

$$\ell_r^{\text{prop}} = a(t)\ell_r^{\text{com}} = a(t)\int_0^r d\zeta/\sqrt{1-\kappa\zeta^2/R^2} \quad (2)$$

Note: $d\ell_r^{\text{prop}}/dt = \dot{a}\,\ell_r^{\text{com}} = H\,\ell_r^{\text{prop}}$ exactly!
→ i.e., at a *fixed cosmic time* t
proper distances increase exactly according to Hubble!
Q: what does this mean for points with $\ell_{\text{r}}^{\text{prop}} > d_H$*?*
Q: is this a problem?

Q: how would you in practice measure ℓ_r^{prop} *for large* r*?*

Luminosity Distance

for a point source (unresolved), observables:
1. redshift z
2. flux (apparent brightness) $F_{\rm obs} = dE_{\rm obs}/dt_{\rm obs}\, dA$
   summed over all wavelengths: "bolometric"

input/assumption: "**standard candle**"
known rest-frame luminosity $L_{\rm em} = dE_{\rm em}/dt_{\rm em}$

Goal: given std candle $L_{\rm em}$, want to relate
observed $z_{\rm em}$ and $F_{\rm obs}$
⇒ find expression for **luminosity distance**
  defined by Newtonian/Euclidean formula:

$$d_{\rm L}(z_{\rm em}) \equiv \sqrt{\frac{F_{\rm obs}}{4\pi L_{\rm em}}} \qquad (3)$$

Q: effects in cosmological setting?
Q: calculation strategies? sanity check(s)?

Strategy: start with observation, work back

Observation:

FO with telescope, area A_{det}
in time interval δt_{obs}
measures total energy $\delta\mathcal{E}_{\text{obs}}$; avg photon energy ϵ_{obs}

observed flux (bolometric, λ-integrated) given by

$$\delta\mathcal{E}_{\text{obs}} = F_{\text{obs}} A_{\text{det}} \delta t_{\text{obs}} \tag{4}$$

F_{obs} is rate of energy flow per unit area
as measured in observer frame

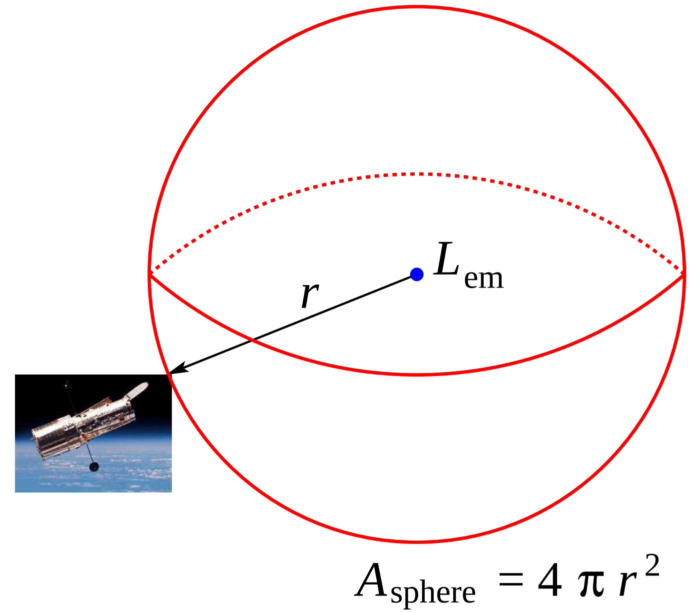
Q: what's invariant/observer independent as signal propagates?

Standard candle emitter:

luminosity L_{em} at $a_{\text{em}}, z_{\text{em}}$

with average photon energy ϵ_{em}

- choose $r_{\text{em}} = 0$ as center
- light "cone" (sphere) today reaches us, has present area $A_{\text{sph}} = 4\pi r^2$



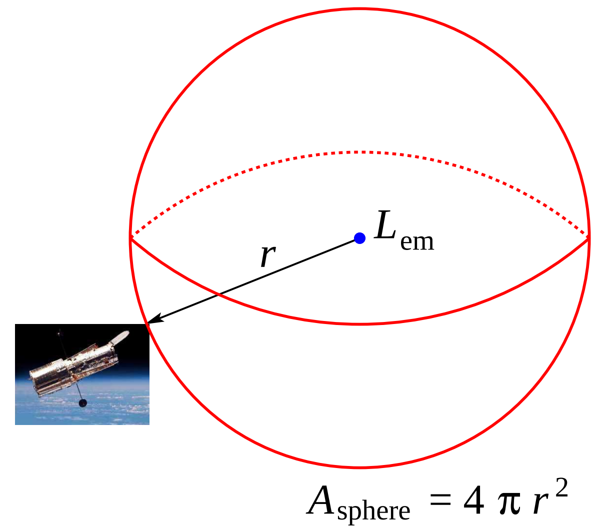
photon counts are invariant

i.e., all agree on how many detector registers

Q: how to quantify photon number conservation?

total photon counts through sphere at r:

$$\delta\mathcal{N}_{\text{obs}} = \frac{F_{\text{obs}} A_{\text{sph}} \delta t_{\text{obs}}}{\epsilon_{\text{obs}}} = 4\pi r^2 \frac{F_{\text{obs}}}{\epsilon_{\text{obs}}} \delta t_{\text{obs}}$$



total photon counts from source

$$\delta\mathcal{N}_{\text{em}} = \frac{L_{\text{em}}}{\epsilon_{\text{obs}}} \delta t_{\text{em}}$$

photon conservation: $\delta\mathcal{N}_{\text{obs}} = \delta\mathcal{N}_{\text{em}}$

$$F_{\text{obs}} = \frac{\epsilon_{\text{obs}}}{\epsilon_{\text{em}}} \frac{\delta t_{\text{em}}}{\delta t_{\text{obs}}} \frac{L_{\text{em}}}{4\pi r^2} \quad (5)$$

Q: and so?

$$F_{\text{obs}} = \frac{\epsilon_{\text{obs}}}{\epsilon_{\text{em}}} \frac{\delta t_{\text{em}}}{\delta t_{\text{obs}}} \frac{L_{\text{em}}}{4\pi r^2} \tag{6}$$

- energy redshifting $\epsilon_{\text{obs}} = a_{\text{em}} \epsilon_{\text{em}}$
- time dilation $\delta t_{\text{obs}} = \delta t_{\text{em}} / a_{\text{em}}$

So we have

$$F_{\text{obs}} = a_{\text{em}}^2 \frac{L_{\text{em}}}{4\pi r^2} = \frac{L_{\text{em}}}{4\pi (1+z)^2 r^2} \tag{7}$$

Q: and so?

Observed flux is

$$F_{\text{obs}} = a_{\text{em}}^2 \frac{L_{\text{em}}}{4\pi r^2} = \frac{L_{\text{em}}}{4\pi(1+z)^2 r^2} \tag{8}$$

identify **luminosity distance** via Newtonian/Euclidean result:

$$d_L \equiv \sqrt{\frac{L_{\text{em}}}{4\pi F_{\text{obs}}}} \tag{9}$$

and so

$$d_L = \frac{r}{a_{\text{em}}} = (1+z)\ r$$

Q: why of practical observational interest?

Q: r unmeasured–how relate to observables?

Q: sanity checks? non-expanding? small z?

Q: why is $d_L \neq \ell_{\text{com}}$?

Q: why is $d_L > r$?

Q: what if measure spectrum $F_\nu = dF/d\nu$?

luminosity distance: $d_L = (1+z)\ r(z)$

Note: relate r to emission redshift z via
trusty photon propagation eq:

$$
\begin{aligned}
\int_0^{r_{\rm em}} \frac{dr}{\sqrt{1-\kappa r^2/R^2}} &= \int_{t_{\rm em}}^{t_{\rm obs}} \frac{dt}{a(t)} \\
&= \int_{a_{\rm em}}^{a_{\rm obs}} \frac{da}{a\dot{a}} = \int_{a_{\rm em}}^{a_{\rm obs}} \frac{da}{a^2 H(a)} \\
&= \int_0^{z_{\rm em}} \frac{dz}{H(z)}
\end{aligned}
$$

where Friedmann gives $H(z)$
$\rightarrow$ r and thus d_L manifestly depends on cosmology
(i.e., cosmic geometry, parameters)
$\star$ d_L for SN Ia $\rightarrow$ cosmic acceleration!

Note: for alt radial variable χ
$d_L = (1+z)/R/S_\kappa(\chi)$

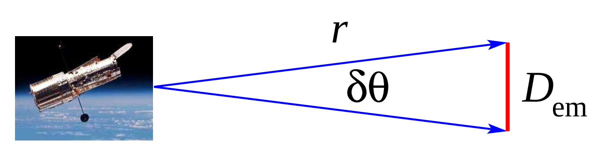
Extended Objects:
Angular Diameter Distance

if object resolved as extended source on sky, not point source
then new observable available:

★ *angular size* $\delta\theta$

- and as usual, redshift z
  and flux (apparent bolometric brightness) F

input/assumption: "standard ruler"
known rest-frame size: diameter $D_{\rm em}$

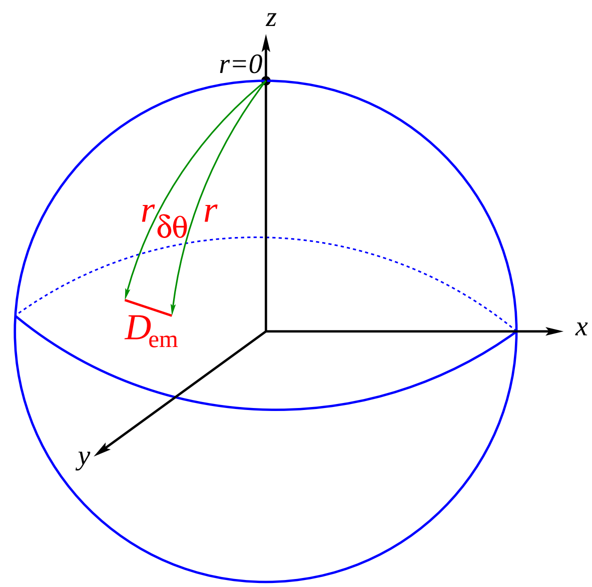


Goal: for std rulers, want to relate
observed z and $\delta\theta$

Q: effects in cosmological setting?
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

To visualize, consider closed universe

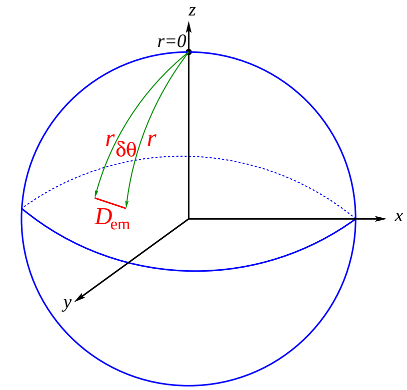
- observer at $r = 0$
- a pair of radial photons
  from edges of source
  trace longitudes



Invariant:
angular (longitude) separation $\delta\theta$ remains same
...while physical separation evolves, due to propagation
and cosmic expansion

At *emission* epoch, physical separation of photons
is standard ruler size $D_{\rm em}$
but also related to $\delta\theta$ and $r = r_{\rm em}$ via RW metric
Q: how?

At *emission* epoch, standard ruler size $D_{\rm em}$ at emission point r fixes angular separation $\delta\theta$:



$$D_{\rm em} = \delta\ell_\theta^{\rm prop,em} = a_{\rm em}\delta\ell_\theta^{\rm com} = a_{\rm em} r\delta\theta$$

But $\delta\theta$ remains fixed over propagation so today we observe

$$\delta\theta = \frac{D_{\rm em}}{a_{\rm em} r}$$

identify angular diameter distance via Newtonian/Euclidean result:

$$d_A \equiv \frac{D_{\rm em}}{\delta\theta} \tag{10}$$

and so

$$d_A = a_{\rm em} r = \frac{r}{1+z} = \frac{S_\kappa(\chi)}{1+z}$$

Angular diameter distance: $d_A = r(z)/(1+z)$

Q: why of practical observational interest?

Q: sanity checks?

Q: why is $d_A < r$?

Q: what if resolve at different λ?

Note:

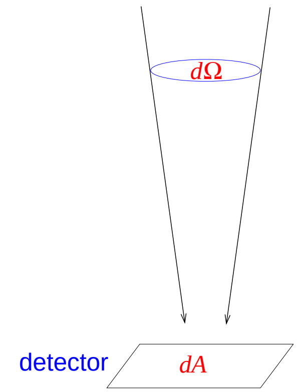
- d_A depends on cosmological history via $r(z)$
- $d_A = a_{\text{em}}^2 d_L = d_L/(1+z)^2$
  different measures!
  but ratio is cosmology independent

Q: implications for CMB fluctuations?

`www: WMAP`

Director's Cut Extras: Surface Brightness

Extended Objects Part Deux: Surface Brightness



if object is resolved, extended source on sky
can measure angular area and determine
surface brightness $I =$ flux/(angular area $\Delta\Omega$)

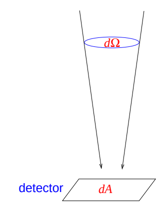
Q: physical effects: "normal" environment?
Q: effects in cosmological setting?
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

Newtonian/Euclidean Surface Brightness

For intuition: review Newtonian/Euclidean result
- flat space
- no redshifting, time dilation

consider an **extended source**, i.e., not pointlike
which is **resolved** by your telescope
  i.e., apparent angular size $>$ point spread function

observables:
- flux $F = dE/dt\ dA$ as before, but also
- angular dimensions $\rightarrow$ angular area $d\Omega$



Wavelength-integrated (bolometric) surface brightness
is wavelength-integrated flux per unit source angular area:

$$I_{\rm obs} = \frac{dE_{\rm obs}}{dA\ dt_{\rm obs}\ d\Omega} = \frac{dF_{\rm obs}}{d\Omega}$$

Dependence on source distance r?

- as usual, $F = L/4\pi r^2$
- source sky area $\Delta\Omega \Rightarrow$ physical area $S = r^2 \Delta\Omega$, so

$$I_{\rm obs} = \frac{F_{\rm obs}}{\Delta\Omega} = \frac{L/4\pi r^2}{S/r^2} = \frac{L}{4\pi S}$$

Newtonian/Euclidean result *independent* of source distance!

"conservation of surface brightness"

fun consequences of surface brightness conservation:

- similar resolved, unobscured Galatic objects (e.g., nebulae) have similar surface brightness
- nearby large Galaxies have similar surface brightness to MW
- in daily life you rarely experience inverse square law e.g., brightness of faces of nearby vs distant classmates

Generalize to cosmological context: observed (bolometric) surface brightness

$$I_{\text{obs}} = \frac{F_{\text{obs}}}{\Delta\Omega_{\text{obs}}}$$

1. already know $F_{\text{obs}} = a_{\text{em}}^2 L_{\text{em}}/4\pi r^2$
2. RW metric says angular area

$$\Delta\Omega_{\text{obs}} \simeq \frac{\delta\ell_\theta^2}{4\pi r^2} = \frac{D_{\text{em}}^2}{4\pi a_{\text{em}}^2 r} = \frac{A_{\text{em}}}{4\pi a_{\text{em}}^2 r^2}$$

Combine:

$$I_{\text{obs}} = \frac{a_{\text{em}}^2 L_{\text{em}}/4\pi r^2}{4\pi A_{\text{em}}/a_{\text{em}}^2 r^2} = a_{\text{em}}^4 \frac{L_{\text{em}}}{A_{\text{em}}} \quad (11)$$

$$= a_{\text{em}}^4 I_{\text{em}} = \frac{I_{\text{em}}}{(1+z)^4} \quad (12)$$

Intensity of resolved, unobsurced source at $z_{\rm em}$:

$$I_{\rm obs} = \frac{I_{\rm em}}{(1+z)^4}$$

- conservation of surface brightness” no longer true!
  vestige: no explicit dependence on r
- *cosmic dimming* $\propto (1+z)^4$
- dimming is independing of cosmology!
  useful consistency check!

Q: implications for CMB brightness?

CMB implications:
for blackbody, Stefan-Boltzmann sez

$$I = \frac{\sigma}{\pi} T^4$$

consider CMB, emitted at $z_{\rm em}$
with temperature $T_{\rm em}$

today, observe surface brightness

$$I_{\rm obs} = (1+z_{\rm em})^{-4} I_{\rm em} = (1+z_{\rm em})^{-4} \frac{\sigma}{\pi} T_{\rm em}^4 = \frac{\sigma}{\pi} \left(\frac{T_{\rm em}}{1+z_{\rm em}} \right)^4$$

still follows blackbody law, but with

$$T_{\rm obs} = \frac{T_{\rm em}}{1+z_{\rm em}}$$

which we have already derived by other means!