

Astro 507
Lecture 28
April 2, 2014

Announcements:

- **PS 5 due Friday**
- must-see Physics Colloquium today, 4pm, Loomis:
Illinois' own Profs. Joaquin Vieira and Peter Adshead
Dr. William D. Watson Memorial Lecture:
“The Birth of the Universe:
The First Direct Evidence for Inflation”

Last time: began inflation

Q: what cosmic puzzles does inflation solve?

↳ *Q: solutions to these puzzles without inflation?*

Q: how does inflation differ from usual cosmic expansion?

Inflation motivation: cosmic puzzles

flatness, horizon, monopoles, lumpiness

Inflation: early period of *rapid, accelerated expansion*

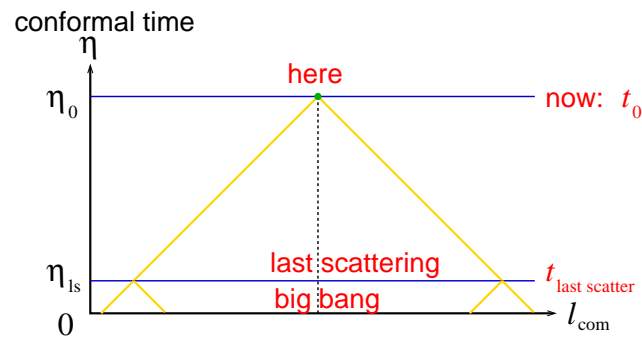
scale factor growth $a_{\text{post-inf}}/a_{\text{pre-inf}} \sim e^{60} \sim 10^{26}$

Simultaneously solves flatness, horizon, monopoles

flatness: acceleration drives $|\Omega - 1| \rightarrow 0$

non-inflationary cosmic spacetime:

universe without inflation



2

Q: how does inflation change this?

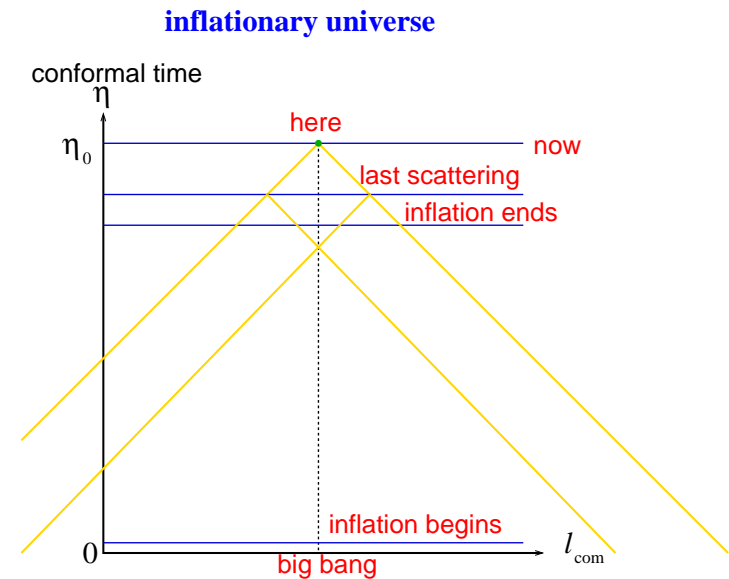
spacetime plot is *conformal time* $\eta = \int_0^t dt/a(t)$
 versus *comoving distance* r_{com}

inflation: over tiny timeframe Δt ,
 scale factor $a(t)$ grows by $a_f/a_i \sim e^N$
 with $N \approx 60$ “e-foldings”
 growth during inflation: \sim exponential

$$\delta\eta \approx \frac{1}{a_i} \int_0^{\Delta t} \frac{dt}{e^{Ht}} \sim e^N t_f$$

\Rightarrow huge growth in conformal time during inflation
 due to acceleration (exponentiation)

ω Q: *but how to make early U accelerate?*



Inflationary Cosmology

recall from Dark Energy discussion: acceleration demands $P < 0$
can't do this with matter or radiation

But:

- ★ *scalar field* ϕ can have $P_\phi < 0$
- ★ scalar fields *required* for electroweak unification and appear in all other unification schemes

Alan Guth (1981)

if early Universe

- ▷ contains a *scalar field*,
- ▷ whose *potential energy* $\rho_\phi \approx V_\phi \approx \rho_{\text{tot}}$

↳ then (in 21st century language) $w_\phi \rightarrow -1$

→ *cosmic acceleration and exponential expansion!*

Cosmic Scalar Fields: Episode II

recall: cosmic scalar field ϕ , “minimally coupled” – i.e.,

- interacts only to itself via potential $V(\phi)$
- and gravity, via ρ_ϕ

Properties: Note $\hbar = c = 1!$ $\Rightarrow [\phi] = [E] = [\ell^{-1}] = [t^{-1}]$

$$\text{Equation of motion} \quad \ddot{\phi} - \nabla^2 \phi + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (1)$$

$$\text{energy density} \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V \quad (2)$$

$$\text{pressure} \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla\phi)^2 - V \quad (3)$$

why? Lagrangian dens $\mathcal{L} = 1/2 \partial_\mu \phi \partial^\mu \phi - V \Rightarrow$ stress-energy

$$T_{\mu\nu} \equiv \text{diag}(\rho_\phi, p_\phi, p_\phi, p_\phi) \quad (4)$$

$$= \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L} = (\partial_\mu \phi \partial_\nu \phi - \frac{g_{\mu\nu}}{2} \partial_\mu \phi \partial^\mu \phi) + g_{\mu\nu} V \quad (5)$$

Scalar Field: Cosmic Equation of Motion

for **homogeneous** field $\phi(t, \vec{x}) = \phi(t)$, so

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V \quad (6)$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V \quad (7)$$

apply “first law of cosmo-thermodynamics” :

$$d(\rho a^3)/dt + p d(a^3)/dt = (\rho + p)d(a^3)/dt + a^3 d\rho/dt = 0$$

$$\begin{aligned} (\rho + p)\frac{d(a^3)/dt}{a^3} + \dot{\rho} &= 3H(\rho + p) + \dot{\rho} \\ &= 3H\dot{\phi}^2 + \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V\right) \\ &= 3H\dot{\phi}^2 + \dot{\phi}\ddot{\phi} + dV/d\phi \dot{\phi} = 0 \end{aligned}$$

◦ which gives $\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$

Recall: for **homogeneous** field $\phi(t, \vec{x}) = \phi(t)$, so $\nabla\phi = 0$, and then we have

$$\ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} = 0$$

formally: same as **Newtonian ball rolling down hill V** but impeded by friction (“Hubble drag”) $3H$

$$\rho_\phi = \dot{\phi}^2/2 + V \tag{8}$$

$$P_\phi = \dot{\phi}^2/2 - V \tag{9}$$

which gives

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \tag{10}$$

and so scalar field can have **$w_\phi \in [-1, 1]$** !

↳ If $w_\phi < 0$, then ϕ is an *accelerant*, has $P_\phi < 0$!

Q: requirements of workable inflation scenario?

Ingredients of an Inflationary Scenario

Recipe:

1. **inflaton field ϕ must exist** in early U.
2. must have $\rho_\phi \approx V$ so that $w_\phi \rightarrow -1$ so that $a \sim e^{Ht}$
3. continue to exponentiate $a \sim e^N a_{\text{init}}$
for at least $N = \int H dt \gtrsim 60$ e-folds
4. stop exponentiating eventually (**“graceful exit”**)
5. convert field ρ_ϕ back to radiation, matter (**“reheating”**)
6. then ϕ must “keep a low profile,” $\rho_\phi \ll \rho_{\text{tot}}$
- 7 (bonus) what about acceleration and dark energy today?
is quintessence a rebirth of inflationary ϕ ?
goal of “quintessential inflation” models

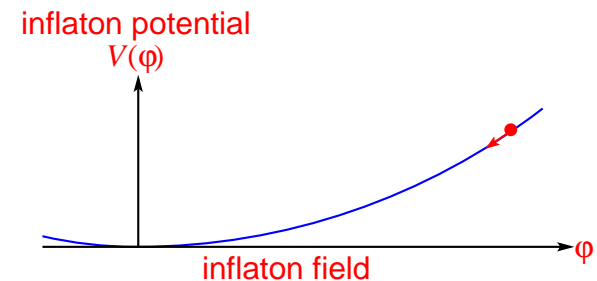
[∞] Q: to meet 2: $\rho_\phi \approx V \rightarrow$ what does this mean?

To inflate, need slow ϕ evolution:

$\ddot{\phi} \ll 3H\dot{\phi}$ \leftrightarrow friction large:

\Rightarrow achieve “terminal speed”

$$\dot{\phi} \approx -\frac{1}{3H}V'$$



Slowness conditions $\dot{\phi}^2/2 \ll V$ and $\ddot{\phi} \ll 3H\dot{\phi}$ constrain “**slow-roll parameters**”:

$$\epsilon(\phi) = \frac{m_{\text{pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \quad (11)$$

$$\eta(\phi) = m_{\text{pl}}^2 \frac{V''}{V} \quad (12)$$

to be **small**: $\epsilon \ll 1$ and $|\eta| \ll 1$

(you’ll get to show this in PS5)

Q: Why is it useful to express slow-roll criteria this way?

Q: What do these imply about the nature of V ?

Q: What about magnitude of ϕ during inflation?

The Charms of a Slow Roll

Usefulness of slow roll parameters ϵ, η

★ ϵ, η quantify conditions for *maintaining* inflation purely in terms of underlying potential V
→ an immediate constraint on inflaton physics
i.e., any workable potential must satisfy slow roll
want derivatives small → need **flat** potential

★ ϵ, η quantify inflaton energy scale

- typically expect $V'/V \sim 1/\phi$
- but slow roll $(V'/V)^2 \sim \epsilon m_{\text{pl}}^2$

together these give $\phi \gtrsim m_{\text{pl}}$ during inflation

generically expect $\phi \gtrsim m_{\text{pl}}$

\Rightarrow for successful inflation, field probes the Planck scale (?)

$;-)$ a good thing?

hints at quantum gravity

if $\Omega_{\text{init}} \gtrsim 1$, inflation prevents U. collapse \rightarrow black hole

$=: -o$ a bad thing?

quantum gravity a prerequisite for inflation models?

moves away Guth's original idea, GUT physics?

★ ϵ, η also can quantify conditions for *ending* inflation

Amount of Inflation

during inflation scale factor grows exponentially (in most models); in any case quantify “amount” of inflation as

$N = \ln(a_{\text{fin}}/a_{\text{init}})$: number of “e-foldings”

What is needed?

to solve horizon, flatness, monopoles back to GUT scale:

$N \gtrsim N_{\text{min}} \sim 60$ (PS6)

What is predicted?

Since $H = \dot{a}/a = d \ln a / dt = \dot{N}$, and $dt = d\phi / \dot{\phi}$, we have

$$N = \int_{t_{\text{init}}}^{t_{\text{fin}}} H dt = \int_{\phi_{\text{init}}}^{\phi_{\text{fin}}} \frac{H d\phi}{\dot{\phi}} \quad (13)$$

slow roll: $\dot{\phi} \simeq -V'/3H$, so

$$N = \int_{\phi_{\text{fin}}}^{\phi_{\text{init}}} \frac{3H^2 d\phi}{V'} = m_{\text{pl}}^2 \int_{\phi_{\text{fin}}}^{\phi_{\text{init}}} \frac{V}{V'} d\phi \quad (14)$$

typically expect $V'/V \sim 1/\phi$, which gives

$$N \sim \frac{\Delta\phi^2}{m_{\text{pl}}^2} \quad (15)$$

amount of inflation set by:

- nature of potential V
- change in ϕ

note also that need $N \gg 1$ and thus

typically expect $\phi_{\text{init}} \gtrsim m_{\text{pl}}$

...but already required by slow roll

Q: what determines inflation end physically? mathematically?

A Graceful Exit from Inflation

inflaton continues until acceleration stops ($w_\phi > 0$)

→ potential energy no longer dominates cosmic ρ

all matter and radiation inflated away, so “rescue” comes from kinetic energy $\dot{\phi}^2/2$ (by itself, has $w = +1!$)

in terms of potential, exit when slow roll stops

quantified by slow-roll parameters

i.e., ϕ evolves until $\epsilon(\phi) \sim 1$

inflaton requirements:

- to achieve slow roll → need flat V far from minimum
- to end slow roll → need non-flat
 $V' \gtrsim V/m_{\text{pl}}$ approaching minimum

14

Q: and then...? What's the Universe like? What happens next?

Reheating: Back to the Hot Big Bang

After $e^{60} \sim 10^{26}$ expansion

radiation, matter particles diluted to negligibility as a^{-3}
temperature drop $T \sim 1/a \rightarrow 0$: “supercooling”

But since $V(\phi) \sim \text{const}$ during inflation

inflaton energy density still large afterwards

must convert to hot, radiation-dominated early U: **reheating**

Details complicated, model-dependent; basic idea:

- ★ ϕ evolves in non-inflationary way
- ★ quantum effects drive energy conversion

Reheating I: Inflaton Oscillations

near minimum $V \simeq \frac{1}{2}V''\phi^2 \equiv \frac{1}{2}m_\phi^2\phi^2$

$$\ddot{\phi} + 3H\dot{\phi} + V' \approx \ddot{\phi} + m_\phi^2\phi = 0 \quad (16)$$

simple harmonic oscillator!

▷ **classically** field oscillates around zero rapidly and coherently: within particle horizon, same oscillation phase

$$\text{so } \langle \dot{\phi}^2/2 \rangle = \langle V \rangle = \langle m_\phi^2\phi^2/2 \rangle$$

• which means $\langle P_\phi \rangle = 0$ *Q: why?*

• which means $w_\phi = 0$, and so

$$\langle \rho_\phi \rangle \sim a^{-3(1+w_\phi)} = a^{-3} \text{ like NR matter!}$$

16 \Rightarrow so ρ_ϕ drops \rightarrow oscillation amplitude decays

Reheating II: Downfall of the Inflaton

- ▷ quantum mechanically field excitations \rightarrow quanta
inflaton particles (mass m_ϕ) created

But the inflaton must be unstable Q: *why?*

\rightarrow decays to particles with Standard Model interactions

- if ϕ only decays to fermions
does so slowly, products made thermally
- if ϕ can decay to bosons, resonances likely
rapid decay far from equilibrium

In either case: decay products interact, exchange energy

thermalize: $\rho_\phi \rightarrow \rho_{\text{rad}} \sim T^4$

$$T_{\text{reheat}} \sim \rho_{\phi, \text{fin}}^{1/4}$$

Q: *what is rock-bottom minimum T_{reheat} ?*

Inflation and the Rest of Cosmology

Reheating Temperature

- ★ All of ‘usual’ hot big bang begins *after* reheat
- ★ Must reheat enough for U to undergo any and all known hot big phases
e.g., have to *at least* heat up to have nucleosynthesis
i.e., successful nuke requires $T_{\text{reheat}} > 1 \text{ MeV}$
earlier phases (if any) demand hotter reheat

What about other physics, cosmology?

Q: How must inflation fit in with, e.g., monopole production, baryon genesis, BBN, CMB?