Astro 507

Lecture 15

Feb. 24, 2014

Announcements:

- **Preflight 3 due Friday at 9am**
- PS1 back at end of class, scores on Compass

Last time: evidence for acceleration

data: SN fainter (lower F) than in coasting, decel. Universe

Q: possible interpretations?

Q: novel property required of any cosmic accelerant?

Q: simplest accelerant?

Q: how much accelerant needed?

The Data: Λ Emerges

SN Ia data in Λ cosmology:

- allow for $\Omega_\Lambda = \Lambda c^2/3H^2 \neq 0$
- find best fit to d_L data:

"concordance universe"

`www:` $\Omega_\Lambda - \Omega_m$ `plane`

$$\boxed{\Omega_\Lambda \simeq 0.7 \qquad \Omega_m \simeq 0.3} \tag{1}$$

This is amazing!

Q: why?

Λ Looms Large

acceleration demands $\Omega_\Lambda \sim 0.7$
roughly independent of CMB

- Einstein-de Sitter expectations of $\Omega_m = \Omega_0 = 1$
  totally ruled out!

- $\Omega_\Lambda \neq 0$: cosmo constant (or worse!) seems to exist!

- $\Omega_\Lambda \gtrsim 2\Omega_m$: U *dominated* by Λ *now*!

- *two mysteries seem related quantitatively:*
  CMB + cluster: $\Omega_0 - \Omega_m = \Omega_{other} \approx 0.7$
  SNe Ia: $\Omega_\Lambda \approx 0.7$
  a consistent picture of a bizarre universe!

Q: if this is all true, cosmic fate?

Λ and Cosmic Fate: Big Chill and Dark Sky

if acceleration is truly due to Λ then:

- already dominates Friedmann
- as a increases, matter & curvature terms drop
  $\rightarrow$ Λ dominates even more!

The bleak Λ-dominated future:

⋆ future $a(t) \simeq e^{\sqrt{\Omega_\Lambda} H_0 (t-t_0)}$ $\rightarrow$ exponential expansion *forever*!
fate is not only *big chill* but *supercooling*

⋆ *event horizon* exists: $d_{\rm event,comov}(t_0) \simeq \Omega_\Lambda^{-1/2} d_H \sim 6400$ Mpc
we will *never* see beyond this!
worse still: later on,

$$d_{\rm event,comov}(t_0 + \Delta t) = e^{-\sqrt{\Omega_\Lambda} H_0 \Delta t} d_{\rm event,comov}(t_0)$$

event horizon shrinks exponentially with time!
$\rightarrow$ ever less to see!
observational astronomy from data mining only!

Λ as Vacuum Energy

Can rewrite Λ as energy density: ρ_Λ:
in Friedmann, put

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2} + \frac{\Lambda c^2}{3} \equiv \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{\kappa c^2}{R^2 a^2}$$

so that

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \quad \text{and} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\rm crit}}$$

Then introduce pressure P_Λ in Fried accel:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda c^2}{3} \equiv -\frac{4\pi G}{3}(\rho + \rho_\Lambda + 3P + 3P_\Lambda)$$

can show:

$$P_\Lambda = -\frac{\Lambda c^2}{8\pi G} = -\rho_\Lambda$$

i.e., $P_\Lambda = w\rho_\Lambda$, with $w = -1$

Note:

- Λ is strict constant $\rightarrow \rho_\Lambda$ constant in space and time
  "energy density of the vacuum" $\rightarrow$ **dark energy**
- $P_\Lambda < 0$: as needed for acceleration
- equation of state parameter $w = -1$ preserves Λ constancy

So: Λ is equivalently a length scale
or an energy density
Q: what sets its value?

Λ and its Discontents

In Classical GR:

- Λ is a (optional) parameter to be measured
- no *a priori* insight as to its value
  (beyond escaping solar system limits)

But quantum mechanics & particle physics
offer a new perspective on vacuum energy

Recall: blackbody radiation
usually write total energy density:

$$\varepsilon_{\rm bb}(T) = \int \overline{n}\hbar\omega \, \frac{d^3p}{h^3} = \frac{1}{2\pi^2c^2}\int_{\omega=0}^{\infty} \frac{\hbar\omega}{e^{\hbar\omega/kT}-1}\omega^2 \, d\omega = a_{\rm Boltz}T^4$$

note that $\varepsilon \to 0$ as $T \to 0$: vacuum has no energy
...but (Λ aside) this was always a cheat!
Q: why? what omitted?

Uncertainty principle $\to$ nothing "at rest"
$\to$ ground state "zero point motion"
$\to$ zero point modes have energy $E_0 \neq 0$

Blackbody result: treats photon modes
as harmonic oscillators
but threw away zero point energy $E_0 = \hbar\omega/2$!
Cheated!

- handwaving excuse:
  E_0 cost of "assembling" oscillators/quanta
  ...and then only energy *differences* count
- in practice, usual Planck result is really
  $\varepsilon_{\rm usual} = \varepsilon_{\rm tot}(T) - \varepsilon_{T=0} = \varepsilon_{\rm tot}(T) - \varepsilon_{\rm zeropoint}$
- but in GR: curvature $\leftrightarrow$ mass-energy density
  absolute energy scales matter!
  e.g., $(\dot{a}/a)^2 \sim 8\pi G/3 \; \varepsilon/c^2$

Q: what if we keep the zero-point energy?

Try keeping zero point energy:

$$\begin{aligned} \varepsilon &\sim \int_0^\infty \langle E(\omega) \rangle \; \omega^2 \; d\omega & (2) \\ &= \int_0^\infty \left(\overline{n} + \frac{1}{2} \right) \hbar\omega \; \omega^2 \; d\omega & (3) \\ &= \int_0^\infty \left(\frac{1}{e^{\hbar\omega/kT} - 1} + \frac{1}{2} \right) \omega^3 \; d\omega & (4) \\ &= \varepsilon_{\text{usual}} + \varepsilon_{\text{zeropoint}} & (5) \end{aligned}$$

where the zero pont contribution is

$$\varepsilon_{\text{zeropoint}} \sim \int_0^\infty \omega^3 \; d\omega = \infty^4$$

"ultraviolet catastrophe"!

Q: possible cures?

Vacuum Energy in Particle Physics

what is cause of catastrophe?

$$\varepsilon_{\text{zeropoint}} \sim \int_0^{\omega_{\text{max}}} \omega^3 \, d\omega \sim \omega_{\text{max}}^4$$

allowed $\omega_{\text{max}} \rightarrow \infty$

$\rightarrow$ included modes of arbitrarily high energy

arbitrarily small wavelength

If quanta *energy has upper limit* E_{max}

i.e., a minimum wavelength $\lambda_{\text{min}} = \hbar c / E_{\text{max}}$

then $\varepsilon_{\text{zeropoint}} \neq \infty$

Q: what might such a limit be?

Q: i.e., at what scale might energies "max out"?

The Planck Scale and Λ

Highest known energy scale in physics: **Planck Scale**
when *quantum effects become important for gravity*

a particle of mass m, energy mc^2
has quantum scale $\lambda_{\rm quantum} = \hbar/mc$ (*Compton wavelength*)
equal to GR scale $\lambda_{\rm GR} = 2Gm/c^2$ (*Schwarzchild radius*)
if $m = M_{\rm Pl}$: the **Planck mass**

$$
\begin{aligned}
M_{\rm Pl}c^2 &= \sqrt{\frac{c}{G\hbar}}c^2 \sim 10^{19} \text{ GeV} && (6) \\
\ell_{\rm Pl} &= \frac{\hbar}{M_{\rm Pl}c} \sim 10^{-33} \text{ cm} && (7)
\end{aligned}
$$

if quanta have $E_{\rm max} = M_{\rm Pl}$ and $\lambda_{\rm min} = \ell_{\rm Pl}$
then estimate vacuum energy density

$$\rho_{\rm vac,Pl} \sim M_{\rm Pl}^4 \sim 10^{110} \text{ erg/cm}^3 \sim 10^{89} \text{ g/cm}^3$$

Q: implications?

Compare to the vacuum density in Λ:

$$\rho_{\rm vac,Pl} \sim 10^{89}\ {\rm g/cm^3} \sim 10^{120} \rho_{\rm Lambda}$$

mismatch is ~ 120 orders of magnitude!!

So the real question is not: *"Why have Λ at all?"*
but rather: *"Why isn't Λ gi-normous?"*

quantum gravity?
maybe some underlying symmetry set $\Lambda = 0$
to avoid "fine-tuning" Λ
if so, then dark energy is not vacuum energy
but some other energy density with negative pressure

high-energy phase transitions/symmetry breaking?
maybe symmetry breaking processes set vacuum energy
e.g., GUT, SUSY, electroweak, QCD
if so, how does each contribute to total vacuum?
run the numbers: best case is QCD

$$\varepsilon_{\text{qcd}} \sim \Lambda_{qcd}^4 \sim (100 \text{ MeV})^4 \sim 10^{30} \varepsilon_{\text{dark energy}} \tag{8}$$

many orders of magnitude improvement, but not quite a fix!

Bottom line:
known quantum fields do not provide viable candidate
for source of vacuum energy $\rho_{\text{vac}} = \rho_{\Lambda}$

Dark Energy: Parameterized Ignorance

Theoretical Ignorance

No good (i.e., pre-existing) candidates for cosmic acceleration
unlike dark matter: high-E theory predicts stable exotic particles

Lacking guidance, look for general way to describe
cosmic substance responsible for acceleration: **dark energy**
recall: matter, radiation, Λ described by $P = w\rho c^2$
with w a constant

Write dark energy density and pressure with

$$P_{\mathrm{DE}} = w\ \rho_{\mathrm{DE}} c^2$$

"parameterize our ignorance" in $\boxed{w}$ (possibly not constant)
cosmo constant is limiting case *Q: Namely?*
Q: what can we say about w values?

Dark Energy: the Little We Know

What is w today?

In DE-only case

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = -\frac{4\pi G}{3}\rho(1+3w) \tag{9}$$

$\rightarrow$ acceleration requires $w < -1/3$ today

Recall: cosmic first law is

$$d(\rho a^3) = -p\ d(a^3) = -w\rho\ d(a^3) \tag{10}$$

For constant w:

$$\rho_{\mathrm{DE}} \propto a^{-3(1+w)} \tag{11}$$

Q: sanity check–results for w = matter, radiation, Λ?
Q: connection between "w" dark energy and Λ?

Data: generalize Ω_Λ limits
to Ω_w and w (now two parameters)

`www: current limits`

$$\Omega_w \sim 0.7 \quad , \quad w < -0.76 \text{ (95\%CL)}$$

- w close to -1: cosmo constant value!
- tests for w change weak but null
  $\rightarrow$ also like cosmo const!

What if w not constant?
Empirical approach: Taylor expand

$$w(a) = w_0 + w_a (1 - a) \tag{12}$$

observations constraint parameters (w_0, w_a)
Q: does this allow for Λ result? if so how?

`www: present data`