Astro 507

Lecture 27

March 31, 2014

Announcements:

- **PF 5 due Friday**

Last time: finished big bang nuke

- CMB + BBN: high-precision light element predictions
  D/H observation agreement spectacular
  ^{4}He agreement good
  ^{7}Li predictions $>$ observation at $4-5\sigma$
- in detail: lithium problem
  observational systematics? `www: extragalactic lithium`
  new physics?
- stepping back–at least rough agreement
  big bang working back to $t \sim 1$ sec
  press on to earlier epochs

Particle Dark Matter

BBN and Particle Dark Matter

BBN motivates dark matter theory & searches two ways:

Quantitative. $\Omega_B \ll \Omega_{\rm m}$: must have non-baryonic dark matter
...and lots of it!

Qualitative. BBN success at $t \sim 1$ s $\rightarrow$ early U as physics lab

"The universe is the poor man's particle accelerator"
– Ya. Zel'dovich

Big implications for–and motivations from–particle physics

Q: what can we say about DM properties generally?

Q: what can we say if DM is in particle form?
lifetime, mass, interactions, quantum #s?

Q: what known particles are candidates for non-baryonic DM?

Q: does particle theory offer dark matter candidates?

Elementary Particle Physics and Dark Matter

Dark matter
dark: no/feeble EM, strong interactions
matter: behaves as nonrelativistic material $\rightarrow \rho \propto a^{-3}$, $P \ll \rho c^2$
naturally leads to hypothesis of DM as
Weakly Interacting Massive Particles: WIMPs

If DM is swarms of WIMPs, what are their properties?

lifetime: must exist today $t_0 \sim 14$ Gyr
$\rightarrow$ stable or very long-lived

mass: don't know!
only know mass dens $\rho_{\mathrm{m},0}$ today on cosmic, galactic scales
but without also knowing # dens $n_{\mathrm{m},0}$, can't get $m = \rho/n$
$\rightarrow$ in fact, with specific model, from m get n_0

Could the Dark Matter be Neutrinos?

interactions/quantum #s:
BBN: dark matter not baryonic

Standard Model of particle physics *does* provide
a candidate for non-baryonic DM
stable + massive: neutrinos; can show (PS5):

$$\Omega_\nu h^2 = \frac{\sum_{\text{species}} m_\nu}{92 \text{ eV}} \tag{1}$$

...but can show (β decay, ν oscillations, CMB, LSS)
$\sum_{\text{species}} m_\nu \lesssim 1$ eV, and so

$$\Omega_\nu \sim 0.01 < \Omega_B \ll \Omega_{\text{m}} \tag{2}$$

we see: νs *are* non-baryonic DM
but *negligible contribution to density*
most dark matter is not neutrinos!

Q: other Standard Model candidates?

no other Standard Model particle candidates viable

non-baryonic DM demands physics beyond the Standard Model

particle candidates available "off the shelf"
in models of physics Beyond the Standard Model
i.e., particle physics models designed to explain
origin of standard model features

examples:
lightest supersymmetric particle, axion, strangelets...

Q: how are WIMPs produced in early U?

Particle Dark Matter: Thermal Relics

Kolb & Turner, Ch. 5; Dodelson Ch. 3.4

Consider stable particle species χ (& antiparticle $\bar{\chi}$)

- nonrelativistic today: $m_\chi \gg T_0 \sim 3 \times 10^{-4}$ meV
- thermally produced in the early universe

What determines its abundance today?

Q: if χ is still in thermal (chemical) eq?

Q: and so?

Relic Particles

for non-relativistic species:
if *still in (chemical) equilibrium*: number density

$$n_\chi = g_\chi \left(\frac{m_\chi T}{2\pi \hbar^2} \right)^{3/2} e^{-(m_\chi - \mu_\chi)/T} \tag{3}$$

chem potential: $\mu \neq 0$ iff conserved particle number

if χ number *not conserved*–i.e., equal numbers of χ and $\bar{\chi}$
then $\mu_\chi = 0$, and so $n_\chi \sim e^{-m_\chi/T} \to 0$
$\Rightarrow$ no relic particles remain – terrible dark matter candidate!

Lessons: relic dark matter particles should

- *either* have *particle/antiparticle asymmetry*
  this is thought to be origin of baryons
- *or* must have dropped *out of equilibrium*
  Q: how might this happen?

Freezeout and Relic Abundance of a Symmetric Species

a *symmetric* species χ has a cosmic abundance
with *equal* numbers of particle and antiparticle
...or particle = antiparticle

thus $n_\chi = n_{\bar{\chi}}$ exactly: no "net χ number"
$\Rightarrow$ complete annihilation would leave no remaining particles

but: annihilation requires particle interactions!
these must compete successfully with expansion & cooling

in cosmic setting, essentially *guaranteed*
that at some point **annihilations freeze out**:
$\Gamma(\chi\bar{\chi} \to \text{stuff}) < H$
$\Rightarrow$ nonzero relic χ abundance, mass density also *guaranteed*!
Q: so does this guarantee that χ is the dark matter?

Annihilation Freezeout

Sketch of calculation appears here; more details in extras

Annihilation rate per χ (and $\bar{\chi}$) particle is

$$\Gamma_{\rm ann}(T) \simeq n_{\chi,\rm eq}(T) \; \langle \sigma_{\rm ann} v \rangle \sim (m_\chi T)^{3/2} e^{-m_\chi/T} \; \langle \sigma_{\rm ann} v \rangle \qquad (4)$$

where σ is the annihilation cross section,
and $\langle \sigma_{\rm ann} v \rangle$ is a thermal average

Freezeout temperature $T_{\rm f}$ set by

$$H(T_{\rm f}) \sim \frac{T_{\rm f}^2}{M_{\rm pl}} = \Gamma_{\rm ann}(T_{\rm f}) \sim (m_\chi T_{\rm f})^{3/2} e^{-m_\chi/T_{\rm f}} \; \langle \sigma_{\rm ann} v \rangle \qquad (5)$$

dominated by exponential: $T_{\rm f} \sim m_\chi$
so freezeout χ density is

$$n_{\chi,\rm f} \simeq \frac{H(T_{\rm f} = m_\chi)}{\langle \sigma_{\rm ann} v \rangle} \sim \frac{m_\chi^2}{M_{\rm pl} \langle \sigma_{\rm ann} v \rangle} \qquad (6)$$

Relic Abundance and Density

relic χ abundance at freezeout $T_{\rm f} \sim m_\chi$:

$$n_{\chi,{\rm f}} \simeq \frac{H(T_{\rm f} = m_\chi)}{\langle \sigma_{\rm ann} v \rangle} \sim \frac{m_\chi^2}{M_{\rm pl} \langle \sigma_{\rm ann} v \rangle} \tag{7}$$

But we want χ abundance and mass density *today*
note that after freeze, χ conserved!
$\rightarrow n_\chi = n_{\chi,{\rm f}}(a_{\rm f}/a)^3 \propto T^3 \propto n_\gamma$
$\rightarrow Y_\chi \equiv n_\chi/n_\gamma$ DM/photon ratio *constant*, set at freeze:

$$Y_\chi = \frac{n_{\chi,{\rm f}}}{n_{\gamma,{\rm f}}} \sim \frac{m_\chi^2/M_{\rm pl} \langle \sigma_{\rm ann} v \rangle}{m_\chi^3} \sim \frac{1}{M_{\rm pl} m_\chi \langle \sigma_{\rm ann} v \rangle} \tag{8}$$

So present number and mass densities are

$$n_{\chi,0} = Y_\chi n_{\gamma_0} \tag{9}$$

$$\rho_{\chi,0} = m_\chi n_{\chi,0} \sim \frac{1}{M_{\rm pl} \langle \sigma_{\rm ann} v \rangle} \tag{10}$$

What have we shown?

if a symmetric stable species ever created

(annihilates but not decays)

then annihilations will freeze, and

inevitably have nonzero relic density today, namely

$$\rho_{\chi,0} = m_\chi n_{\chi,0} \sim \frac{1}{M_{\rm pl} \langle \sigma_{\rm ann} v \rangle} \tag{11}$$

This calculation is of the highest interest to particle physicists

Q: why?

We have calculated a relic density

Q: Notable aspects about what it does, doesn't depend on?

Q: To what should it be compared?

Cold Relics: Present Abundance

⋆ $\rho_{\psi,0}$ indep of m_ψ

⋆ $\rho_{\psi,0} \propto 1/\sigma$: the weak prevail!
Q: what sort of cross section is relevant here?

⋆ To get "interesting" present density:
$\Omega_\psi \sim 1 \rightarrow \rho_\psi \sim \rho_{\rm crit}$ *demands* a *specific* cross section

$$\sigma_{\rm ann} \sim \frac{n_{\gamma,0}}{\Omega_\psi M_{\rm p} \rho_{\rm crit}} \tag{12}$$

$$\sim 10^{-38}\ {\rm cm}^2 \tag{13}$$

scale of the Weak interaction! $[\sigma_{\rm weak}(E \sim {\rm GeV}) \sim 10^{-38}\ {\rm cm}^2]$

The WIMP Miracle

Dark Matter candidate:
if DM is a cold symmetric relic
needed *annihilation cross section* is at Weak scale!
corresponding energy: if $\sigma \sim \alpha/E^2$
then $\sigma \sim 10^{-36}\ \mathrm{cm}^2 = 10\ \mathrm{pb} \rightarrow E \sim 1\ \mathrm{TeV}$

deeper reason for DM as
Weakly Interacting Massive Particle: **WIMP**

that weak-scale annihilations $\rightarrow \Omega_\chi \sim \Omega_{\mathrm{nbdm}}$: **"WIMP Miracle"**

How to find them?
What if we do? What if we don't?

Director's Cut Extras

Freezeout and Relic Abundanceof a Symmetric Species

for *conserved* species ψ (chem. pot. $\mu_\psi \neq 0$)
constant comoving number: $d(na^3) = 0$
$\Rightarrow \dot{n}_\psi + 3\frac{\dot{a}}{a}\, n_\psi = 0$

for *non-conserved* species: $d(n_\psi a^3) = q a^3 \; dt \neq 0$, where
q = *source/sink* rate = creation/destruction rate per unit vol
$\Rightarrow$ $\boxed{\dot{n}_\psi + 3\frac{\dot{a}}{a}\, n_\psi = q}$
assume annihilation $\psi\bar{\psi} \to X\bar{X}$ product X thermal,
with chem. pot. $\mu_X \ll T \;\; \Rightarrow \;\; n_X = n_{\bar{X}}$

$$q = q_{\rm net} = q_{\rm prod} - q_{\rm ann} \quad (14)$$
$$= \langle \sigma v \rangle_{\rm prod}\, n_X n_{\bar{X}} - \langle \sigma_{\rm ann} v \rangle_{\rm ann}\, n_\psi n_{\bar{\psi}} \quad (15)$$
$$= \langle \sigma v \rangle_{\rm prod}\, n_X^2 - \langle \sigma v \rangle_{\rm ann}\, n_\psi^2 \quad (16)$$

in equilib, *Q: what condition holds for q?*

chem equilib: $q = 0 \Rightarrow \boxed{q_{\text{prod}} = q_{\text{ann}}}$
so in general

$$\dot{n}_\psi + 3Hn_\psi = q = -\langle \sigma v \rangle_{\text{ann}} \left[n_\psi^2 - (n_\psi^{\text{eq}})^2 \right] \qquad (17)$$

and a similar expression for $\bar{\psi}$

Change variables:
(1) use comoving coords:
photon density $n_\gamma \propto T^3 \propto a^{-3}$,
so put $Y = n_\psi / n_\gamma$ to remove volume dilution
then $\dot{n_\psi} + 3\dot{a}/a \; n_\psi = n_\gamma \dot{Y}$
(2) put $x = m_\psi / T \propto a$
since $t \propto 1/T^2 \propto x^2$,
$dY/dt = dY/dx \; \dot{x} = H \; x \; dY/dx$

Then:

$$Hx \frac{dY}{dx} \;=\; -n_\gamma \langle \sigma v \rangle_{\text{ann}} \left(Y^2 - Y_{\text{eq}}^2 \right) \qquad (18)$$

$$\qquad (19)$$

finally

$$\frac{x}{Y_{\rm eq}} \frac{dY}{dx} = -\frac{\Gamma_A}{H} \left[\left(\frac{Y}{Y_{\rm eq}} \right)^2 - 1 \right] \tag{20}$$

where $\Gamma_A = n_\psi^{\rm eq} \langle \sigma v \rangle_{\rm ann}$: annihil. rate

So: change in comoving ψ controlled by
(1) annihil. effectiveness Γ/H
(2) deviation from equil

when $\Gamma/H \gg 1$

Q: what if $Y < Y_{\rm eq}$? $Y > Y_{\rm eq}$?

when $\Gamma/H < 1$

Q: how does Y change?

Q: how you you expect Y to evolve?

when $\Gamma/H \gg 1$, Y driven to Y_{eq}
when $\Gamma/H < 1$, Y change is small $\rightarrow$ freezeout!

relic abundance at $T \rightarrow 0$ or $x \rightarrow \infty$ is
$Y_\infty \simeq Y_{\text{eq}}(x_f)$: value at freezeout

Step back:
How can a symmetric species have
$n_\psi = n_{\bar{\psi}} \neq 0$ at $T \ll m$?

physically: expansion is key

if $H = 0$, $Y_\infty = Y_{\rm eq}(\infty) = 0$:

$\rightarrow$ all ψ find $\bar{\psi}$ partner, annihilate

but $H \neq 0$: when U dilute enough,

ψ never finds $\bar{\psi}$: i.e., $\Gamma \ll H$

nonzero relic abundance

hot relics: $x_f \ll 1$ ($T_f \gg m$)

cold relics: $x_f \gg 1$

Note: hot/cold *relics* refers to freezeout conditions

But: hot/cold *dark matter* refers to structure formation criteria

(namely, m vs temp $T_{\rm eq}$ at matter-rad equality)

Cold Relics: WIMPs

cold relic: non-relativistic at freezeout

so $x_f = m/T_f \gg 1 \to T_f \ll m$

$\Rightarrow n_{\rm eq} \sim e^{-m/T}(mT)^{3/2}$

$\Rightarrow Y_{\rm eq} \sim e^{-x}x^{3/2}$

Freezeout:

$\Gamma_{\rm ann} = H$ at $T = T_f$

$\Rightarrow n_{\rm eq} \langle \sigma v \rangle_{\rm ann} \sim \sqrt{G} T^2$

what needed to find value of T_f?

To solve:

- need annihilation cross section
  for many models, $\langle \sigma v \rangle \propto v^n$ (S-wave: $n = 0$)
  $\Rightarrow \langle \sigma_{\rm ann} v \rangle (x) = \sigma_1 c x^{n/2}$, where $\sigma_1 = \sigma(E = m)$
- convenient rewrite $1/\sqrt{G} = M_{\rm Pl} \simeq 10^{19}$ GeV
  (Planck Mass)

set $\Gamma_{\rm ann}(T_f) = H(T_f)$, and solve for T_f
Find: $x_f \sim \ln(m M_{\rm Pl} \sigma_1) \Rightarrow T_f = m/x_f$
So

$$Y_\infty \simeq Y_{\rm eq}(x_f) \quad (21)$$

$$\sim \frac{x_f^{3/2}}{m M_{\rm Pl} \sigma_1} \quad (22)$$

→ present relic number density

$$n_{\psi,0} = Y_\infty n_{\gamma,0} = 400 \; Y_\infty \; \mathrm{cm}^{-3} \tag{23}$$

present relic mass density

$$\rho_{\psi,0} = m n_{\psi,0} \; \simeq \; \frac{x_f^{3/2} n_{\gamma,0}}{M_{\mathrm{Pl}} \sigma_1} \tag{24}$$

What have we shown?
if a symmetric stable species ever created
(annihilates but not decays)
then annihilations will freeze, and
inevitably have nonzero relic density today.

This calculation is of the highest interest to particle physicists
Q: why?
We have calculated a relic density
Q: To what should this be compared?