Astro 507

Lecture 37

April 25, 2014

Announcements:

- **Problem Set 6 due next Wednesday April 30**
- **ICES** available online – please do it!

Last time: Newtonian perturbation theory in expanding U

Q: dark matter perturbations before matter-rad eq? after?

Q: baryonic perturbations before recomb? after?

Q: physical origin of CMB ΔT *on scales* $\gg d_{\rm hor,recomb}$*?*

dark matter: pressureless

$\rightarrow$ all k modes unstable if inside Hubble length

but: perturbations grow verry slooooowly during radiation era

$\rightarrow$ DM structures begin formation at matter-radiation equality

then $\delta_{\rm m}(t) = \delta_{\rm m,init}\ D(t)$ with $D(t) \propto a(t) \propto t^{2/3}$

baryons: until recomb, tightly coupled to photons

$\rightarrow$ feel huge photon pressure $P_\gamma \propto T^4$

$\rightarrow$ sound speed $c_s \sim c/\sqrt{3}$ huge!

so all sub-horizon modes stable! just oscillate

$\rightarrow$ relativistic pressure-mediated (i.e., acoustic) standing waves!

oscillation frequency $\nu = c_s/\lambda$:

small-scale modes oscillate many times

largest-scale modes $\lambda = c_s \eta_{\rm hor}$ oscillates only once

Pre-Recombination: Acoustic Oscillations

Baryons in DM-dominated background

$$\ddot{\delta}_b + 2\frac{\dot{a}}{a}\dot{\delta}_b \simeq 4\pi G\rho\delta_{dm} - \frac{k^2 c_s^2}{a^2}\delta_b \sim \frac{\delta_{dm}}{t^2} - \frac{k^2 c_s^2}{a^2}\delta_b \tag{1}$$

key comparison: mode scale $\lambda \sim k^{-1}$
vs **comoving sound horizon** $c_s t/a = d_{s,com}$

for *large scales* $kc_s t/a \ll 1$: *baryons follow DM*
for *small scales* $kc_s t/a \gg 1$: *baryons oscillate*, as

$$\delta_b \sim \frac{1}{\sqrt{ac_s k}} e^{i\int kc_s d\eta} \tag{2}$$

(PS 6) where $d\eta = dt/a$ is conformal time

Q: for fixed k, what is δ time behavior?
Q: at fixed t, what is δ pattern vs k?
Q: what sets largest λ that oscillates?

baryonic perturbations do not grow, but oscillate:

$$\delta_b \sim \frac{1}{\sqrt{a c_s k}} e^{i \int k c_s d\eta} \tag{3}$$

to simplify, imagine constant c_s, $\delta_b \sim e^{i k c_s \eta}$

at fixed k, sinusoidal oscillations

phase counts number of cycles $N = k c_s \eta / 2\pi = c_s \eta / \lambda$

oscillation frequency: $\omega \sim k c_s \sim c_s / \lambda \propto 1/\lambda$

at fixed $t \to$ fixed η:

small λ and large $k \to$ rapid oscillations

largest oscillations at scale $\lambda \sim c_s \eta \sim c_s t / a$: *sound horizon*

Q: when do oscillations stop? observable signature?

Recombination: Snapshot Taken

At recombination, free e^- abundance drops

baryons quickly decouple from photons

huge drop in pressure $\rightarrow c_s \rightarrow 0$

begin to collapse onto DM potentials

photons travel freely (typically) afterwards

fluctuation pattern at recomb is "frozen in"

δ vs scale records different # of cycles at recomb

$$P(k) = \|\delta_k\|^2 \sim \frac{\sin(2kc_s\eta_{\rm rec})}{2kc_s\eta_{\rm rec}} P_{\rm init}(k) \tag{4}$$

written onto temperature pattern ("say cheese!")

Recomb fast $\rightarrow$ CMB is image of last scattering surface

Q: on small scales, is an overdensity a hot spot or cold spot? why?

Spots Cold and Hot: Small Scales

Define temperature fluctuation $\Theta = \delta T/T$

On Small Scales: Adiabatic

standing waves lead to fluctuations in $\rho_b \sim T^3$, so

$$\Theta \equiv \frac{\delta T}{T} = \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_b \tag{5}$$

$\Rightarrow$ extrema in density $\to$ extrema in $\Theta \propto \delta_\gamma$

$\star$ photon T contrast reflects T distribution at source
$\to$ hot is hot and cold is cold

- but both high *and* low density give *large* $(\delta T/T)^2$!
  photon climb out of potential doesn't change $\delta T/T$ much
  $\to$ CMB hot spots are high density, cold are low

Q: what about on large scales?

Very Large Scales: Sachs-Wolfe

beyond horizon: no oscillations, main effects gravitational (GR):

- gravitational redshift: photon climbs out of potential $\delta\Phi < 0$
  redshift $\delta\lambda/\lambda = \Phi_0 - \Phi_{\rm ls} = -\delta\Phi$
  and since $T \sim 1/\lambda$, $(\delta T/T)_{\rm redshift} = \delta\Phi$: photons cooled!
- time dilation: takes longer to climb out of overdensity
  looking at younger, hotter universe
  $\delta t/t = \delta\Phi$, and since $a \sim t^{2/3}$ and $T \sim 1/a$
  then $T \sim t^{-2/3}$, and $(\delta T/T)_{\rm dilation} = -2/3\ \delta\Phi$

net effect: Sachs - Wolfe

$$\left(\frac{\delta T}{T}\right)_{SW} = \left(\frac{\delta T}{T}\right)_{\rm redshift} + \left(\frac{\delta T}{T}\right)_{\rm dilation} = \frac{1}{3}\delta\Phi \qquad (6)$$

⋆ overdensities are cold spots, underdensities hot

¬ Note: this regime is what tests inflation
Q: what predicted?

Inflation and Sachs-Wolfe

Inflation: quantum fluctuations $\rightarrow$ density fluctuations

- adiabatic (all species)
- Gaussian
- scale invariant–what does this mean?

In detail: inflation predicts that the dimensionless fluctuations in the *gravitational potential* $\leftrightarrow$ *local curvature* are independent of scale
$\rightarrow$ this was what we really calculated in Inflation discussion

inflationary scale-invariance is for grav potential:
i.e., Fourier mode contribution $\Delta_\Phi^2 \sim k^3 |\Phi_k|^2 \sim const$ indep of k
∞ $\rightarrow$ scale invariant: $|\Phi_k|^2 \sim k^{-3}$
Q: how related ot $P(k)$?

need to connect gravitational potential/curvature perturbations to density perturbations

But in Newtonian regime, know how to do this:
Poisson relates potential and density:

$$\nabla^2 \delta\Phi = 4\pi G \delta\rho \quad \rightarrow \quad \Phi_k \sim \delta_k / k^2 \tag{7}$$

and so $P(k) = |\delta_k|^2 \sim k^4 |\Phi_k|^2$

thus scale invariant gravitational potential
gives power spectrum:

$$P_{\text{scale-inv}}(k) \sim k^4 \left|\Phi_{\text{scale-inv}}(k)\right|^2 \sim k \tag{8}$$

i.e., scale invariance: $P(k) \sim k^n$, $n_{\text{scale-inv}} = 1$

Angular vs Linear Scales

So far: decomposed fluctuations in (3-D) $\vec{k}$-space
but observe on sky: 2-D angular distribution

Transformation: projection of plane waves
at fixed k: see intersection of wave with last scattering shell
`www: Wayne Hu animation`

appears on a range of angular scales
but typical angular size is $\theta \sim \lambda / d_{\text{rec,com}} \sim (k d_{\text{rec,com}})^{-1}$

large angles $\to$ large λ (check!)
for large angular scales $\theta > \theta_{\text{hor,diam}} \sim 1^\circ$, superhorizon
perturbations not affected by oscillation
for small angular scales, see standing waves

- peaks at extrema, harmonics of sound horizon
  k are in ratios 1:2:3:...

The CMB Observed

- Observe 2-D sky distribution of $\frac{\Delta T}{T}(\hat{n}) \equiv \Theta(\hat{n})$ in direction $\hat{n}$
- Decompose into spherical harmonics

$$\Theta(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \tag{9}$$

with Y_{lm} spherical harmonics *Q: why not* $\ell = 0, 1$*?*
Q: angular size vs ℓ*?* λ *vs* ℓ*?*

Form angular correlation function *Q: what is this physically?*

$$\langle \Theta(\hat{n}_1)\Theta(\hat{n}_2) \rangle = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell + 1) \left\langle |a_{\ell m}|^2 \right\rangle P_\ell(\hat{n}_1 \cdot \hat{n}_1) \tag{10}$$

$$= \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell + 1) C_\ell P_\ell(\cos\vartheta) \tag{11}$$

where $\cos\vartheta = \hat{n}_1 \cdot \hat{n}_1$
Q: averaged over the m *azimuthal modes–why?*

all interesting anisotropy information encoded in

$$C_\ell = \left\langle |a_{\ell m}|^2 \right\rangle \tag{12}$$

isotropy $\rightarrow$ azimuthal dependence averages to zero

Note: analog of Δ^2 (variance per log scale) is
$\mathcal{T}^2(\ell) = \ell(\ell+1)C_\ell$: usually what is plotted

Since $P_\ell(\cos\theta) \sim (\cos\theta)^\ell \sim \cos(\ell\theta)$
at fixed ℓ, angular size $\theta \sim 2\pi/\ell = 180^\circ/\ell$
e.g., $\ell = 2$ quadrupole $\rightarrow \theta \sim 90^\circ$
and horizon size $\theta \sim 1^\circ$ is at $\ell \sim 200$

and since $\theta \sim \lambda/d_{\rm rec} \sim 1/dk$:
multipoles scale as $\ell \sim 1/\theta \sim k \sim 1/\lambda$
low $\ell \rightarrow$ big angular, physical scales $\rightarrow$ small k

CMB Anisotropy Observations: Strategy

- achieve high sensitivity, remove systematics
  make a "difference experiment"
  i.e., measure δT directly, don't subtract
- observe as much of the sky as possible (or as needed!)
  balloons/ground: limited coverage
  satellites (COBE, WMAP, Planck): all-sky
- remove Galactic contamination: "mask" plane
- recover Θ for observed region

- decompose into spherical harmonics $Y_{\ell m}$
- construct power spectrum $\ell(\ell+1)C_\ell$
- report results
- collect thousands of citations, prominent Prizes

CMB Temperature Anisotropies: Results

COBE (1993)

- first detection of $\delta T/T \neq 0$
- receiver horn angular opening $\sim 8^\circ$
  $\rightarrow$ only sensitive to large angular scales
  i.e., superhorizon size
- found $(\delta T/T)_{\rm rms} \sim 10^{-5}$
- power $\ell(\ell+1)C_\ell$ flat $\rightarrow$ implies $P(k) \sim k$!
  $n = 1$ spectrum: scale invariant!

Interregnum (late 90's, early 00's)

- ground-based, balloons confirmed anisotropy
- acoustic peaks discovered
  strong indication of first peak

WMAP (2003-)

- first all-sky survey of small angular scales
- $n = 1$ confirmed, indication of small tilt $n - 1 \neq 0$?
  consistent with inflation! and non-trivially so!
- acoustic peaks mapped: good measurement of 1st, 2nd
  detection of third
- first peak: $\ell \sim 200$ horizon at recomb!
- power dropoff seen at large ℓ
  $\rightarrow$ nonzero thickness of last scattering
  due to photon diffusion, non-instantaneous decoupling