Astro 507

Lecture 6

Feb. 3, 2014

Announcements:

- PS1 due next Friday, Feb. 7
- Office Hours: 3–4 pm Thursday, or by appointment

Last time: Cosmodynamics I–Newtonian Cosmology
result: the right answer–Dr. Friedmann's famous equation
Suitable for framing, tweets, T-shirts, tattoos...
Q: what's the Friedmann eq? who cares–i.e., why is it useful?
Einstein-de Sitter solution: $\rho = \rho_{\rm m}$, $\kappa = 0$
Q: fate? expansion rate?
Q: what if $\kappa = +1$?

Matter and Curvature

What if $\kappa = +1$?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{c^2}{R^2}a^{-2}$$

a cannot grow without bound *Q: why?*

Q: what is $a_{\rm max}$*?*

Q: evolution after $a = a_{\rm max}$*? cosmic fate?*

What if $\kappa = -1$?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} + \frac{c^2}{R^2}a^{-2}$$

Q: what is $a_{\rm max}$*? cosmic fate?*

Matter and Curvature

if $\kappa = +1$: *positive curvature*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{c^2}{R^2}a^{-2}$$

- $H = \dot{a}/a = 0$ when $a = a_{\rm max} = 8\pi G R^2/3c^2$
- but for all t, all a: $\ddot{a}/a = -4\pi G\rho/3 < 0$

  $\rightarrow$ after maximum, $H < 0 \rightarrow$ *universe contracts*

fate: collapse continues back to $a = 0$: **"big crunch!"**

if $\kappa = -1$: negative curvature

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} + \frac{c^2}{R^2}a^{-2}$$

$H > 0$ for all $a \rightarrow a$ grows without bound

fate: expand forever–**"big chill"!**

at large t, "curvature-dominated": $a(t) \rightarrow ct/R$

Q: how can we tell what our κ value is?

Geometry, Density, and Dynamics

rewrite Friedmann

$$1 = \frac{8\pi G\rho}{3H^2} - \frac{\kappa c^2}{R^2}(aH)^{-2} = \Omega - \frac{\kappa c^2}{R^2}(aH)^{-2} \quad (1)$$

where the density parameter is

$$\boxed{\Omega = \frac{\rho}{\rho_{\text{crit}}}} \quad (2)$$

where the critical density is

$$\boxed{\rho_{\text{crit}} = \frac{3H^2}{8\pi G}} \quad (3)$$

Note: for a particular density component ρ_i
corresponding density parameter is $\Omega_i = \rho_i/\rho_{\text{crit}}$
total $\Omega \equiv \Omega_{\text{tot}}$ sums all species: $\Omega = \sum_i \Omega_i$

Note that

$$\kappa = \left(\frac{aHR}{c}\right)^2 (\Omega - 1) = (\text{pos def}) \times (\Omega - 1)$$

geometry (and fate*) of Universe $\Leftrightarrow \kappa \Leftrightarrow \Omega - 1$

if $\Omega = 1$ ever:

- $\Omega = 1$ always; $\kappa = 0 \rightarrow$ no curvature, expand forever

if $\Omega < 1$ ever:

- $\Omega < 1$ always; $\kappa = -1 \rightarrow$ negative curvature, expand forever

if $\Omega > 1$ ever:

- $\Omega > 1$ always; $\kappa = +1 \rightarrow$ positive curvature, recollapse

Q: but if Ω just a stand-in for κ, why useful?

*κ always gives geometry, but κ and fate decoupled if $\Lambda \neq 0$

Geometry and Fate are Knowable!

we saw: κ found from Ω

and: we can determine $\Omega \propto \rho/H^2$
from *locally measurable quantities* ρ and H:
$\rightarrow$ cosmic fate & geometry knowable!
...and become *experimental questions!*

But recall:
so far, only have considered non-relativistic matter
definitely an incomplete picture
$\rightarrow$ at minimum, must include photons!

To Be or Not to Be Relativistic

for a particle ("species") of mass m
relativistic status set by comparison: typical speed v vs c
equivalent to comparing: typical E_{kin} vs mc^2
but if thermal, $E_{\text{kin}} \sim kT$
$\rightarrow$ relativistic: $kT \gg mc^2$ $\rightarrow$ non-relativistic: $kT \ll mc^2$

massless particles

if $m = 0$: always have $v = c \rightarrow$ forever relativistic

massive particles

if $m > 0$: *always* a time in Early U when $kT \gg mc^2$
$\rightarrow$ massive particles born relativistic, become non-rel!
$\rightarrow$ relativistic status is time-dependent!

Q: are there species which are always *relativistic? non?*
Q: what is relativistic, non-rel today?

Today: $kT_{\text{CMB},0} \sim 10^{-4}$ eV

always: photons relativistic because $m_\gamma = 0$

gravitons also massless (if they exist)

clearly: $m_e c^2, m_p c^2 \gg kT_0 \rightarrow$ non-relativistic today!

but *were* relativistic in early U

but what about *neutrinos?*

we know: 3 massive species exist

do not (yet!) know mass of any species

but we *do* know their mass differences

for experts: oscillation experiments measure $\delta m_{ij}^2 = m_i^2 - m_j^2$

which set a laboratory-based *lower limit*:

heaviest neutrino must have $m_\nu > 0.04$ eV

$\rightarrow$ *at least one ν species non-relativistic today!*

$\rightarrow$ contributes to Ω_{matter}

Redshifts I

quick-n-dirty: wavelengths are lengths! ..it's right there in the name!
$\rightarrow$ expansion stretches photon λ

$$\lambda \propto a$$

if *emit* photon at $t_{\rm em}$, then at later times

$$\lambda(t) = \lambda_{\rm emit}\frac{a(t)}{a(t_{\rm em})} \tag{4}$$

if *observe* later, $\lambda_{\rm obs} = \lambda_{\rm em}\ a_{\rm obs}/a_{\rm em}$
measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \tag{5}$$

high z $\leftrightarrow$ *small* a $\leftrightarrow$ *distant past*

Newtonian Derivation of Redshift: Hubble & Doppler

slower-n-cleaner: non-relativistic Doppler
non-rel Doppler sez:

$$\frac{\delta\lambda}{\lambda} \equiv z = \frac{v}{c} \tag{6}$$

Hubble sez:

$$cz = Hr \tag{7}$$

Together

$$\frac{\delta\lambda}{\lambda} = \frac{Hr}{c} \tag{8}$$

But light travels distance r in time $\delta t = r/c$, so

$$\frac{\delta\lambda}{\lambda} = H\delta t = \frac{\dot{a}\delta t}{a} = \frac{\delta a}{a} \tag{9}$$

for arriving light, fractional λ change = fractional a change!

Scale Factor and Redshift

$$a = \frac{1}{1+z}$$
$$z = \frac{1}{a} - 1$$

recordholders to date–most distant objects `www: recordholders`

- farthest quasar: $z = 7.085$
- farthest gamma-ray burst: $z \approx 9.2$
- farthest galaxy: $z \sim 12$ (photometric data only)

For $z = 12$, *when light emitted:*

$\rightarrow$ scale factor was $a = 0.08$

interparticle (intergalactic) distances 8% of today!

$\rightarrow$ galaxies were 13 times closer

squeezed into volumes 2200 times smaller!

$\rightarrow$ age: $t = 2/3\ \Omega_{\rm m}^{-1/2} t_{\rm H}/(1+z)^{3/2} = 0.026\ t_{\rm H} = 370\ {\rm Myr}$

Q: implications of seeing galaxies and GRBs at such z?

Redshifts and Photon Energies

in photon picture of light: $E_\gamma = hc/\lambda$

so in cosmological context photons have

$$E_\gamma \propto \frac{1}{a} \tag{10}$$

$\rightarrow \gamma$ energy redshifts

Consequences:

▷ *Q: photon energy density $\varepsilon(a)$?*

▷ if thermal radiation,

Q: $T \leftrightarrow \lambda$ connection?

Q: expansion effect on T?

Relativistic Species

Photon energy density: $\varepsilon_\gamma = E_\gamma \, n_\gamma$
average photon energy: $E_\gamma \propto a^{-1}$
photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)
$\Rightarrow$ $\varepsilon_\gamma \propto a^{-4}$

Thermal (blackbody) radiation:
Wien's law: $T \propto 1/\lambda_{\text{max}}$
but since $\lambda \propto a$ $\rightarrow$ then $T \propto 1/a$

Consequences:

- $\varepsilon_\gamma \propto T^4$: Boltzmann/Planck!
- T decreases $\rightarrow$ U cools!
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but "garden variety" quasar: $z = 3$
  "feels" $T = 8$ K (effect observed!)