Astro 507

Lecture 9

Feb. 10, 2014

Announcements:

- Preflight 2 due next Friday, 9am

  I. Discussion (public)

  II. Reading response

Cosmology talks this week:

- Astro Colloquium, Tues. **3:45pm**, Astr 134
  Kim Coble, Chicago State University
  cosmology in the transformation of undergraduate teaching
- Astro Theory Seminar, Wed. *noon*, Loomis 464
  Keith Bechtol, U. Chicago
  extragalactic γ-ray background, MW satellite galaxies

Last time:

- spacetime–Aristotle to Galileo/Newton to Einstein
- the **interval** *Q: namely? significance?*
- equivalence principle *Q: namely?*
- rocket gedankenexperiment
  Q: implications for light trajectory?
  Q: implications for photon frequency/wavelength/energy?
  Q: implications for clocks?

Equivalence Principle: in uniform gravity g

$\to$ same results as rocket accelerating with $a = g$

- gravity bends light!

`www: strong lensing`

- gravitational redshift/blueshift!

$$\frac{\delta\lambda}{\lambda} \approx \frac{\delta v_{\text{obs}}}{c} \approx \frac{g\,\delta h}{c^2} = \frac{\delta\phi}{c^2} \tag{1}$$

- gravitational time dilation:

$$\frac{\delta\tau}{\tau} = \frac{\delta\lambda}{\lambda} \approx \frac{g\delta h}{c^2} = \frac{\delta\phi}{c^2} \tag{2}$$

Lesson: gravity distorts

- light path = space
- apparent frequency = time

$\to$ *gravity alters spacetime!*

Einstein (1915): *include gravity in spacetime*

General Relativity

Newton (1687): Universal Gravitation
gravity is a force (field) that couples to mass
▷ matter tells gravity how to force
▷ gravity force tells matter how to move

Einstein (1915): General Relativity
gravity is spacetime curvature: not a force!
★ "matter tells spacetime how to curve
★ spacetime tells matter how to move" –J. .A. Wheeler

Curved Spacetime?

Curved space: geometric constructions in space
(triangles, rectangles, circles... *Q: how define?*)
give non-Euclidean results *Q: namely?*

Q: so–curved spacetime?

Spacetime Curvature

Test: (Feynman Lectures II, Chapter 42)

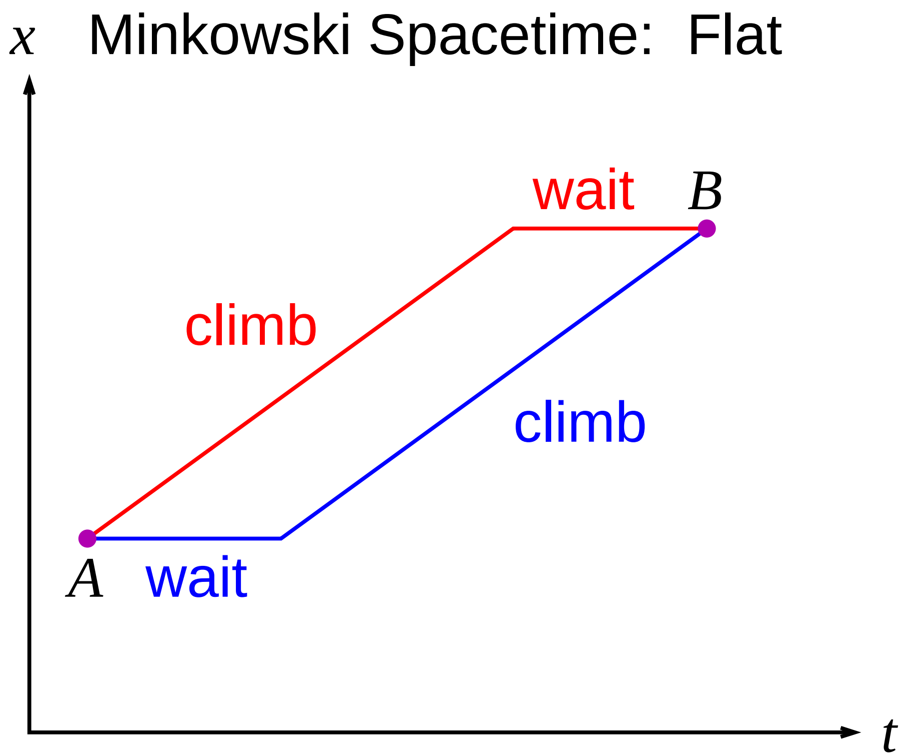
- construct geometric object in spacetime
- are properties Euclidean?

Case 1: Minkowski Space (i.e., special relativity, no accel)
(1-D) interval ("line element") for events separated by (dt, dx)

$$ds^2 = dt^2 - dx^2 \tag{3}$$

Construct rhombus: in *spacetime*
two observers go from events A to B
▷ obs 1: go left at $v = 0.5c$ for 10 s, then wait 10 s
▷ obs 2: wait 10 s, then go left at $v = 0.5c$ for 10 s

Q: spacetime diagram?



result **is Euclidean** *Q: why?*

$\Rightarrow$ Minkowski spacetime is **not curved = flat**

Case 2: Surface of Earth (i.e., const accel: gravity)
(1-D) line element:

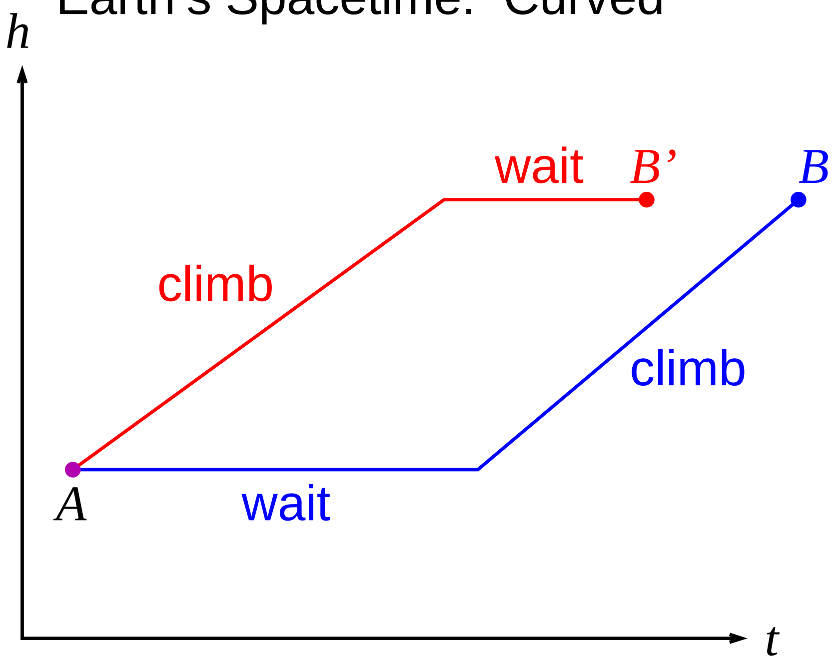
$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) dt^2 - \left(1 + \frac{2\phi}{c^2}\right)^{-1} dx^2 \tag{4}$$

where $\phi = \phi(x)$: time-independent Newtonian potential

Construct rhombus in *spacetime*
two observers go from events A to B
▷ obs 1: go up 1 km, then wait 10 s
▷ obs 2: wait 10 s, then go up 1 km

Q: spacetime diagram?

Earth's Spacetime: Curved



result is **not Euclidean**:

$$(\text{wait time}) = (\delta s)_{\text{wait}} = \sqrt{1 + 2gh/c^2}\ (\delta t)_{\text{wait}} \tag{5}$$

why? waiting time "advance differently" − *time dilation*!

∞ ⇒ Earth's spacetime is **curved**!

gravity ⇔ spacetime curvature

GR on a T-Shirt

General Relativity spirit and approach:
like special relativity, only moreso

Special Relativity concepts retained:

- **spacetime**: events, relationships among them
- **interval** gives observer-independent (invariant) measure of "distance" between events
- Special Relativity is a special case of GR
  SR: no gravity $\rightarrow$ no curvature $\rightarrow$ "flat spacetime"
  GR limit: gravity sources$\rightarrow$0 give spacetime$\rightarrow$Minkowski

GR: Special Relativity concepts generalized

- gravity encoded in spacetime structure
- spacetime can be curved
- coordinates have no intrinsic meaning

The Metric

Fundamental object in GR: **metric**
consider two nearby events, separated by
coordinate differences $dx = (dx^0, dx^1, dx^2, dx^3)$
GR (in orthogonal spacetimes) sez:
interval between them given by **"line element"**

$$
\begin{aligned}
ds^2 &= A(x)\,(dx^0)^2 - B(x)\,(dx^1)^2 - C(x)\,(dx^2)^2 - D(x)\,(dx^3)^2 \\
&\equiv \sum_{\mu\nu} g_{\mu\nu} dx^\mu dx^\nu \equiv g_{\mu\nu} dx^\mu dx^\nu
\end{aligned}
$$

where the **metric tensor** $g_{\mu\nu}$

- in this case (orthogonal spacetime): $g = \text{diag}(A, B, C, D)$
- components generally are functions of space *& time* coords
- is symmetric, i.e., $g_{\mu\nu} = g_{\nu\mu}$
- encodes all physics (like wavefunction in QM)

Q: if no gravity=Minkowski, what's the metric?

physical interpretation of interval: like in SR

$$ds^2 = (\text{apparent elapsed time})^2 - (\text{apparent spatial separation})^2$$

⋆ observers have *timelike* worldlines: $ds^2 > 0$

⋆ light has *null* $ds = 0$ worldlines

Simplest example: Minkowski space (Special Relativity)

$g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$: constant values proper spatial distances:

- i.e., results using meter sticks
- measured simultaneously ($dx^0 = 0$)

length element:

$$d\ell^2 = -ds^2 = d\ell_1^2 + d\ell_2^2 + d\ell_3^2 = g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2$$

space (3-)volume element:

$$\begin{aligned} dV_3 &= d\ell_1 d\ell_2 d\ell_3 \\ &= \sqrt{|g_{11}g_{22}g_{33}|}\; dx^1 dx^2 dx^3 \end{aligned}$$

spacetime 4-volume element:

$$\begin{aligned} dV_4 &= d\ell_0 dV_3 = \sqrt{|g_{00}g_{11}g_{22}g_{33}|}\; dx^0 dx^1 dx^2 dx^3 \\ &= \sqrt{|\det g|}\; dx^0 dx^1 dx^2 dx^3 \end{aligned}$$

Example: Minkowski space, Cartesian coords

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

length: $d\ell^2 = dx^2 + dy^2 + dz^2$

3-volume: $dV_3 = dx\, dy\, dz$

4-volume: $dV_4 = dx\, dy\, dz\, dt$

Example: Minkowski space, spherical coords

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

length: $d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

3-volume: $dV_3 = r^2 \sin\theta\, dr d\theta d\phi \equiv r^2 dr d\Omega$

4-volume: $dV_4 = r^2 dr d\Omega dt$

Relativistic Cosmology

Cosmological Spacetimes

Want to describe spacetime of the universe
to zeroth order: homogeneous, isotropic

1. at each spacetime point
   exactly *one* observer sees isotropy*
   call these fundamental observers
   roughly: "galaxies" i.e., us
   (strictly speaking, we don't qualify) *Q: why?*

2. isotropy at each point $\rightarrow$ homogeneity
   but can be homogeneous & not isotropic

*We will see: observers moving w.r.t. FOs eventually come to rest w.r.t. FOs

3. homogeneity and isotropy $\rightarrow$ symmetries

U. is **“maximally symmetric”**

$\rightarrow$ greatly constrain allowed spacetimes

i.e., allowed metrics

The Cosmic Line Element

cosmological principle:
can divide spacetime into time "slices"
i.e., 3-D spatial (hyper) surfaces

▷ populated by fundamental observers

▷ with coords, e.g., (t, x, y, z)

▷ choose FO's to have $d\vec{x} = 0$
i.e., spatial coords are **comoving** ("fixed to expanding grid")
on surface: fundamental observers must all have
$ds^2 = dt^2 \rightarrow$ i.e., $g_{tt} = const = 1$ *Q: why?*
$\rightarrow g_{tt}$ *indep of space, time*

these give:

$$ds^2 = dt^2 - g_{ii}(dx^i)^2 \tag{6}$$

Cosmological Principle and the Cosmic Metric

homogeneity and time

no space dependence on $d\ell_0 = dt$

- can define cosmic time t (FO clocks)
- at fixed t, time lapse dt not "warped" across space

homogeneity and space

- at any t, properties invariant under translations
- no center
- can pick arbitrary point to be origin
- e.g., here!

Cosmological spacetime encoded via cosmic **metric**
which determines how the interval depends on coordinates
any observer computes interval between events as
$ds^2 =$ (elapsed time)^2 – (spatial displacement)^2

Cosmic metric so far:

$$ds^2 = dt^2 - g_{ii}(dx^i)^2 \tag{7}$$

where: t is cosmic time

now impose *isotropy*

- at any cosmic t, interval invariant under rotations
- pick arbitrary origin, then (comoving) spherical coords the usual r, θ, ϕ, with $r^2 = x^2 + y^2 + z^2$ and arbitrary origin (usually, but not always, here!)

Q: now that does metric look like?

For *fundamental* observers, maximal symmetry demands metric which can* be written as:

$$ds^2 = dt^2 - a(t)^2 d\ell^2_{\text{com}} \quad (8)$$

$$= dt^2 - a(t)^2 \left[f(r) dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (9)$$

$a(t)$ is the cosmic scale factor

$f(r)$ is as yet undetermined

- for flat (Euclidean) space, $f(r) = 1$
- so $f \neq 1 \rightarrow$ non-Euclidean spatial geometry = curved space!

Q: why same time dep for radial and angular displacements?

Note power of cosmo principle

$\rightarrow$ only allowed dynamics is uniform expansion $a(t)$!

*other space & time coordinates possible and sometimes useful

but in all cases space and time must *factor* in this way