

Astro 507  
Lecture 17  
Feb. 28, 2014

Announcements:

- **Problem Set 3 available; due next Friday**
- *Preflight 3 was due today*

Last time: dark energy evolution and scalar fields

*Q: what's the DE coincidence problem?*

*Q: what's a scalar field?*

*Q: what makes scalar fields appealing dark energy candidates?*

# The Physics of Scalar Fields

**scalar field:**  $\phi(\vec{x}, t)$

*scalar* → single-valued object = *function*

no directionality → kosher with cosmo principle

*field* → function takes values at all points in space(time)

Scalar fields abound in all areas of physics

*Q: examples of known, physical scalar fields?*

in particle physics, scalar fields arise in

force unification, origin of mass (Higgs!)

in cosmology: DE, inflation → can't avoid!

*“Scalar fields are the cosmologist's blunt instrument.”*

– J. Frieman

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Note: for scalar novices, today's Extras have additional info

## Scalar Fields: Motion and Energy

need equations to govern scalar field  $\phi(\vec{x}, t)$

- field “motion” = variation in space & time  
will seek 2nd-order eq of motion  
→ schematically, “  $\partial\partial\phi = \text{stuff} = \text{source of } \phi$ ”
- energy: “kinetic” and “potential”

*Q: Relativistic considerations? Impact on eq of motion?*

# Scalar Field Dynamics

Special Relativity demands:

- $\phi$  disturbances (“signals”) propagate at speeds  $\leq c$
- space & time on equal footing

⇒ if second-order equation of motion, must have

$$\partial^2\phi = \partial_t^2\phi - \nabla^2\phi = \text{sources/interactions of } \phi \quad (1)$$

lead to  $\phi$  **energy density**

$$\varepsilon_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \varepsilon_{\text{int}} = \frac{1}{2}\dot{\phi}^2 + \nabla\phi^2 + \varepsilon_{\text{int}} \quad (2)$$

Simplest **interactions**: only with self  $\rightarrow \varepsilon_{\text{int}} = V(\phi)$   
gives equation of motion with “force”  $-\partial_\phi V \equiv -V'(\phi)$

$$\partial^2\phi = \partial_t^2\phi - \nabla^2\phi = -\partial_\phi V(\phi) \quad (3)$$

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Q: *cosmological considerations?*

# Cosmological Scalar Fields

## Equation of Motion

in Minkowski space: relativistic scalar field

$$\partial_t^2 \phi - \nabla^2 \phi = \text{sources/interactions}$$

*in FRW, include coupling to gravity → redshifting effects*

...experts see note in Director's Cut Extras below

$$\partial_t^2 \phi - \nabla^2 \phi = -3 \frac{\dot{a}}{a} \dot{\phi} + \text{sources/interactions}$$

but cosmo principle → at any time  $\phi$  homogeneous:  
same at all  $x$ , so no space derivatives

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + V' = 0$$

5 where  $V'(\phi) \equiv \partial_\phi V$

Q: similarities with Newtonian particle equation of motion?

Scalar field energy density:

$$\varepsilon = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

note kinetic, potential terms

scalar field momentum density  $\rightarrow$  pressure

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

*Q: crucial, important difference between these expressions?*

*Q: how could we exploit this?*

## Scalar Field “Equation of State”

For cosmic  $\phi$  field,  
can write expression for  $w = P/\rho$  parameter:

$$w_\phi \equiv \frac{P_\phi}{\epsilon_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \quad (4)$$

Limiting cases:

- if kinetic term dominates:  $\frac{1}{2}\dot{\phi}^2 \gg V(\phi)$   
Q: then  $w_\phi \rightarrow ?$
- if the potential term dominates:  
Q: then  $w_\phi \rightarrow ?$

∟

Q: implications?

## Scalar Potentials as Accelerants

Relativistic scalar field potential contributes negative pressure!

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \xrightarrow{V \gg \dot{\phi}^2} -1 \quad (5)$$

candidate for dark energy

Good news:

$w_\phi \rightarrow -1$  independent of details of  $V$ !

as long as  $V \gg \dot{\phi}^2$ : potential dominates

cosmic acceleration “natural” if

▷ scalars present and

▷  $\varepsilon_\phi$  large

∞

Q: bad news?



## Bad news:

$w_\phi \rightarrow -1$  independent of details of  $V$

- ▷ not guided as to details, physics of  $V$
- ▷ need to know/guess/measure interactions with itself, other matter/energy

## Quintessence

Simplest assumption:  $\phi$  feels only itself  
i.e., interacts only with itself  
→  $V(\phi)$  independent of matter, radiation  
but: still has gravitational interactions  
→ indirectly communicates with matter, rad  
via  $-3H\dot{\phi}$  term

Also: will see later—we *need* past epochs of  
matter, radiation dom: else no light elements,  
no grow of cosmic structures  
→ in past,  $\rho_\phi \ll \rho_{\text{tot}}$

## Quintessence: Ingredients

- cosmic scale field  $\phi$  (or  $Q$ )
- gravity + self-interactions  $V = V(\phi)$  only

Outcome (for some choices of  $V$ ):

- $\phi$  evolves slowly, small at early times:  $\rho_\phi/\rho_{\text{tot}} \ll 1$
  - $\phi$  evolution coupled to matter, rad
    - “attractor” solutions for  $\rho_\phi/\rho$
    - ★  $\rho_\phi$  slowly becomes dominant
    - ★ acceleration sets in
- www: tracker model
- addresses the coincidence problem  
or at least trades it for  $\phi$  and  $V$

## Alternative to Dark Energy: Modified Gravity

so far: *we have assumed GR → Friedmann is correct*

and thus cosmic acceleration demands accelerant: dark energy

not a crazy approach:

General Relativity successful in laboratory solar system tests

but: GR not independently tested at cosmological scales

→ *what if GR wrong or incomplete?*

*modified gravity*: no longer GR → no longer Friedmann

the game: describe acceleration without dark energy

Q: *constraints on modified gravity?*

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Q: *how to distinguish mod grav vs DE with SN measurements?*

Q: *how to break degeneracy?*

## Modified Gravity vs Dark Energy: Degeneracies

dark energy: GR still valid, but new substance  $\rho_{\text{DE}}(a)$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} (\rho_{\text{m+r}} + 3P_{\text{m+r}} + \rho_{\text{DE}} + 3P_{\text{DE}}) \quad (6)$$

modified gravity: GR invalid, but no new substance

$$\frac{\ddot{a}}{a} = F(\rho_{\text{m+r}}, P_{\text{m+r}}, a) \quad (7)$$

given observed  $\ddot{a}/a$ , can always explain with either

- arbitrary  $\rho_{\text{DE}}, P_{\text{DE}}$ , or
- arbitrary  $F(\rho_{\text{m+r}}, P_{\text{m+r}}, a)$

How to break acceleration degeneracy?

look at predictions for *inhomogeneous* universe

→ growth of structure from density perturbations

tests spatial character of gravity inaccessible in homogeneous U

# Cosmic Acceleration/Dark Energy

## The Future

### Challenges for theory:

*Q: open questions?*

*Q: what tools needed to address them?*

*Q: possible implications of answers?*

### Challenges for observations:

*Q: open questions?*

*Q: what tools needed to address them?*

*Q: possible implications of answers?*

# Cosmic Acceleration/Dark Energy

## The Future: Job Security!

### Challenges for theory: Formidable!

- a failure of Friedmann  $\rightarrow$  General Relativity?  
hints at quantum gravity?
- ...or... Friedmann/GR okay, dark energy exists  
what is it?  
is it connected to dark matter?  
how does it fit into the rest of physics?
- is it truly a cosmo constant  $\Lambda$ ?  
if so, why this value? why nonzero?
- if DE  $\neq \Lambda$ , what is it?  
how does it evolve?  
how does it couple to matter, rad?  
participation in structure formation?  
what is the fate of the universe?  
is there an early U. connection–inflation?

## Challenges for observation: **Formidable!**

- is the universe accelerating?  
SNIa are (for now) most direct evidence  
systematic errors?
- what is precise value of  $w_0$ ?
- is  $w \neq -1$ , i.e., can we rule out cosmo constant  
and rule *in* an evolving cosmic energy component?
- does  $w$  itself evolve? was  $w < -1$  ever (or will it be)?



# Future Dark Energy Experiments

**Dark Energy Survey** Illinois, FNAL, Chicago, ...

**Strategy** Multi-pronged attack:

- optically monitor  $4000 \text{ deg}^2 \sim 1\text{sr}$  of sky  
discover  $\sim 1900$  SNIa in  $0.3 < z < 0.8$
- overlaps with CMB (S-Z) survey  
discover, map 20,000 clusters

**Scientific Payoff**

- ▷ luminosity distances
- ▷ galaxy distribution vs  $z$   
angular correlations  
angular distortions: “weak lensing”
- ▷ cluster counting and spatial distribution, BAO
- ▷  $w_0$  to  $\sim 5\%$ ,  $dw/dz$  to  $\sim 30\%$
- ▷ multiple methods: checks on systematics

**Large Synoptic Survey Telescope (LSST)** Illinois et al  
repeated scan of  $\Omega_{\text{lsst}} \sim 20,000 \text{ deg}^2 \simeq \text{half of sky}$ , yields:

*The Sky: The Movie*

transient universe on minute–year timescales

down to  $24 - 27^{\text{mag}}$  in 5 bands

SN count approaching  $10^6$ , bazillion Type Ia's to  $z \sim 1.3$

also gamma-ray burst afterglows, exoplanet transits,

Galactic & extragalactic microlensing, stellar parallax,

variable stars, Kuiper belt objects, killer asteroids, ...

*The Sky: The Wallpaper*

SDSS-type digital photometric map

down to  $m_{AB} \sim 29^{\text{mag}}/\text{arcsec}^2$  in 5 bands

∞ galaxy clusters, strong and weak lensing, BAO, galaxy clustering,

Galactic structure ...

# Space Missions

advantages:

- diffraction limited  $\rightarrow$  high angular resolution
- uniform sky coverage
- with low-res spectrograph:  $z$  errors negligible

science:

- supernovae
- weak lensing: expansion rate and structure growth
- BAO (standard ruler):  $d_A$ , expansion history, structure growth

**Euclid** ESA, NASA

- 1.2 m telescope, 0.5 deg<sup>2</sup> field of view
- $\sim$  2020 launch

**WFIRST** NASA, NSF

- 2.4 m telescope
- sin 2024 launch?

## Dark Energy: Epilogue

Jim Peebles, *The Large Scale Structure of the Universe* (1980)  
end of the final paragraph (on a different topic, but still apt):

... we must still bear in mind Bondi's caution that "there are probably few features of theoretical cosmology that could not be completely upset and rendered useless by new observational discoveries." For the present subject we might add, "or by a good new idea."

## Director's Cut Extras: Scalar Field Dynamics

# Scalar Field Dynamics: Plausibility Argument

from A. Zee, *Quantum Field Theory in a Nutshell*

Picture an array of masses connected by springs  
“bedspring” model of a field

$N$ -oscillator model: discrete version

masses  $m$ , positions  $q_n$ ,

equilibrium when  $q_n = na$ : lattice spacing  $a$   
acceleration due to offset from equilibrium:

$$m\ddot{q}_n = k(q_{n+1} - q_n) - k(q_n - q_{n-1}) \quad (8)$$

continuum limit:  $N \rightarrow \infty$ ,  $a \rightarrow 0$ , and then

$$\rho \frac{\partial^2 \phi}{\partial t^2} = Y \frac{\partial^2 \phi}{\partial x^2} \quad (9)$$

put  $c^2 = Y/\rho$ : “sound speed”

$$\partial_t^2 \phi - 1/c^2 \partial_x^2 \phi = 0 \quad (10)$$

smoothed version of oscillator network:

$$\partial_t^2 \phi - \frac{1}{c^2} \partial_x^2 \phi = 0 \quad (11)$$

“equation of motion” for  $\phi$

- note equal footing of  $x, t$
- supports wave solutions:  $\phi = \phi(x \pm c_s t)$

if additional forces between masses

$$\partial_t^2 \phi - 1/c^2 \partial_x^2 \phi = F = -\partial V / \partial \phi \quad (12)$$

where **potential**  $V$  can depend on  $\phi$ ,  
other matter fields

harmonic oscillator model obviously not literal

≈ cosmos not made of bedsprings!

*Q: why is it still useful?*

## Scalar Fields in an FLRW Universe

In special relativity Minkowski space, scalar field has:

$$T_{\mu\nu} = \partial_\mu\phi \partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu} [\partial_\lambda\phi\partial^\lambda\phi - V(\phi)] \quad (13)$$

and  $\partial^\mu T_{\mu\nu} = 0$  gives equation of motion

$$\partial_\mu\partial^\mu\phi + \partial_\phi V(\phi) = 0 \quad (14)$$

make “minimal substitution”

- $\eta_{\mu\nu} \rightarrow g_{\mu\nu}$
- $\partial_\mu \rightarrow \nabla_\mu$  covariant derivative

$$T_{\mu\nu} = \nabla_\mu\phi \nabla_\nu\phi - \frac{1}{2}g_{\mu\nu} [\nabla_\lambda\phi\nabla^\lambda\phi - V(\phi)] \quad (15)$$

so that in Cartesian FO frame  $T_{\mu\nu} = \text{diag}(\rho_\phi, -P_\phi, -P_\phi, -P_\phi)$



FLRW equation of motion: evaluate

$$\nabla^\mu T_{\mu\nu} = 0 \quad (16)$$

in FO frame, get

$$\nabla_\mu \nabla^\mu \phi + \partial_\phi V(\phi) = 0 \quad (17)$$

Note that  $\nabla^\mu \phi = \partial^\mu \phi$ , but

$$\nabla_\mu (\nabla^\mu \phi) = \partial^2 \phi + \Gamma_{\mu\mu}^\lambda \partial_\lambda \phi \stackrel{\text{FO}}{=} \ddot{\phi} + \Gamma_{\mu\mu}^0 \dot{\phi} \quad (18)$$

and since  $\Gamma_{\mu\mu}^0 = 3\dot{a}/a$ , we find

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \partial_\phi V(\phi) = 0 \quad (19)$$