Astro 507
Lecture 38
April 30, 2014

Announcements:

- **Problem Set 6 due Friday**
- office hours tomorrow, 3–4pm
- **ICES** available online – please do it!

Leftover issues:

- CMB spherical harmonic decomposition
  $T(\theta, \phi) = \sum_{\ell,m} a_{\ell m} \, Y_{\ell m}(\theta, \phi)$

amplitude $a_{\ell m}$ expected m dependence at fixed ℓ

`www: hint--spherical harmonic maps for different` m

- WMAP/Planck frequency coverage

Last time:

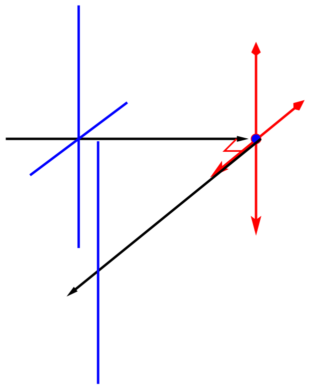
- CMB temperature anisotropies
  Q: what quantity is plotted to show CMB “wiggles”?
  Q: what is the physical origin of CMB “wiggles”?

- began CMB polarization
  Q: how and under what conditions does
  Thomson scattering produce polarization?
  Q: pol’n signal from a region scattering isotropic radiation?

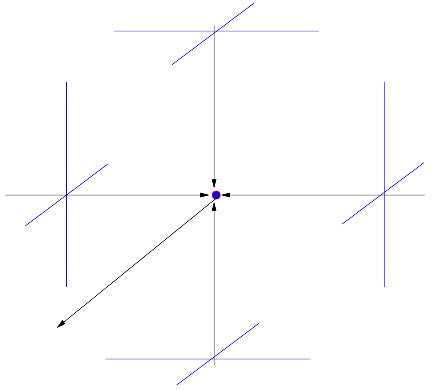
classical picture: e^- as dipole antenna

incident polarized wave accelerates e^-

$\rightarrow$ azimuthally symmetric radiation,
peaks in $\theta = 0$ plane



for isotropic radiation:

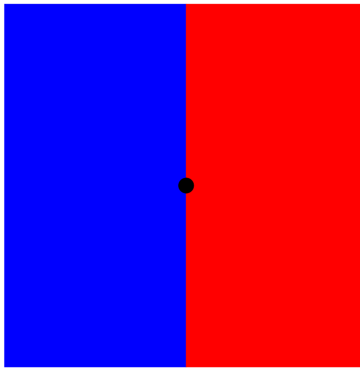


unpolarized!

point on hot-cold "wall"

Q: T pattern seen at point?

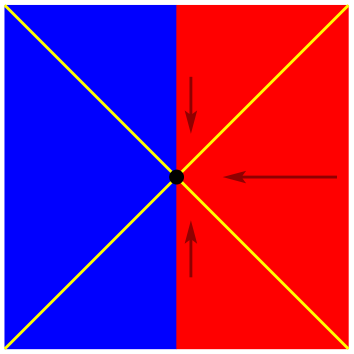
Q: what's scattered pol'n?



pattern seen at point:

dipole anisotropy

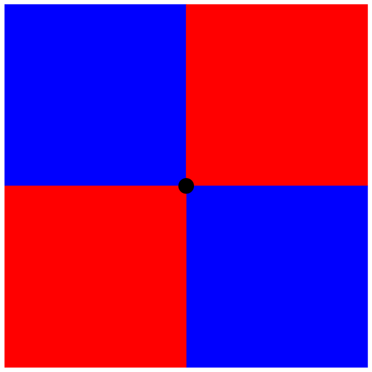
extra polarized radiation from hot region cancels


dipole anisotropy:
unpolarized

Now consider point on “ checkerboard vertex”

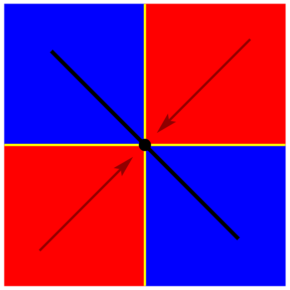
Q: what is scattered polarization? why?

Q: what temperature pattern seen at point?



point sees *quadrupole* anisotropy
extra polarization from hot regions
doesn't cancel

quarupole anisotropy:
linear polarization

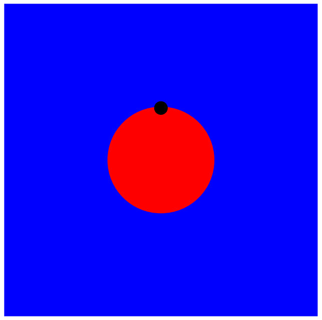


→ net linear polarization towards us, aligned w/ "cold" axis

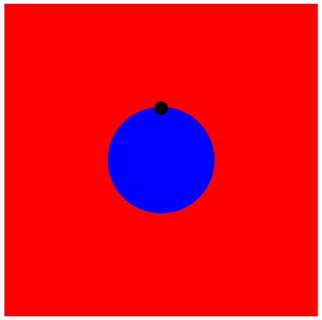
`www: cool Wayne Hu movie`

Q: what about edge of circular hot spot? cold spot?

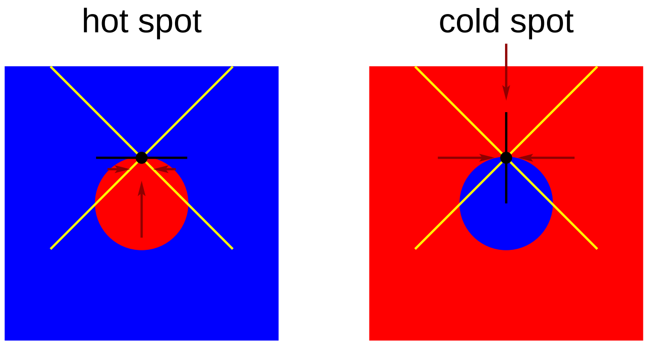
hot spot



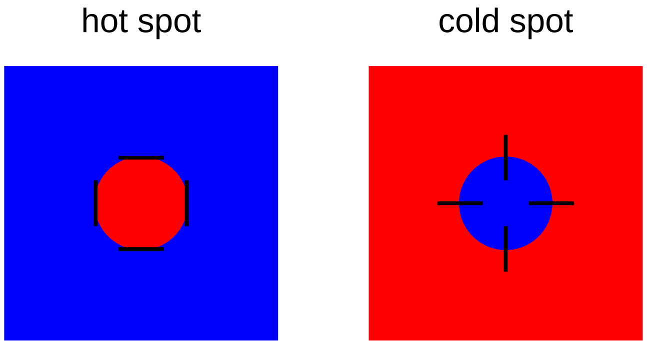
cold spot



at a single point on edge:



so by symmetry:



polarization tangential (ring) around hot spots
radial (spokes) around cold spots
(superpose to “+” $=$ zero net polarization–check!)

www: WMAP polarization observations of hot and cold spots

Note: polarization & T anisotropies *linked*
→ consistency test for CMB theory and hence hot big bang

Polarization: E and B Modes

CMB polarization makes headless vector field on sky
i.e., at each point, polarization vector (possibly zero)
but vector has no "forward/backward" arrow

can decompose polarization field into
- E **modes**: div$\vec{P} \neq 0$ and curl$\vec{P} = 0$
- B **modes**: div$\vec{P} = 0$ and curl$\vec{P} \neq 0$

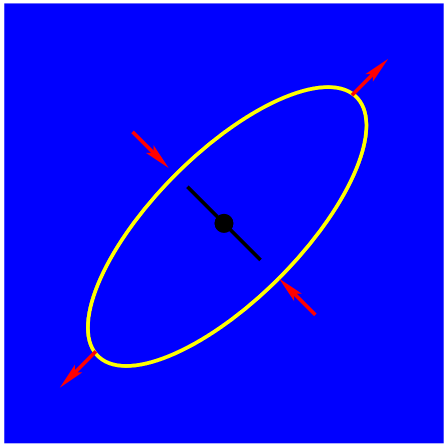
Q: which modes from hot spots? cold spots?

can show:
- *temperature (scalar) perturbations only excite E modes*
- *tensor (gravity wave) perturbations excite both E and B modes*

B Modes and Gravity Waves

recall: gravity waves preserved volume
but stretch and squeeze in + and × modes

gravity wave:
linear polarization



effect on CMB:
velocity perturbation
leads to linear polarization

Polarization Observed

First detection: pre-WMAP!
⋆ DASI (2002) ground-based interferometer
at level predicted based on T anisotropies! Woo hoo!

WMAP (2003): first polarization-T correlation function

WMAP (2006):
- better statistics
- also polarization autocorrelation

⋆ used T-pol'n links to get model-independent
3-D density power spectrum: consistent with scale invariant!

BICEP2: The Revolution Begins?

March 17, 2014:
BICEP2 announces detection of primordial CMB B modes

- B modes measured on large scales (low ℓ)
  should be dominated by primordial gravity wave signal

- B modes already seen by SPT at small scales
  but these are due to CMB lensing by large scale structure

if confirmed:
- crowning achievement of inflation
- opens window to first instants of the Big Bang!

CMB Summary and Outlook

What has the CMB done for us?

- confirmed hot, dense, smooth early universe
- measured primordial power spectrum, consistent w/ inflation
- seen acoustic peaks
- measured a wealth of cosmological parameters
- seen polarization: confirms underlying physics model
- BICEP2: inflationary gravity wave signal!!?!

What will the CMB do for us?

- very soon (this year and next):
  confirmation(?) of gravity wave signal from inflation!
- CMB as background illumination for structure formation
  SZ effect, 21-cm, ...
- stay tuned!

Structure and Horizons

Particle horizons set range for causal physics
including growth of structure
so two requirements for perturbation growth
⋆ perturbation must be inside "horizon," i.e., $\lambda \leq d_H = H^{-1}$
⋆ U. must be matter-dominated: $z < z_{\rm eq}$

Choreography:
inflation lays down perturbations at z enormous
all frozen in until matter domination , then

- on scales already inside Hubble length at $z_{\rm eq}$
  δ_m growth stalled until matter-domination
- on superhorizon scales at $z_{\rm eq}$, δ_m growth begins immediately after $d_H > \lambda$

Today: observe scales in both regimes
Q: What should be the difference?
What characteristic scale divides these regimes?

Key scale in cosmic structure distribution:
comoving Hubble length at matter-rad equality

$$d_{\mathrm{H,com}}(z_{\mathrm{eq}}) = \frac{1}{a_{\mathrm{eq}} H_{\mathrm{eq}}} = \frac{a_{\mathrm{eq}}^{1/2} d_{\mathrm{H},0}}{\sqrt{2\Omega_{\mathrm{m}}}} \sim 60\ h^{-1}\ \mathrm{Mpc} \tag{1}$$

corresponding to $k_{\mathrm{eq}} = 1/d_{\mathrm{H,com}} = 0.02\ h\ \mathrm{Mpc}^{-1}$
Q: sound familiar?

How do does perturbation growth differ
on scales sub/super horizon at at z_{eq}?

in linear regime ($\delta \ll 1$)
linear growth factor: $D(t) = \delta_k(t)/\delta_k(t_{\mathrm{init}})$; k-independent

- large scales have linear growth factor D_0/D_{enter}
- small scales have grown more in absolute terms but less than linear extrap from horizon entry only grown by $D_0/D_{\mathrm{eq}} < D_0/D_{\mathrm{enter}}$

Dividing scale at equality horizon:

$\lambda_{\text{eq}} = d_{\text{com,hor}}(z_{\text{eq}}) \sim \eta_{\text{eq}}$ and corresponding k_{eq}
if smaller scale, horizon entry at pre-eq redshift z_{enter}
such that $d_{\text{hor,com}}(z_{\text{enter}}) = \eta_{\text{enter}} = \lambda$
→ small scales have growth "stunted" by factor

$$\frac{D_{\text{small}}}{D_{\text{large}}} = \frac{a_{\text{enter}}}{a_{\text{eq}}} = \left(\frac{\eta_{\text{enter}}}{\eta_{\text{eq}}}\right)^2 = \left(\frac{\lambda}{\lambda_{\text{eq}}}\right)^2 = \left(\frac{k_{\text{eq}}}{k}\right)^2 < 1 \tag{2}$$

where we used $D \propto a \propto \eta^2$ in matter-dom

Different scales have not grown by same amount!
→ to recover initial power spectrum need to account for this

Transfer Function

Theory (initial power spectrum) connected with
Observation (power spectrum processed by growth)
via transfer function–measures "stunting correction"

$$T_k(z) = \frac{\text{present density spectrum}}{\text{extrapolated initial spectrum}} = \frac{\delta_{k,\text{today}}}{D(z)\delta_k(z)} \tag{3}$$

$$\rightarrow \begin{cases} 1 & k < k_{\text{eq}} \\ (k_{\text{eq}}/k)^2 & k > k_{\text{eq}} \end{cases} \tag{4}$$

Note: since $\delta_{k,\text{init}} \sim \delta_{k,0}/T_k$
power spectrum goes as $P_{k,\text{init}} \sim P_{k,0}/T_k^2$

Now apply to observations

Recovering the Initial Power Spectrum

Apply transfer function to invert observed spectrum

Observed power spectrum

- peak at ~ 30 Mpc $\simeq \lambda_{\rm eq}$ (check!)
- for $k < k_{\rm eq}$, $P_{\rm obs}(k) \sim k^1 = P_{\rm init}(k)$
  → scale invariant! (check!)
- for $k > k_{\rm eq}$, turnover in power spectrum (check!)
  quantitatively: $P_{\rm obs}(k) \rightarrow k^{-3}$
  so $P_{\rm init} \sim P_{\rm obs}/T^2 \sim k^4 P_{\rm obs} \sim k$
  also scale invariant (check!)

observed power spectrum consistent with gravitational growth of scale-invariant spectrum!

Dark Matter–Cold and Hot

Perturbation *growth* & clustering depends on dark matter
internal motions–i.e., “temperature” or *velocity dispersion*
key idea: velocity dispersion (spread) is like pressure
→ stability criterion is Jeans-like

Cold Dark Matter (CDM)

slow velocity dispersion–trapped by gravitational potentials
no lower (well, very small) limit to structure sizes
perturbation growth only limited by onset of matter dom
→ small, subhorizon objects form first, then larger
→ **hierarchical structure formation**: “bottom-up”

Hot Dark Matter (HDM)

high velocity dispersion–escape small potentials
small objects can’t form–large must come first
then fragment to form smaller: “top down”

Q: particle candidate for HDM?

Q: physical implications for HDM structure formation?

Q: how can this be tested?

Q: how does HDM alter the power spectrum (transfer function)?

Hot Dark Matter: Neutrino Cocktail

HDM classic candidate: massive ($m_\nu \sim 1$ eV) neutrinos
if light enough, relativistic before $z_{\rm eq}$
→ "free streaming" motion out of high-density regions
→ characteristic streaming scale: horizon size when $\nu \rightarrow$ nonrel

$$\lambda_{{\rm FS},\nu} \sim 40\ \Omega_m^{-1/2}\ \sqrt{1\ {\rm eV}/m_\nu}\ {\rm Mpc} \tag{5}$$

★ perturbations on scales $\lambda < \lambda_{\rm FS}$ suppressed
★ $\lambda_{{\rm FS},\nu}$ sensitive to absolute ν masses!

If HDM is dominant DM: expect *no* structure below $\lambda_{\rm FS}$
→ a pure HDM universe already ruled out!

If "mixed dark matter," dominant CDM, with "sprinkle" of HDM
HDM reduces structure below $\lambda_{\rm FS}$
→ $\lambda_{\rm FS}$ written onto power spectrum (transfer function)
→ accurate measurements of, e.g., $P(k)$ sensitive to m_ν
cosmic structure can weigh neutrinos! (goal of DES, et al)

ΛCDM

"Standard" Cosmology today: ΛCDM ...namely:

- FLRW universe
- today dominated by cosmological constant $\Lambda \neq 0$
- with cold dark matter
  $\Rightarrow$ hierarchical, bottom-up structure formation
- ...and usually also inflation: scale invariant, Gaussian, adiabatic

This is the "standard" model but not the only one
Q: arguments in favor?
Q: arguments for other possibilities?
Q: which pieces most solid? which shakiest?

At minimum: ΛCDM is *fiducial* / *benchmark* model
standard of comparison for alternatives

...and so we will adopt ΛCDM the rest of the way