Astro 507
Lecture 40
May 2, 2014

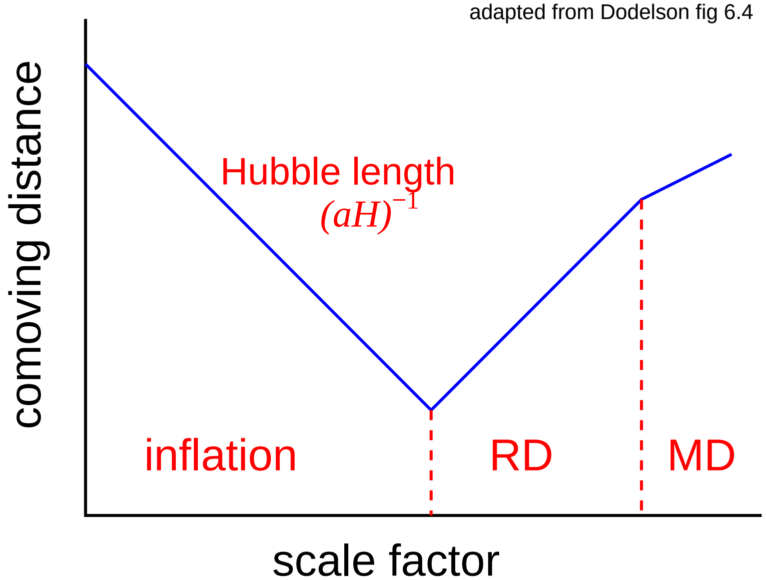
Announcements:

- **Problem Set 6 due Fnow**
- Final Preflight posted, due next Wednesday 9am
  fun, optional, easy bonus points
- **ICES** available online – please do it!

Last time:

consider dark matter perturbations at scale $k = 2\pi/\lambda_{\text{comov}}$

- initially laid down in early universe (by inflation?)
- but "left horzion" when $\lambda > d_{H,\text{comov}}^{\text{inf}} = 1/aH_{\text{inf}}$



adapted from Dodelson fig 6.4

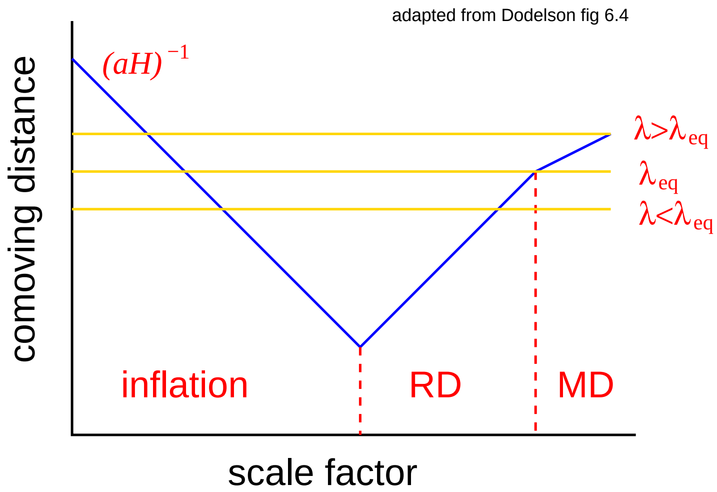
causal physics begins when "re-enter horizon" after inflation

Q: key factor determining further growth history?

Key scale in cosmic structure distribution:
comoving Hubble length at matter-rad equality

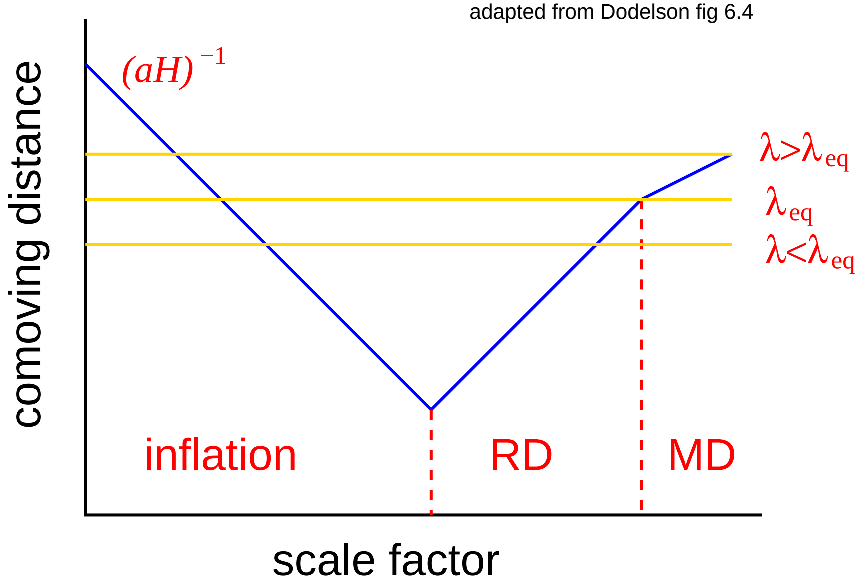
$$d_{\rm H,com}(z_{\rm eq}) = \frac{1}{a_{\rm eq} H_{\rm eq}} = \frac{a_{\rm eq}^{1/2} d_{\rm H,0}}{\sqrt{2\Omega_{\rm m}}} \sim 60\ h^{-1}\ {\rm Mpc} \tag{1}$$

corresponding to $k_{\rm eq} = 1/d_{\rm H,com} = 0.02\ h\ {\rm Mpc}^{-1}$
Q: sound familiar?



Q: How do does perturbation growth differ on scales sub/super horizon at at $z_{\rm eq}$*?*

adapted from Dodelson fig 6.4



in linear regime ($\delta \ll 1$)

linear growth factor: $D(t) = \delta_k(t)/\delta_k(t_{\rm init})$; k-independent

- large scales have linear growth factor $D_0/D_{\rm enter}$
- small scales have grown more in absolute terms
  but less than linear extrap from horizon entry
  only grown by $D_0/D_{\rm eq} < D_0/D_{\rm enter}$

Dividing scale at equality horizon:

$\lambda_{\rm eq} = d_{\rm com,hor}(z_{\rm eq}) \sim \eta_{\rm eq}$ and corresponding $k_{\rm eq}$

if smaller scale, horizon entry at pre-eq redshift $z_{\rm enter}$

such that $d_{\rm hor,com}(z_{\rm enter}) = \eta_{\rm enter} = \lambda$

$\rightarrow$ small scales have growth "stunted" by factor

$$\frac{D_{\rm small}}{D_{\rm large}} = \frac{a_{\rm enter}}{a_{\rm eq}} = \left(\frac{\eta_{\rm enter}}{\eta_{\rm eq}}\right)^2 = \left(\frac{\lambda}{\lambda_{\rm eq}}\right)^2 = \left(\frac{k_{\rm eq}}{k}\right)^2 < 1 \qquad (2)$$

where we used $D \propto a \propto \eta^2$ in matter-dom

Different scales have not grown by same amount!

$\rightarrow$ to recover initial power spectrum need to account for this

Transfer Function

Theory (initial power spectrum) connected with
Observation (power spectrum processed by growth)
via transfer function–measures "stunting correction"

$$T_k(z) = \frac{\text{present density spectrum}}{\text{extrapolated initial spectrum}} = \frac{\delta_{k,\mathrm{today}}}{D(z)\delta_k(z)} \quad (3)$$

$$\rightarrow \begin{cases} 1 & k < k_{\mathrm{eq}} \\ (k_{\mathrm{eq}}/k)^2 & k > k_{\mathrm{eq}} \end{cases} \quad (4)$$

Note: since $\delta_{k,\mathrm{init}} \sim \delta_{k,0}/T_k$
power spectrum goes as $P_{k,\mathrm{init}} \sim P_{k,0}/T_k^2$

Now apply to observations

Recovering the Initial Power Spectrum

Apply transfer function to invert observed spectrum

Observed power spectrum

- peak at ~ 30 Mpc $\simeq \lambda_{\rm eq}$ (check!)

- for $k < k_{\rm eq}$, $P_{\rm obs}(k) \sim k^1 = P_{\rm init}(k)$
  $\rightarrow$ scale invariant! (check!)

- for $k > k_{\rm eq}$, turnover in power spectrum (check!)
  quantitatively: $P_{\rm obs}(k) \rightarrow k^{-3}$
  so $P_{\rm init} \sim P_{\rm obs}/T^2 \sim k^4 P_{\rm obs} \sim k$
  also scale invariant (check!)

$\searrow$ observed power spectrum consistent with
gravitational growth of scale-invariant spectrum!

Dark Matter–Cold and Hot

Perturbation *growth* & clustering depends on dark matter
internal motions–i.e., "temperature" or *velocity dispersion*
key idea: velocity dispersion (spread) is like pressure
→ stability criterion is Jeans-like

Cold Dark Matter (CDM)

slow velocity dispersion–trapped by gravitational potentials
no lower (well, very small) limit to structure sizes
perturbation growth only limited by onset of matter dom
→ small, subhorizon objects form first, then larger
→ **hierarchical structure formation**: "bottom-up"

Hot Dark Matter (HDM)

high velocity dispersion–escape small potentials
small objects can't form–large must come first
then fragment to form smaller: "top down"

Q: particle candidate for HDM?

Q: physical implications for HDM structure formation?

Q: how can this be tested?

Q: how does HDM alter the power spectrum (transfer function)?

Hot Dark Matter: Neutrino Cocktail

HDM classic candidate: massive ($m_\nu \sim 1$ eV) neutrinos
if light enough, relativistic before $z_{\rm eq}$
$\to$ "free streaming" motion out of high-density regions
$\to$ characteristic streaming scale: horizon size when $\nu \to$ nonrel

$$\lambda_{{\rm FS},\nu} \sim 40\ \Omega_m^{-1/2}\ \sqrt{1\ {\rm eV}/m_\nu}\ {\rm Mpc} \quad (5)$$

★ perturbations on scales $\lambda < \lambda_{\rm FS}$ suppressed
★ $\lambda_{{\rm FS},\nu}$ sensitive to absolute ν masses!

If HDM is dominant DM: expect *no* structure below $\lambda_{\rm FS}$
$\to$ a pure HDM universe already ruled out!

If "mixed dark matter," dominant CDM, with "sprinkle" of HDM
HDM reduces structure below $\lambda_{\rm FS}$
$\to$ $\lambda_{\rm FS}$ written onto power spectrum (transfer function)
$\to$ accurate measurements of, e.g., $P(k)$ sensitive to m_ν
cosmic structure can weigh neutrinos! (goal of DES, et al)

ΛCDM

"Standard" Cosmology today: ΛCDM ...namely:

- FLRW universe
- today dominated by cosmological constant $\Lambda \neq 0$
- with cold dark matter
  $\Rightarrow$ hierarchical, bottom-up structure formation
- ...and usually also inflation: scale invariant, Gaussian, adiabatic

This is the "standard" model but not the only one
Q: arguments in favor?
Q: arguments for other possibilities?
Q: which pieces most solid? which shakiest?

At minimum: ΛCDM is *fiducial* / *benchmark* model
standard of comparison for alternatives

...and so we will adopt ΛCDM the rest of the way

Nonlinear Reality

So far: much success in understanding structures in the linear regime $\delta \ll 1$

But the real universe is nonlinear!
What happens when perturbations become large?
$\Rightarrow$ both theory and observations become challenging!

Theory: nonlinear dynamics rich = interesting = hard
some ingenious analytical approximations, special cases
but serious calculations require numerical solution

Observation: collapsed objects can be easy to find
e.g., bright galaxies–but more to the picture than meets the eye
- can't see the DM halos (usually!); mass doesn't trace light
- how to define a halo? measure its mass?

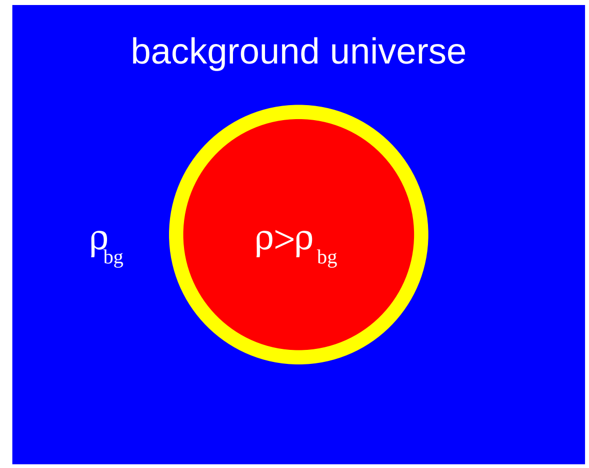
Q: why would this be ambiguous?

Spherical Collapse

consider idealized initial conditions

"top hat" Universe

- spherical, uniform density ρ
- embedded in flat, matter-dom universe with "background" density $\rho_{\rm bg}$ ("compensated" by surrounding underdense shell)



spherical collapse model a cosmological workhorse

a nonlinear problem with analytic solution!

Given: initial density contrast $\delta_i \ll 1$ at some t_i
Want to calculate: density contrast $\delta(t)$
lucky break–Newton's "iron sphere"/Gauss' law/Birkhoff's:
in spherical matter distribution, interior ignorant of exterior
$\Rightarrow$ overdense region evolves exactly as closed universe!

PS6: solution is parametric (cycloid)

$$a(\theta) = \frac{a_{\mathrm{max}}}{2}(1 - \cos\theta) \quad (6)$$

$$t(\theta) = \frac{t_{\mathrm{max}}}{\pi}(\theta - \sin\theta) \quad (7)$$

$$(8)$$

- "development angle" $\theta \propto \eta$ conformal time!
- formally, collapse (to a point!) at $t_{\mathrm{coll}} = 2t_{\mathrm{max}}$

Q: describe overdensity evolution qualitatively?

Q: what really happens when $t \gtrsim t_{\mathrm{coll}}$?

Spherical Collapse: Qualitative Lessons

Formal solution

$$a(\theta) = \frac{a_{\mathrm{max}}}{2}(1 - \cos\theta) \quad ; \quad t(\theta) = \frac{t_{\mathrm{max}}}{\pi}(\theta - \sin\theta) \tag{9}$$

- initially expand with Universe
- but extra gravity in overdensity slows expansion
- reach max expansion at t_{max}, then begin collapse
  "turnaround" epoch
- in reality: after turnaround, infalling matter virializes
  marks birth of halo as collapsed object
- Note: Brooklyn is not expanding! Nor is SS, MW, LG
  Q: what is criterion not to expand?

Beyond the formal solution:

- halo still overdense → neighboring shells fall in
  → mass continues to grow by accretion!
- in real life: mergers too