Astro 507

Lecture 28

April 2, 2014

Announcements:

- **PS 5 due now**
- **Preflight 6 posted today–last PF!**

Last time: slow-roll inflation

- scalar field dynamics in an expanding universe
  slow roll conditions constrain inflaton potential

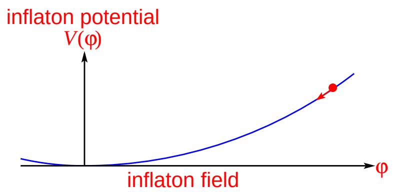
Q: what's rolling? why must roll be slow?
what is required to make it slow?

Q: what is ϕ? $V(\phi)$?
how does inflation begin? end?

Ingredients of an Inflationary Scenario

Recipe:

1. **inflaton field ϕ must exist** in early U.
2. must have $\rho_\phi \approx V$ so that $w_\phi \rightarrow -1$
   so that $a \sim e^{Ht}$
3. continue to exponentiate $a \sim e^N a_{\text{init}}$
   for at least $N = \int H\, dt \gtrsim 60$ e-folds



4. stop exponentiating eventually (**"graceful exit"**)
5. convert field ρ_ϕ back to radiation, matter (**"reheating"**)
6. then ϕ must "keep a low profile," $\rho_\phi \ll \rho_{\text{tot}}$

7 (bonus) what about acceleration and dark energy today?

Q: what can we say about how inflation fits
in the sequence *of cosmic events, e.g.*
monopole production, baryon genesis, BBN, CMB?

Cosmic Choreography: The Inflationary Tango

Inflation must occur such that it
solves various cosmological problems, then
allows for the universe of today, which *must*

- ▷ contain the known particles, e.g., a net baryon number
- ▷ pass thru a radiation-dominated phase (BBN)
  and a matter-dominated phase (CMB, structure formation)

⇒ this forces an ordering of events

Cosmic Choreography: Required *time-ordering*

1. monopole production (if any)
2. inflation
3. baryogenesis (origin of $\eta \neq 0$)
4. radiation $\to$ matter $\to$ dark energy eras

Electroweak woes: hard to arrange baryogenesis afterwards!

Models for Inflation

Inflation model: specifies inflaton potential $V(\phi)$
[+ initial conditions, reheat prescription]

good news:
involves physics at extremely high energy scales
probed by observable signatures of inflation

bad news:
involves physics at extremely high energy scales
far beyond the reach of present-day or planned accelerators
no laboratory guidance or checks of inflationary physics

Q: possible physically reasonable choices for $V(\phi)$*?*

A Sample of Single-Field Potentials

Polynomial Potentials

e.g., Klein-Gordon $V = m^2\phi^2/2$, quartic $V = \lambda\phi^4$

- simplest models giving inflation
- require *Planck-scale* initial conditions for ϕ
- but to achieve sufficient inflation (enough e-foldings N) and perturbations at observed (CMB) scale demands *tiny coupling* $\lambda \sim 10^{-13}$ (!)
  $\rightarrow$ potential energy scale $V \ll m_{\text{pl}}^4$
  but why is coupling so small?

Illustrates characteristics of (successful) inflation models:

▷ large initial $\phi \gtrsim m_{\text{pl}}$ value

▷ small coupling in V $\rightarrow$ scale $V^{1/4} \sim 10^{15-16}$ GeV (GUT?)

Exponential Potentials: Power-Law Inflation

for potentials of the form

$$V = V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{m_{\rm pl}}\right) \tag{1}$$

then can solve equations of motion exactly:

$$a \sim t^p \tag{2}$$

if $p > 1$, U. accelerates, but not exponentially

Designer Potentials

can customize V to give desired $a(t)$, e.g.,
to get $a \sim \exp(At^f)$, with $0 < f < 1$
then choose

$$V(\phi) \sim \left(\frac{\phi}{m_{\rm pl}}\right)^{-\beta}\left[1 - \frac{\beta^2}{6}\left(\frac{m_{\rm pl}^2}{\phi^2}\right)\right] \tag{3}$$

How about the Higgs?

from electroweak unification, we know of one scalar
$\rightarrow$ Higgs field H^0, $M_H \approx 125$ GeV

same symbol as Hubble, right kind of field $\rightarrow$ is it ϕ?
i.e., what about inflation at the electroweak scale?

not a bad idea–possibly correct!–but nontrivial at best
problem not with inflation, but its place in the cosmic dance

Inflation, Inhomogeneities, and Quantum Mechanics

Thus far: classical treatment of inflaton field

(except for inflaton decays during reheating)

- ϕ described by classical equations of motion
- taken to hold for arbitrarily small ϕ

In this picture:

when exit inflation, universe essentially

▷ perfectly flat, and

▷ perfectly smooth–i.e., density spatially uniform

regardless of initial conditions (as long as they allowed inflation)

Q: why?

Classical Inflation and Smoothness

expect initial spatial inhomogeneities in $\phi(\vec{x})$
but evolves classically as

$$\ddot{\phi} - \nabla^2\phi + 3H\dot{\phi} - V' = 0 \tag{4}$$

where

$$\nabla^2 = \sum \frac{\partial^2}{\partial x^2_{\rm phys}} = \frac{1}{a^2} \sum \frac{\partial^2}{\partial x^2_{\rm com}} \tag{5}$$

inhomogeneities $\delta\phi(\vec{x})$ measured by nonzero gradients
but since $\nabla^2 \propto 1/a^2 \rightarrow 0$ exponentially, classically: $\delta\phi(\vec{x}) \rightarrow 0$
$\Rightarrow$ after inflation ϕ and $\rho = V(\phi)$ exponentially smooth in space

good news: solved flatness, smoothness problems
bad news: we have done too much! too smooth!
can't form structures if density perfectly uniform

Quantum Mechanics to the Rescue

but quantum mechanics exists and is mandatory
classical ϕ field $\rightarrow$ quantized as inflatons
think $\vec{E}, \vec{B}$ vs photons

inflaton field must contain quantum fluctuations
before, during inflation

uncertainty principle: $\Delta x \Delta p \sim \hbar$
causal region at time t: Hubble length $\Delta x \sim d_H = c/H(t)$
expect momentum and energy fluctuations

$$c\Delta p \sim \Delta E \sim \hbar H \tag{6}$$

Q: implications?
Q: fate of fluctuations born a given scale λ_{init}?
Q: analogy with Hawking radiation?

A Quantum Perturbation Factory

quantum mechanics: perturbations in energy $\rightarrow \delta\phi$
$\rightarrow$ different regions start inflation at different $V(\phi)$

quantum fluctuations born at scale λ_{init}

- exponentially stretched until $\lambda > d_H$ "horizon crossing"
- then no longer causally connected
  $\rightarrow$ cannot "fluctuate back to zero"
- *"frozen in" as real density perturbations!*
  cosmic structures originate from quantum fluctuations!

Hawking radiation analogy:
uncertainty principle: $\Delta E \Delta t \sim \hbar$, so in timscale
$\Delta t \lesssim \hbar / m_{\psi} c^2$: particle pairs $\psi\bar{\psi}$ born and annihilate
black hole: one falls in, other emitted as thermal Hawking rad.
inflation: pair separated by expansion, "frozen" as fluctuation

Implications

If the inflationary model is true
density fluctuation “seeds” of cosmic structures are inflated quantum mechanical fluctuations

Q: how does this limit what we can know about them?

Q: what can *we hope to know?*

Q: what do we need to calculate?

Inflationary Fluctuations: What we need to know

quantum fluctuations are *random*

$\rightarrow$ impossible to predict locations, amplitudes of overdensities

$\rightarrow$ cannot predict location, mass, size of *any particular* cosmic object: galaxy/cluster/supercluster ...

but quantum mechanics does allow *statistical predictions*

What we want: *statistical* properties of fluctuations

- typical magnitude of fluctuations $\delta\phi$
- how $\delta\phi$ depends on lengthscales
- corresponding fluctuations in ρ_ϕ
- correlations at different length scales

Fluctuation Amplitude: Rough Estimate

quantum fluctuation $\rightarrow$ turn to uncertainty principle

$$\delta E \ \delta t \sim \hbar \sim 1 \tag{7}$$

recall: energy density is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V \tag{8}$$

if perturbation from classical: $\phi(t, \vec{x}) = \phi_{\rm cl}(t) + \delta\phi(t, \vec{x})$,
then for small $\delta\phi \ll \phi_{\rm cl}$,

$$\delta\rho \sim (\nabla\delta\phi)^2 + V'(\phi_{\rm cl})\delta\phi \approx (\nabla\delta\phi)^2 \tag{9}$$

since slow roll $\rightarrow V'$ small (flat potential)

Q: what is characteristic volume for fluctuation?
Q: what is characteristic timescale δt?

H^{-1} is only lengthscale in problem
so $\nabla \delta\phi \sim \delta\phi / H^{-1} \Rightarrow \delta\rho \sim H^2 (\delta\phi)^2$
so in Hubble volume $V_H = d_H^3 = H^{-3}$, energy fluctuation is

$$\delta E = \delta\rho \ V_H = \frac{(\delta\phi)^2}{H} \tag{10}$$

characteristic timescale is $\delta t \sim 1/H$, so

$$\delta E \ \delta t \sim \frac{(\delta\phi)^2}{H^2} \sim 1 \tag{11}$$

and typical (root-mean-square) inflaton fluctuation is

$$\boxed{\delta\phi \sim H} \tag{12}$$

had to be! H is the only other
dimensionally correct scale in the problem!

Note: $H \sim const$ during inflation
all fluctuations created with $\sim$ same amplitude

What Just Happened?

To summarize:

- *classically*, inflaton field ϕ_{cl} quickly inflates away any of its initial perturbations
- but *quantum fluctuations* $\delta\phi$ unavoidable created and persist throughout inflation
- in any region, amplitude $\delta\phi(\vec{x})$ random but *typical* value $\delta\phi \sim H$

Q: what do the presence of inflaton fluctuations mean for inflationary dynamics in different regions?

Q: what consequences/signatures of fluctuations might remain after inflation?

Fluctuation Evolution and the Cosmic Horizon

in presence of fluctuations $\delta\phi$ and $\delta\rho_\phi$
can view inflationary universe as ensemble of "sub-universes"
evolving independently–same slow roll, but
with different ϕ, ρ_ϕ at a fixed t
classical discussion $\rightarrow$ ensemble average
now want behavior typical deviation from mean

particle horizon $\sim H^{-1}$ critical
- already saw: sets scale for fluctuation
- also "shuts off" fluctuation evolution

consider perturbation of lengthscale λ
- leaves horizon when $H \sim 1/\lambda$
- then can't evolve further: keeps same $\delta\rho/\langle\rho\rangle$
- until after inflation, when re-enters horizon

Bottom line: at any given scale λ
relevant perturbation is the one born
during inflation when $\lambda \sim 1/H$

dimensionless perturbation amplitude:
fraction of mean density in horizon $\delta_H \equiv \delta\rho/\langle\rho\rangle$

on scale λ, amplitude fixed at horizon exit
$\delta\phi \sim H$ (in fact, $H/2\pi$)
$\rightarrow$ perturbed universe starts inflating at higher ϕ
or undergoes inflation for different duration $\delta t \simeq \delta\phi/\dot{\phi}$
this gives an additional expansion

$$\delta \ln a = \frac{\delta a}{a} = H\delta t = \frac{H^2}{2\pi\dot{\phi}} \qquad (13)$$

but inflation exit is set at fixed $\phi_{\rm end}$
and fixed potential value $V_{\rm end} \sim \rho_{\rm end}$

perturbed energy density at end of inflation set by different expansion at inflation exit:

$$\delta_{\rm H} \equiv \frac{\delta\rho}{\langle\rho\rangle} \tag{14}$$

$$\sim \frac{\delta(a^3 V_{\rm end})}{\left\langle a^3 V_{\rm end} \right\rangle} \sim \frac{\delta a}{a} = \frac{H^2}{2\pi\dot{\phi}} \tag{15}$$

evaluated at any scale λ at horizon crossing
i.e., when $\lambda_{\rm com} \sim 1/aH$
$\Rightarrow$ *density perturbations created at all lengthscales!*

caution: quick-n-dirty result
but gives right answer
in particular, fluctuation indep of lengthscale

What Just Happened? ...Part Deux

the *classical* behavior of a slow-rolling ϕ
lead to homogeneity, isotropy
regardless of initial conditions
$\Rightarrow$ fixes horizon, flatness, monopole problem

the *quantum* fluctuations in ϕ
lead to density perturbations on all lengthscales
including scales $\gg d_{\text{hor}}$ today
these perturbations form the "seeds" for cosmic structures!

quantum mechanics & uncertainty principle
essential for the existence of cosmic structure

"The Universe is the ultimate free lunch."
– Alan Guth