Astro 507

Lecture 22

March 12, 2014

Announcements:

- **Preflight 4 due 9am Friday**

Last time:

recombination $\rightarrow$ huge drop in free e^- $\rightarrow$ CMB freeze/decouple

to calculate in detail: need cosmic statistical mechanics

key inputs: uncertainty principle, Boltzmann factor,

and baryon-to-photon ratio $\eta = n_{\rm b}/n_\gamma$

Q: whatsa baryon? η order of magnitude?

key outputs: non-rel, non-degen $n = g(mT/2\pi\hbar^2)^{3/2} e^{-(m-\mu)/T}$

for reaction in ("chemical") equilibrium: $\sum \mu_i = \sum \mu_f$

Q: apply to recombination?

The Mighty Saha Equation

Recombination: equal forward and reverse rates for

$$p + e \leftrightarrow \mathrm{H} + \gamma$$

and so chem potentials have

$$\mu_p + \mu_e = \mu_{\mathrm{H}} \tag{1}$$

for non-rel species $n = g(mT/2\pi\hbar^2)^{3/2} e^{-(m-\mu)/T}$
thus we have **Saha equation**

$$\frac{n_e n_p}{n_{\mathrm{H}}} = \frac{g_e g_p}{g_{\mathrm{H}}} \left(\frac{m_e m_p}{m_{\mathrm{H}}}\right)^{3/2} \left(\frac{T}{2\pi\hbar^2}\right)^{3/2} e^{-(m_e+m_p-m_{\mathrm{H}})/T} \tag{2}$$

$$\approx \left(\frac{m_e T}{2\pi\hbar^2}\right)^{3/2} e^{-B/T} \tag{3}$$

where $B \equiv m_e + m_p - m_{\mathrm{H}} = 13.6$ eV introduce "free electron fraction" $X_e = n_e/n_B$

use $n_B = \eta n_\gamma \propto \eta T^3$

from Extras last time: $n_\gamma = 2\zeta(3)/\pi^2 \, T^3$, with $\zeta(3) = \sum_1^\infty 1/n^3 = 1.20206\ldots$

and note that $n_p = n_e$ *Q: why?*, so

$$\frac{n_e^2}{n_{\mathrm{H}} n_B} = \frac{X_e^2}{1 - X_e} = \frac{\sqrt{\pi}}{4\sqrt{2}\zeta(3)} \frac{1}{\eta} \left(\frac{m_e}{T}\right)^{3/2} e^{-B/T} \qquad (4)$$

Q: sanity checks? what sets characteristic T scale?

Q: when is $X_e = 0$ (exactly)?

At last–recombination!

Q: how define physically?

Q: how define operationally, in terms of X_e?

Q: given some $X_{e,\mathrm{rec}}$, how to get z_{rec}?

The Epoch of Recombination

Saha gives

$$\frac{1 - X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\pi^{1/2}} \, \eta \, \left(\frac{B}{m_e}\right)^{3/2} \left(\frac{T}{B}\right)^{3/2} e^{B/T} \tag{5}$$

if always equilib, then strictly $X_e = 0$ only at $T = 0$
but note $e^{B/T}$: X_e exponentially small when $T \ll B$

viewed as a function of $B/T \equiv u$

$$\frac{1 - X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\pi^{1/2}} \, \eta \, \left(\frac{B}{m_e}\right)^{3/2} \, u^{3/2} e^{u} \equiv A \, u^{3/2} e^{u} \tag{6}$$

where $A = 4\sqrt{2}/\pi^{1/2}\zeta(3) \, \eta \, (B/m_e)^{3/2}$

Q: what is order-of-magnitude of A?
Q: implications for recombination?
Q: physical picture?

in recombination Saha expression $(1 - X_e)/X_e = A(B/T)^{3/2} e^{B/T}$
prefactor is tiny!
$A \sim \eta(B/m_e)^{3/2} \sim 10^{-9}(10^{-5})^{3/2} \sim 10^{-16}$!
why? largely due to *tiny baryon-to-photon ratio*

but when recombine: $1 - X_e \simeq X_e$
so require $1 \sim 10^{-16}(B/T_{\rm rec})^{3/2} e^{B/T_{\rm rec}}$
$\Rightarrow$ so need $B/T_{\rm rec} \gg 1$ to offset prefactor
$\Rightarrow$ and thus $T_{\rm rec} \ll B$!

more carefully define recomb: $X_e = X_{e,\rm rec} = 0.1$
(arbitrary, but not crazy; see PS4)
then solve for $T_{\rm rec}$:

$$\frac{B}{T_{\rm rec}} = \ln\left(\frac{\pi^{1/2}}{4\sqrt{2}\zeta(3)}\right) + \ln\left(\frac{1 - X_{e,\rm rec}}{X_{e,\rm rec}^2}\right) + \ln \eta^{-1} + \frac{3}{2}\ln\frac{m_e}{B} + \frac{3}{2}\ln\frac{B}{T}$$

$$\sim 40 \quad (\gg 1)$$

(ignore or iterate $\ln B/T$ term)

Recombination Quantified

and so

$$T_{\rm rec} \approx \frac{B}{40} \simeq 0.3 \text{ eV} \ll B \tag{7}$$

$$z_{\rm rec} \approx 1400 \ll z_{\rm rec,naive} \tag{8}$$

$$t_{\rm rec} \approx \frac{2}{3\sqrt{\Omega_{\rm m}}} H_0^{-1}(1+z_{\rm rec})^{-3/2} = 350,000 \text{ yrs} \tag{9}$$

PS4: try it yourself!

Implications for CMB frequency spectrum:

- at recomb: emission lines created at $h\nu_{\rm rec} \gtrsim 3B/4$ and thus at $h\nu_{\rm rec} \gtrsim 30kT_{\rm rec}$
- post-recomb: T and ν both redshift the same way, so
- CMB spectrum *distorted* from Planck at high freq: $h\nu \gtrsim 30kT$
- small signal, difficult to observe, but tantalizing `www: predictions`

Q: what physically is responsible for $T_{\rm rec} \ll B$?

Recombination "Delay"

Why is $T_{\text{rec}} \ll B$?

▷ because for small X_e, Saha says $X_e \propto 1/\eta^{1/2} \gg 1$

▷ many photons per baryon: even if typically $E_\gamma \ll B$,
high-E tail of Planck distribution not negligible (at first)
lots of ionizing photons with $E_\gamma \geq B$
H dissociated as soon as formed

When does dissociation stop?
can show that fraction of photons with $E_\gamma > B$
is roughly $f_{\text{ionizing}} \sim e^{-B/T}$
so ratio of ionizing photons per baryon is

$$\frac{n_{\gamma,\text{ionizing}}}{n_B} \sim \frac{e^{-B/T}}{\eta} \tag{10}$$

estimate recombination when $n_{\gamma,\text{ionizing}}/n_B \sim 1$
$\rightarrow T \sim B/\ln \eta^{-1} \ll B$ (check!)
$\Rightarrow$ recombination "delayed" to huge photon-to-baryon ratio

Recombination: Hydrogen Level Population

recall: Boltzmann expression for atomic hydrogen (H I):

$$\frac{n(2P)}{n(1S)} = 3e^{-3B_{\mathrm{H}}/4T} = 3e^{-120{,}000\,\mathrm{K}/T} \qquad (11)$$

Q: implications for H populations?

consider recombining $p + e \rightarrow H + \gamma$ throughout recomb:

Q: what is γ energy at emission?

Q: what happens to γ?

Q: implications?

∞

Recombination: Nonequilibrium Effects

for $p + e \to \mathrm{H}(n = 1) + \gamma$:

- $E_\gamma = B_\mathrm{H}$ "Lyman limit"
- H atoms *absorption cross section huge* at this energy
  photon mean free path $\ell = 1/n_\mathrm{H}\sigma_\mathrm{abs}$ tiny
  universe optically thick to Lyman photons

$\Rightarrow$ quickly reionizes another H atom! *no net change!*

To overcome delay

- recombine to 1st excited state: $p + e \to \mathrm{H}(2p) + \gamma$
- single photon $\mathrm{H}(2p) \to H(1s) + \gamma$ Lyα transition
  also optically think, also no net progress
- *two-photon transition* $\mathrm{H}(2p) \to H(1s) + \gamma + \gamma$ can go
  but probability & rate smaller than for single photon
- eventually redshifting takes Lyman photons off resonance

net effect: *delays recombination relative to Saha*

Last Scattering: Photons Decouple from Matter

"recombination" a smooth transition in X_e, not instantaneous
`www: equilibrium` X_e `plot`
nevertheless, exponential drop in X_e around $z_{\rm rec}$

photons interact with gas via Thomson scattering: $\gamma e \to \gamma e$
rate per photon of scattering with e:

$$\Gamma_e(\gamma) = n_e \sigma v = n_e \sigma_T c = X_e n_{\rm b} \sigma_T c \tag{12}$$

drop in $X_e \rightarrow$ abrupt slowdown in scattering

as usual, competition between interaction and expansion
interactions "stop" when

$$\Gamma_e(\gamma) \lesssim H \tag{13}$$

and solving for $\Gamma_e(T) = H(T)$ gives last scattering :

$$\boxed{z_{\rm ls} \sim 1100} \tag{14}$$

After last scattering:

- photons "decoupled" from gas
- but $X_e \neq 0$: some free e, p remain

  Q: what is X_e as $T \to 0$? why?

Freezing of Recombination

when typical photon has last scattering with e
still some residual ionization: i.e., some free e, p
can they recombine? yes!
do they recombine? yes, for a short while...then no!

Why? recombination rate per p: $\Gamma_{\text{rec},p} \sim n_e \sigma_{\text{rec}} v_{\text{therm}}$
with $\sigma_{\text{rec}} \sim (m_e/T)\sigma_{\text{T}}$ and $v_{\text{therm}} \sim \sqrt{T/m_e}$
recombination stops when $\Gamma_{\text{rec},p} \lesssim H$

after this: cooling does not reduce ionization
fixed value of $X_{e,\text{freeze}} \sim 10^{-4}$: "freeze-in of residual ionization"
at

$$z_{\text{ri}} \simeq 1000 \tag{15}$$

Q: cosmological implications of $X_{e,\text{freeze}} \neq 0$?

Recombination Timeline Summarized

The large drop in free electron density around $z \sim 1000$ leads to three distinct but related events:

(1) recombination U. **ionized $\to$ neutral**

$X_e \to X_{e,\mathrm{rec}} \sim 0.1$: $z_{\mathrm{rec}} \sim 1300$

...but photons still coupled to gas, and vice versa

(2) last scattering typical photons no longer interacts with e

U. **opaque $\to$ transparent**

$\Gamma_e(\gamma) \sim H$: $z_{\mathrm{ls}} \sim 1100$

...but gas still coupled to photons *Q: how can this be?*

$T_{\mathrm{gas}} = T_{e,p,\mathrm{H}} = T_\gamma$

(3) residual ionization freeze-in

free e and p diluted until "can't find each other"

But even still: photons scatter off residual ionization
e and thus p, H still exchange energy
with thermal photon bath: $T_{e,p,\mathrm{H}} = T_\gamma$ still!
when does this stop?

(4) gas decoupling

typical residual e no longer has photon interactions
gas decouples from photons
when? Thomson scattering rate *per* e: $\Gamma_e = n_\gamma \sigma_\mathrm{T} c \lesssim H$
at $z_{\mathrm{dec,gas}} \sim 500$
note: scatter rate *per* e=$\Gamma_e \gg \Gamma_\gamma$=*scatter rate per CMB photon*

Summary of CMB Highlights

CMB Observed

can make precision observations of spectrum, sky distribution thanks to sophisticated radio techniques and instruments

- CMB fantastically isotropic: $\delta T/T \sim few \times 10^{-5}$
- CMB exquisitely thermal

CMB Theory

detailed, precise calculations of recomb, last scattering, thanks well-known atomic physics

- isotropic CMB $\rightarrow$ U. was once very homogeneous
- Planckian CMB spectrum $\rightarrow$ U. was once thermalized
  $\rightarrow$ plasma hot, dense enough to equilibrate

CMB $\rightarrow$ demands hot big bang in FLRW universe!

Extrapolated current U to $t \sim 400,000$ yr
and $z \sim 1000$ $\rightarrow$ **great success!**
Emboldens us to push earlier!

Primordial Nucleosynthesis

Prelude to Nucleosynthesis

Q: what sets T scale for element (nuclei) synthesis?

Q: what component dominates cosmic density, expansion then?

Q: what is the particle content of the universe then?

Q: what form(s) do the baryons take then? mesons?

Nucleosynthesis: Setting the Stage

⋆ light elements formed in nuclear reactions
relevant scale: nuclear binding energies $\sim$ MeV

⋆ $T \sim$ MeV at redshift $z_{\rm bbn} = T/T_0 - 1 \sim 10^{10}$!
since $z_{\rm bbn} \gg z_{\rm eq}$ (matter-rad equality)
well into radiation dominated era: $\rho \approx \rho_{\rm rad}$
`www:` Ω `vs` a `plot`
will see: $t(1 \text{ MeV}) \sim 1$ sec

⋆ particle content at BBN
relativistic species: photons, neutrinos, $e^{\pm}$ when $T \gtrsim m_e$
non-relativistic species: baryons, e^{-} when $T \ll m_e$
what about dark matter? energy?
DM presumably non-rel, weakly interacting: inert during BBN
DE: also assume not important for dynamics, microphyiscs
...but can later relax these assumptions and test them!

Who Feels What? Particles and Forces

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{matrix} \text{charm quark} \\ \text{strange quark} \\ \text{mu lepton (muon)} \\ \\ \end{matrix} \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{matrix} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \\ \end{matrix} \qquad (16)$$

quarks: feel all fundamental forces (strong, EM, weak, gravity)
carry conserved quantum number: **baryon number**
leptons: do *not* feel strong force
but also carry conserved quantum number: **lepton number**

- ▷ *charged* leptons: feel EM, weak, gravity
- ▷ neutrinos: only feel weak, gravity

More bragging rights:
in BBN, *all four* fundamental forces play a crucial role!