Astro 507
Lecture 23
March 14, 2014

Announcements:

- **Preflight was due this morning**
- **PS4 posted today, due next Friday**
- CfA Mystery announcement 11am Monday March 17
  `www: rumor` inflation gravity waves in CMB?

PF4 Discussion Question: A "Heliocentric" Universe
Vote: $T_0^{\text{He-dom}}$ *vs* $T_0^{\text{H-dom}}$*?*
Vote: in the alt universe, what is CMB spectrum I_ν*?*
Vote: $\Omega_\gamma^{\text{He-dom}}$ *vs* $\Omega_\gamma^{\text{He-dom}}$*?*

Last time: began big bang nuke & particle cosmology
Q: BBN vs CMB similarities? differences?
Q: characteristic T*? what cosmic dynamics?*
Q: what will be relativistic? nonrelativistic?

Neutrinos: Essential Ingredient yet Barely There

antineutrinos: $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$
since electric charge $Q(\nu) = 0$, possible that ν is own antiparticle
Q: is it?

masses: known that m_ν are nonzero (oscillations observed)
mass values not known (but for sure $\lesssim few \times 10$ eV $\ll m_e$)
Q: implications for BBN?
for quarks and charged leptons, masses increase with each family
$\rightarrow$ same for νs??

weak interaction: qualitative characteristics
(1) "signature" is transformation of quark, lepton flavor
e.g., β decays like $n \rightarrow p + e^- + \bar{\nu}_e$
really a quark change $d(ud) \rightarrow u(ud) + e^- + \bar{\nu}_e$
(2) for $E \lesssim 100$ GeV $(= M_W, M_Z)$, rxn strength is weak (duh!)
e.g., $\nu_e e \rightarrow \nu_e e$ scattering ~ 1 MeV: $\sigma_{\nu_e e} \sim 10^{-44}\ \mathrm{cm}^2 \sim 10^{-20} \sigma_T$

Nucleosynthesis: Particle Content Revisited

relativistic species:
γ, $\nu_i \bar{\nu}_i$ ($i \in e\mu\tau$), $e^{\pm}$ (for $T \gtrsim m_e$)

non-relativistic species:
baryons in BBN: when $T \gtrsim$ MeV: p, n only
when $T \leq m_e \to e$ non-rel too

⋆ neutrinos in BBN
Q: what sets n_ν, ρ_ν, T_ν? how do they evolve?
Q: assumptions needed?

BBN Initial Conditions: Ingredients of Primordial Soup

Begin above nuke binding: $T > 1$ MeV

EM reactions fast: typical rate $\Gamma_{\rm EM} \sim n_\gamma \sigma_{\rm T} c \gg H$
$\Rightarrow$ baryon, photon, $e^\pm$ pair plasma in thermal equilib:
$T_B = T_e = T_\gamma \equiv T$

weak int fast too (for now)! $\Gamma_{\rm weak} \sim n_\nu \sigma_{\rm weak} c \gg H$
all ν species coupled to each other, and plasma
$\rightarrow T_\nu = T_\gamma$

What sets densities n_ν, ρ_ν?
not only T_ν, but also dreaded chem potential μ_ν
physics issue: is there a net neutrino excess: $n_\nu \neq n_{\bar{\nu}}$?
c.f. net baryon excess $\rightarrow$ exists: $n_B \neq n_{\bar{B}}$, but small: $n_B/n_\gamma \ll 1$
if net lepton number $n_L \sim n_B$, turns out $\mu_\nu/T \sim \eta$ negligible
we will assume $\mu_\nu \ll T \Leftrightarrow$ no large lepton/baryon excess
if otherwise, changes story!

BBN Initial Conditions: Radiation Domination

Neutrino densities: for sure $m_\nu \ll T$
assume $\mu_\nu \ll T$ $\rightarrow$ absolute n_ν, ρ_ν, P_ν set by T_ν
$\rightarrow$ each ν species i has $n_{\nu_i} = n_{\bar{\nu}_i}$ and

$$n_{\nu\bar{\nu},i} \propto T^3 = \frac{3}{4} n_\gamma \quad \rho_{\nu\bar{\nu},i} \propto T^4 = \frac{7}{8} \rho_\gamma \tag{1}$$

total relativistic energy density:

$$\rho_{\rm rel} = \rho_\gamma + \rho_{e^\pm} + N_\nu \rho_{1\nu\bar{\nu}} \equiv g_* \frac{\pi^2}{30} T^4 \tag{2}$$

where g_* counts *"effective # of relativistic degrees of freedom"*
at $T \gtrsim 1$ MeV, $g_* = 43/4 = 10.75$, and Friedmann:

$$\frac{t}{1\ \text{sec}} \approx \left(\frac{1\ \text{MeV}}{T} \right)^2 \tag{3}$$

Q: simple way to see $t \sim 1/T^2$ *scaling is right?*

now focus on baryons *Q: what sets* n_B*?* n/p*?*

BBN Initial Conditions: The Baryons

Cosmic baryon density n_B, and thus $\eta = n_B/n_\gamma$
not changed by reactions with $T \lesssim E_{\text{Fermilab}} \sim 1$ TeV $= 10^6$ MeV
i.e., baryon non-conservation not observed to date

▷ n_B set somehow in early universe ("cosmic baryogenesis")

▷ don't *a priori* know n_B, treat as free parameter (η)

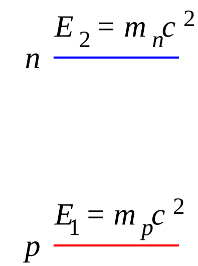
neutron-to-proton ratio n/p *can* and *does* change at ~ 1 MeV
weak int fast: $n \leftrightarrow p$ interconversion

$$n + \nu_e \leftrightarrow p + e^- \quad (4)$$
$$p + \bar{\nu}_e \leftrightarrow n + e^+ \quad (5)$$

also recall $m_n - m_p = 1.29$ MeV: close in mass but not same!

Q: implications for n/p?

n/p ratio "thermal"
think of as *2-state* system: the "nucleon,"



- nucleon "ground state" is the *proton*: $E_1 = m_p c^2$
- nucleon "excited state" is the *neutron*: $E_2 = m_n c^2$

when in equilibrium, Boltzmann sez:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2 - E_1)/T} = e^{-(m_n - m_n)/T} \qquad (6)$$

with $\Delta m = m_n - m_p = 1.293318 \pm 0.000009$ MeV

at $T \gg \Delta m$: $n/p \simeq 1$
at $T \ll \Delta m$: $n/p \simeq 0$

Equilibrium maintained until weak interactions freeze out
i.e., competition between weak physics, gravity physics
Q: how will weak freezeout scale compare to
nuclear binding energy scale ~ 1 *MeV?*

Weak Freezeout Temperature

Weak interactions freeze when $H = \Gamma_{\mathrm{weak}}$, i.e.,

$$\sqrt{G_{\mathrm{N}}}\,T^2 \;\sim\; \sigma_0 m_e^{-2} T^5 \tag{7}$$

$$\Rightarrow T_{\mathrm{weak\ freeze}} \;\sim\; \frac{(G_{\mathrm{N}})^{1/6}}{(\sigma_0/m_e^2)^{1/3}} \sim 1\ \mathrm{MeV} \tag{8}$$

gravity & weak interactions conspire to give $T_{\mathrm{f}} \sim m_e \sim B_{\mathrm{nuke}}$!

for experts: note that $G_{\mathrm{N}} = 1/M_{\mathrm{Planck}}^2$, so

$$\frac{T^2}{M_{\mathrm{Pl}}} \;\sim\; \alpha_{\mathrm{weak}} \frac{T^5}{M_W^2} \tag{9}$$

$$\Rightarrow T_{\mathrm{freeze}} \;\sim\; \left(\frac{M_W}{M_{\mathrm{Pl}}}\right)^{1/3} M_W \sim 1\ \mathrm{MeV} \tag{10}$$

freeze at nuclear scale, but by accident!

Q: what happens to n, p *then? what else is going on?*

Element Synthesis

first step in building complex nuclei: $n + p \to d + \gamma$
but $d + \gamma \to n + p$ until $T \ll B(d)$; see Extras

when photodissocation ineffective, $n + p \to d + \gamma$ fast
rapidly consumes all free n and builds d
which can be further processed to mass-3:

$$d + p \to {}^3\mathrm{He} + \gamma \quad d + d \to {}^3\mathrm{H} + p \quad d + d \to {}^3\mathrm{He} + n \tag{11}$$

and to ^{4}He

$$^3\mathrm{H} + d \to {}^4\mathrm{He} + n \quad {}^3\mathrm{He} + d \to {}^4\mathrm{He} + p \tag{12}$$

some of which can then make mass-7:

$$^3\mathrm{H} + {}^4\mathrm{He} \to {}^7\mathrm{Li} + \gamma \quad {}^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma \tag{13}$$

Q: what limits how long these reactions can occur?
Q: which determines which products are most abundant?

BBN Reaction Flows

Binding Energy

nuclei are bound quantum structures, confined by nuclear forces among the "nucleons" n, p

can quantify degree of stability–i.e., resistance to destruction via binding energy: for nucleus with Z protons, N neutrons, $A = N + Z$ nucleons

$$\begin{aligned} B_A &= \text{energy of individual parts} - \text{energy of bound whole} \\ &= (Z m_p + N m_n - m_A) c^2 \\ &> 0 \ \text{ if bound} \end{aligned}$$

note: generally B_A increases with A

but that's not the whole story on stability

binding shared among all A nucleons,
so binding per nucleon is B_A/A

nuclear stability $\leftrightarrow$ high B_A/A

`www: plot of` B_A/A `vs` A
lowest binding/nucleon: d!
highest: ^{56}Fe, but among light elements, ^{4}He highest by far
Q: implications for BBN

Reaction flows: tightest binding favored

$\rightarrow$ essentially all pathways flow to ^{4}He

`www: nuke network`

almost all $n \rightarrow {}^4$He:

$n(^4\text{He})_{\text{after}} = 1/2\ n(n)_{\text{before}}$

$$Y_p = \frac{\rho(^4\text{He})}{\rho_B} \simeq 2(X_n)_{\text{before}} \simeq 0.24 \qquad (14)$$

$\Rightarrow \sim 1/4$ of baryons into ^{4}He, 3/4 $p \rightarrow$H

result weakly (log) dependent on η

Robust prediction: large universal ^{4}He abundance

But $n \to {}^4$He incomplete: as nuke rxns freeze,
leave traces of:

- D
- ^{3}He (and ^{3}H$\to$^{3}He)
- ^{7}Li (and ^{7}Be$\to$^{7}Li)

abundances $\leftrightarrow$ nuke freeze T
trace species D, ^{3}He, ^{7}Li: strong $n_B \propto \eta$ dependence

BBN theory predictions summarized in **"Schramm Plot"**

Lite Elt Abundances vs η

`www: Schramm plot`

Note: no $A > 7$...so no C,O,Fe... *Q: why not?*

Why no elements $A > 7$?

1. Coulomb barrier

2. nuclear physics: "mass gaps"
no stable nuclei have masses $A = 5, 8$
$\rightarrow$ with just p & ^{4}He, can't overcome via 2-body rxs
need 3-body rxns (e.g., $3\alpha\rightarrow^{12}$C) to jump gaps
but ρ, T too low

Stars *do* jump this gap, but only because have higher density a long time compared to BBN

Testing BBN: Warmup

BBN Predictions: Lite Elements vs η

To test: measure abundances

Where and when do BBN abundances (Schramm plot) apply?

Look around the room–not 76% H, 24% He.
Is this a problem? Why not?

Solar system has metals not predicted by BBN
Is this a problem? Why not?

So how test BBN? What is the key issue?

When does first non-BBN processing start?

Testing BBN: Lite Elements Observed

Prediction:

BBN Theory $\rightarrow$ lite elements at $t \sim 3$ min, $z \sim 10^9$

Problem:

observe lite elements in astrophysical settings

typically $t \gtrsim 1$ Gyr, $z \lesssim few$

stellar processing alters abundances

Q: If measure abundances in a real astrophysical system, can you unambiguously tell that stars have polluted?

Q: How can we minimize (and measure) pollution level?

stars not only alter light elements

but also make heavy element = “metals”

stellar cycling: metals $\leftrightarrow$ time

Solution:

$\rightarrow$ measure lite elts and metals

low metallicity $\rightarrow$ more primitive

in limit of metals $\rightarrow$ 0: primordial abundances!

look for regions with low metallicity $\rightarrow$ less processing

Directors' Cut Extras

Elementary Particles for the Minimalist
Antimatter

fundamental result of Relativistic QM
every particle has an antiparticle
e.g., $\overline{e^-} = e^+$ positron
e.g., $\bar{p}$ = antiproton; Fermilab: $p\bar{p}$ collisions

note: mass $m(\bar{x}) = m(x)$
decay lifetime $\tau(\bar{x}) = \tau(x)$
spin $S(\bar{x}) = S(x)$
electric charge $Q(\bar{x}) = -Q(x)$

sometimes particle = own antiparticle (must have charge 0)
e.g., $\bar{\gamma} = \gamma$, but note: $\bar{n} \neq n$

Cosmic Antimatter: rule of thumb
$x, \bar{x}$ abundant when thermally produced: $T > m_x$

Baryons

n and p not fundamental particles
made of 3 pointlike particles: "quarks"
two types ("flavors") in n, p: u "up," d "down"
$p = uud$, $n = udd \to$ quark electric charge $Q_u = +2/3$, $Q_d = -1/3$
spin $S(u) = 1/2 = S(d)$

baryon $\equiv$ made of 3 quarks

baryon conservation:
assign "baryon number" $A(q) = +1/3$, $A(\bar{q}) = -1/3$
$\to A(n) = A(p) = +1$
in all known interactions: baryon number conserved:
$\sum A_{\text{init}} = \sum A_{\text{fin}}$
$\to$ guarantees stability of the proton *Q: why?*
but free n unstable, decay to p *Q: why not* n *decay in nuclei?*

Periodic Table of Elementary Particles

known fundamental particles (& antipartners): 3 **families**

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{matrix} \text{charm quark} \\ \text{strange quark} \\ \text{mu lepton (muon)} \\ \ \end{matrix} \quad \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{matrix} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \ \end{matrix} \tag{15}$$

all of these are spin-1/2: matter made of fermions!

Family Resemblances

1st family: quarks, charged lepton (e) comprise ordinary matter

2nd, 3rd family particles

- same electric charges, same spins, (mostly) same interactions as corresponding 1st family cousins
- but 2nd, 3rd family quarks, charged leptons more massive and & *unstable* $\rightarrow$ decay into 1st family cousins

lifetimes very short, e.g., longest is $\tau(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = 2 \times 10^{-6}$ s

Q: implications for BBN epoch?

Weak $n \leftrightarrow p$ Rates

example: want rate Γ_n per n of $\nu + n \to e^- + p$
as func. of T

Generally,

$$\Gamma_n = n_\nu \langle \sigma v \rangle \sim T^3 \langle \sigma \rangle \quad (16)$$

since $v_\nu \simeq c$

can show: cross section $\sigma \sim \sigma_0 (E_e/m_e)^2$
where $\sigma_0 \sim 10^{-44}$ cm^2 very small!
so thermal avg: $\langle \sigma \rangle \sim \sigma_0 (T/m_e)^2$

for experts: $\sigma \sim G_F^2 T^2 \sim \alpha_{\rm weak} T^2 / M_W^4$
so $\Gamma_{\rm weak} \sim \alpha_{\rm weak} T^5 / M_W^4$