Astro 507

Lecture 19

March 5, 2014

Announcements:

- **Problem Set 3 due Friday**
  office hours: 3:10-4pm Thurs., or by appt
- **PS3 Addendum to problem 3(b)**
  - ▷ "coasting Universe" means $\ddot{a} = 0$
  - ▷ Note that in most of $\Omega_{\rm m}$ vs Ω_Λ plot
    universe is *not flat* – need curvature term in $H(z)$!

Last time: observed CMB–discovery and isotropic component

Q: lessons from CMB discovery?

Q: properties of the observables?

Q: implications?

The Observed Isotropic CMB

CMB spectrum has Planck form:

$$I_{\nu,\text{obs}} = B_\nu(T_0) = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT_0} - 1} \tag{1}$$

- $T_0 = 2.7255 \pm 0.0006$ K – *precision cosmology*
- no nonthermal spectral features found, to a level $|I_{\nu,\text{obs}} - B_\nu|/B_\nu \lesssim 10^{-4}$: *the CMB is a blackbody*

sky pattern: if we write

$$I_\nu(\theta, \phi) = I_{\nu,\text{isotropic}} + \Delta I(\theta, \phi) \tag{2}$$

then $\Delta I(\theta, \phi)/I_{\nu,\text{isotropic}} \lesssim 10^{-5}$

$\Rightarrow$ *the CMB is highly isotropic: cosmo principle vindicated!*

Q: what about redshifting effect on T? I? I_ν?

CMB redshifting:

- Wien says $\lambda_{\mathrm{max}} T = const$, and since $\lambda \propto a$,
  $T = T_0/a = (1+z)T_0$

- total (integrated) intensity $I = \sigma_{\mathrm{SB}} T^4/\pi$
  and thus observers at z would see $I(z) = I_0/a^4 = (1+z)^4 I_0$
  and conversely, $I_0 = I(z)/(1+z)^4$
  $\Rightarrow$ cosmological dimming of surface brightness
  (true for any I)

Q: what sets ε_γ*?* Ω_γ*?* n_γ*?*

Derived CMB Properties

the CMB is a blackbody, and thus:
the temperature completely determines its properties!

energy density

$$\varepsilon_{\gamma,0} = \frac{\pi^2}{15} \frac{(kT_0)^4}{(\hbar c)^3} = 0.26057 \text{ eV/cm}^3 \tag{3}$$

evolving as $\varepsilon_\gamma = \varepsilon_{\gamma,0}/a^4 = (1+z)^4\ \varepsilon_{\gamma,0}$
Q: c.f. starlight? `www: cosmic radiation backgrounds`

equivalent mass density

$$\rho_{\gamma,0} = \frac{\varepsilon_{\gamma,0}}{c^2} = 4.6451 \times 10^{-34} \text{ g/cm}^3 \tag{4}$$

and thus

$$\Omega_{\gamma,0} = 5.04 \times 10^{-5} \left(\frac{0.7}{h}\right)^2 \tag{5}$$

number density

$$n_{\gamma,0} = \frac{2\zeta(3)}{\pi^2}\left(\frac{kT_0}{\hbar c}\right)^3 = 410.73 \text{ photons/cm}^3 \qquad (6)$$

with $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 = 1.202\ldots$

Q: is this a lot or a little? what's a useful comparison?

Q: physical implications of blackbody form of CMB?

Planck Form: Implications

The observed CMB is consistent, at high precision, with
a purely Planckian form
that is: to high precision, **the CMB is a perfect blackbody**

but a blackbody spectrum:

- characterizes a system in thermodynamic equilibrium at T
- is independent of the size, shape, or composition of the system in equilibrium
- see extras below for more on this

thus the CMB implies that
the Universe once attained thermodyanmic equilibrium
i.e., the Universe was once "*in good thermal contact*"
...we'll make this notion more precise

Note also that the *present* universe
must be *transparent* to the CMB

Q: why is this? what's the evidence?

Q: what does this imply about epoch probed by CMB?

The present Universe is transparent to the CMB

e.g., high-redshift radio sources (quasars) are visible
thus the CMB is now *decoupled* from cosmic matter
and has been, at least to largest observed sources $z \gtrsim 10$

thus: for at least $z \lesssim 10$, matter and radiation
in the Universe were *not held in equilibrium*

the equilibrium and thermalization needed to come earlier

- higher density
- higher temperature

the Planckian CMB points to a hot, dense early Universe

 Q: what technology needed to calculate transparency?

For Radiation Transfer Fans

ignoring for now cosmological dimming,
and ignoring scattering (isotropic universe still!)
radiation transfer says

$$\frac{dI_\nu}{ds} = -n_{\mathsf{abs}}\sigma_\nu \; I_\nu + j_\nu \tag{7}$$

with *absorbption cross section* σ_ν and *emission coefficient* j_ν

as usual, rewrite as

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \tag{8}$$

with *optical depth* $d\tau_\nu = n_{\mathsf{abs}}\sigma_\nu \; ds$
and *source function* $S_\nu = j_\nu / n_{\mathsf{abs}}\sigma_\nu$

the solution to transfer equation has character

$$I_\nu \stackrel{\tau \gg 1}{\longrightarrow} S_\nu \stackrel{\text{thermal}}{\longrightarrow} B_\nu(T) \tag{9}$$

- *if sightline is optically thick* then
  observed intensity is source function, and furthermore
- *if source is thermal at T* then
  source function is Planckian

in other words:
a blackbody spectrum implies
an optically thick source in theromdynamic equilibrium

and so the Planckian CMB spectrum tell us
- *the Universe was once optically thick*
- *the Universe was once in thermodynamic equilibrium*

The CMB as a Scattering Problem

recall: *any* observed photon has this life cycle:

- emission
- scattering (possibly none, possibly many times)
- absorption (i.e., detection)

thus: any *detected* = absorbed photon
points back to emission or most recent scattering event
e.g., daytime sky: Sun's emission disk vs off-source scattered blue light

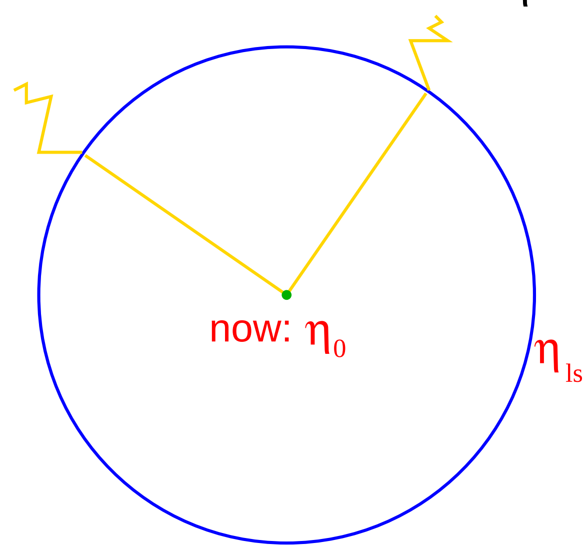
the fact that the CMB is a *background*
to low-z objects $\rightarrow$ late-time U. is *transparent* to CMB

thus: the CMB probes exactly the epoch
when the universe was last able to scatter photons
i.e., the last time U. was *opaque* to its thermal photons

CMB as Cosmic "Baby Picture": Last Scattering Surface

CMB created by (and gives info about)
epoch of cosmic transition: *opaque* $\rightarrow$ *transparent*

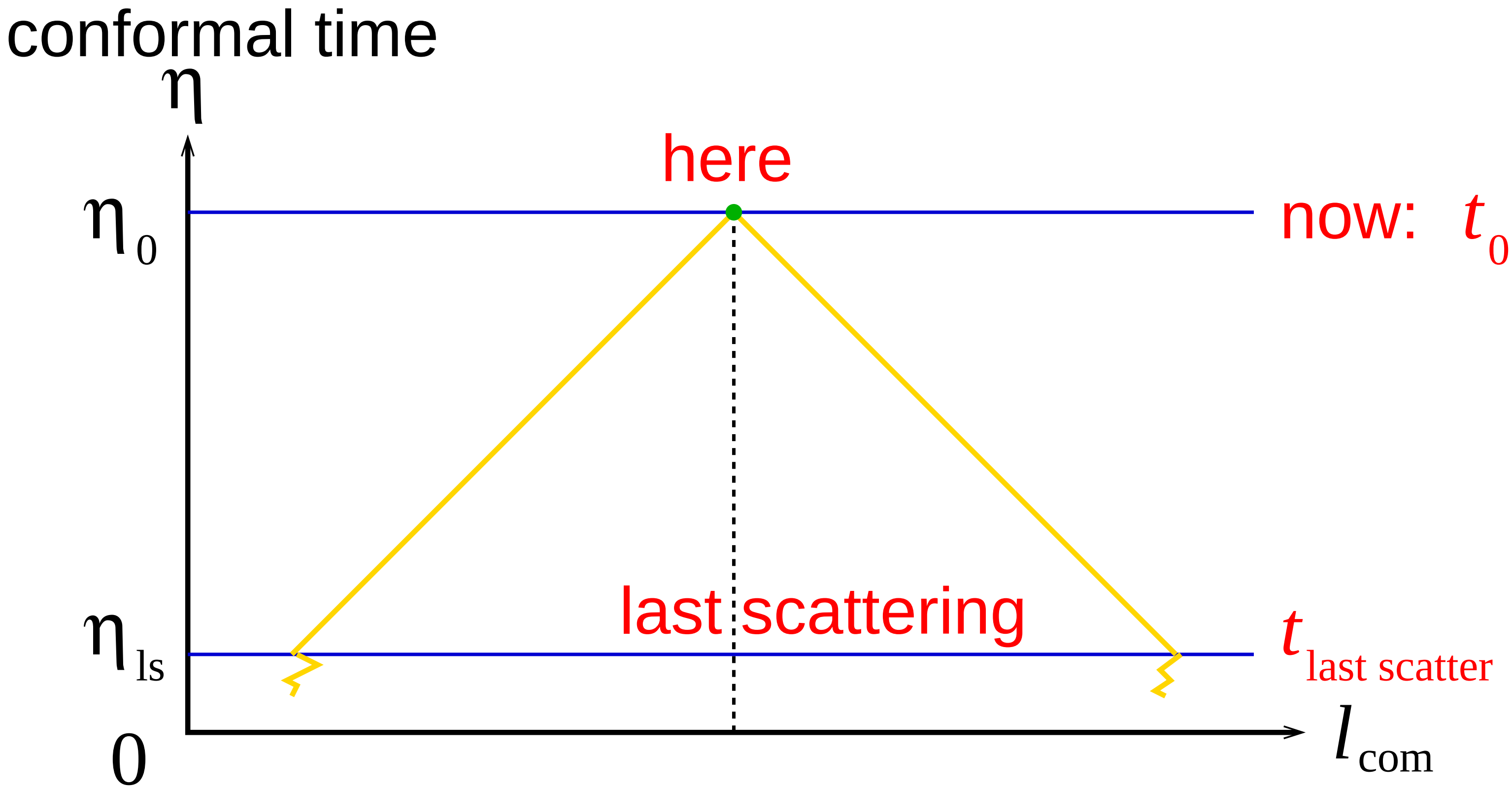
but transparent/opaque transition is
controlled by photon *scattering*



CMB "released" to "free stream"
at epoch of **"last scattering"** $z_{\rm ls}$
$\rightarrow$ CMB sky map is a picture of U. then:
"**surface of last scattering**"

akin to photosphere of the Sun, but
"cosmic photosphere" is seen from inside!

Q: CMB photons in spacetime diagram?



For more detail, e.g., when is z_{ls}?

$\rightarrow$ need scattering technology

Highlights from Scattering 101

Collisions: $a + b \to$ stuff

Consider particle beam:
"projectiles," number density n_a
incident w/ velocity v
on targets of number density n_b

Due to interactions, targets and projectiles "see" each other
as spheres of projected area $\sigma(v)$: the

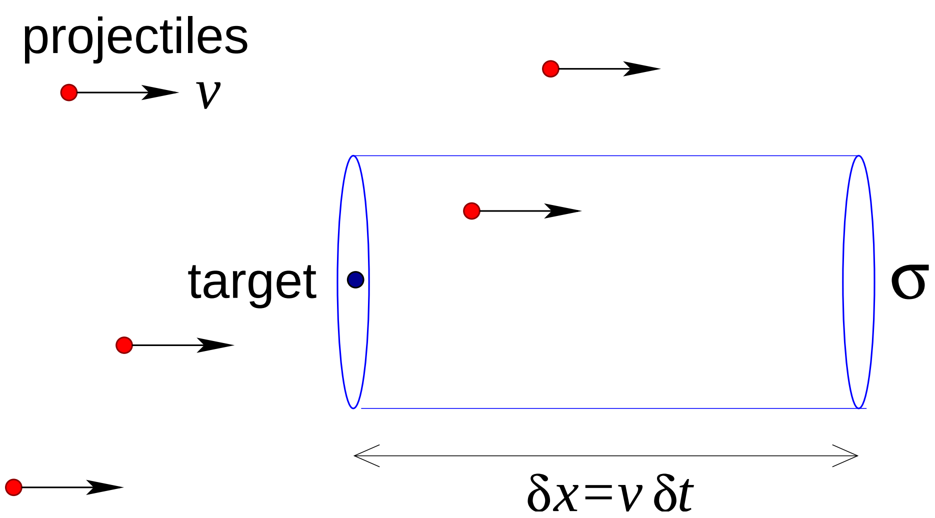
cross section

⋆ fundamental measure interaction strength/probability
⋆ *atomic, nuke & particle physics meets astrophysics via* σ

in time δt, what is avg # collisions on one target?
Q: what defines "interaction zone" around target?

interaction zone: particles sweep out "scattering tube"

- *projectiles see targets as "bulls-eyes" of size* σ
  ...and vice versa!
  sets tube cross-sectional area
- tube length $\delta x = v\delta t$



interaction volume swept around target:
$\delta V = \sigma\delta x = \sigma v\delta t$

collide: if a projectile is in the volume

Cross Section, Flux, and Collision Rate

in tube volume δV, # projectiles = $\mathcal{N}_{\rm proj} = n_a \delta V$
so ave # collisions in δt:

$$\delta\mathcal{N}_{\rm coll} = \mathcal{N}_{\rm proj} = n_{\rm a} \ \sigma \ v \ \delta t \tag{10}$$

so $\delta\mathcal{N}_{\rm coll}/\delta t$ gives

avg collision rate *per target* b $\boxed{\Gamma_{{\rm per}\, b} = n_a \ \sigma \ v = \sigma \ j_a}$

where $j_a = n_a v$ is incident **flux**
Q: Γ units? sensible scalings n_a, σ, v*? why no* n_b*?*

Q: average target collision time interval?
 Q: average projectile distance traveled in this time?

estimate avg time between collisions on target b:

mean free time τ

collision rate: $\Gamma = d\mathcal{N}_{\text{coll}}/dt$

so wait time until next collision set by $\delta N_{\text{coll}} = \Gamma_{\text{per}\,b}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\text{per}\,b}} = \frac{1}{n_a \sigma v} \tag{11}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\text{mpf}} = v\tau = \frac{1}{n_a \sigma} \tag{12}$$

no explicit v dep, but still $\ell(E) \propto 1/\sigma(E)$

Q: physically, why the scalings with n, σ?

Q: what sets σ for billiard balls?

Q: what set σ for $e^- + e^-$ scattering?

Cross Section vs Particle "Size"

if particles interact only by "touching"
(e.g., classical, macroscopic billiard balls)
then $\sigma \leftrightarrow$ particle radii: $\sigma = \pi(r_a + r_b)^2$

but: if interact by force field
(e.g., gravity, EM, nuke, weak)
cross section σ *unrelated* to physical size!

For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
"touch-free scattering"

Reaction Rate Per Volume

recall: collision rate *per target* b is $\Gamma_{\text{per}\,b} = n_a \sigma_{ab} v$
total collision rate *per unit volume* is

$$r = \frac{dn_{\text{coll}}}{dt} = \Gamma_{\text{per}\,b} n_b = \frac{1}{1+\delta_{ab}} n_a n_b \sigma v \tag{13}$$

Kronecker δ_{ab}: 0 unless particles a & b identical
Note: *symmetric* w.r.t. the two particles

What if particles have more than one relative velocity?