Astro 210

Lecture 11

Feb 9, 2018

Announcements

- **HW3 due online in PDF, today 5:00 pm**
- Good News: *No HW next week!*
- Bad News: **Hour Exam 1 in class next Friday**
  info and old exams online
- HW1 grades posted; all solutions posted before exam
- Planetarium shows next **Tuesday** and **Thursday**
  online: **reservations**, schedules, directions, **report form**

Last time: light=electromagnetic radiation as astronomer's tool

Q: why "electromagnetic"? why "radiation"?

Q: wave-like properties of light?

Q: particle properties of light?

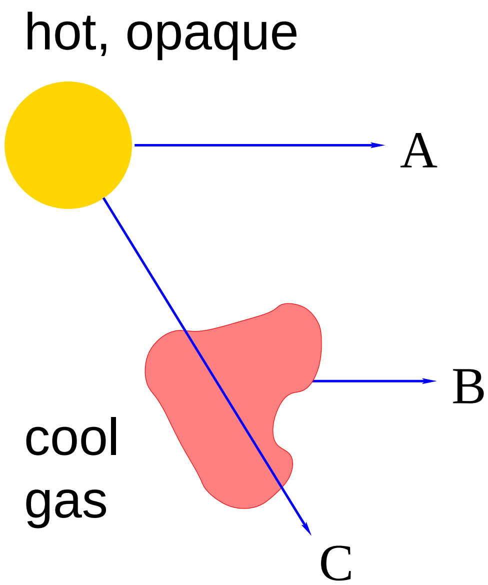
apparent brightness:
flux $F = (dE/dt)/A =$ incident power/collecting area
measures light energy flow

for experts: $F = \varepsilon\, c$ where ε is EM energy density
so flux is EM energy current density

Q: how does your eye interpret flux?

Q: what's "luminosity"? What is L for a typical lightbulb?
Q: flux from a spherical spherical source at r?

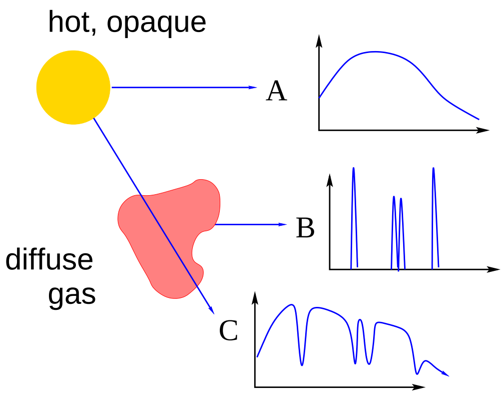
can classify three basic kinds of spectra: **Kirchoff's Rules**



Q: what does A see? hint–space heater demo

Q: what does B see? hint–neon light

Q: what about C?



A. a hot and opaque solid, liquid or dense gas emits
a *continuous spectrum* (A)

B. a hot low-density (transparent) gas produces
an *emission line spectrum*
note: *pattern of lines specific to element*

C. Continuous radiation viewed though cooler gas produces
an em absorption line spectrum, note: the lines absorbed have
same color/wavelength as the emission lines in B

these effects are godsends for astrophysics! *Q: why?*

Matter

Recall:

matter
↓
molecules
↓
atoms
↙ ↘
nucleus electrons
↙ ↘
protons neutrons

atoms come in **elements**

92 natural, 23+ artificial

`www: periodic table`

determined by nuclear charge $Z = \#$ protons

e.g., hydrogen H: $Z = 1$

uranium U: $Z = 92$

same element (same # p) can have different # neutrons

$\rightarrow$ "isotopes"

examples: most hydrogen is $^1\text{H} = \boxed{1p,\ 0n}$

but $\sim 10^{-4}$ of hydrogen is deuterium $^2\text{H} = \boxed{1p,\ 1n}$

most U is $^{238}\text{U} = \boxed{92p,\ 146n}$; about $\sim 1\%$ is $^{235}\text{U} = \boxed{92p,\ 143n}$

atom net charge fixed by # electrons

$\# e = \# p \rightarrow$ neutral

$\# e = \# p - 1 \rightarrow$ singly ionized

Note: all p, n, e are absolutely *identical* and *indistinguishable*

this turns out to be crucial for the understanding of matter

in a quantum mechanical way

Atoms & Spectra

how are spectral lines ("barcode") related to atom structure?

Balmer hydrogen gas $\rightarrow$ emission line spect. (visible λ)
found empirical pattern to lines

$$\lambda = 3.65 \times 10^{-7} \, \frac{n^2}{n^2 - 4} \text{ m} \quad n \text{ integer } \geq 3 \tag{1}$$

(1) only these lines seen and no others
(2) simple mathematical structure cries out for explanation!

try it! for $n = 3$:
$\lambda_{n=3} = 3.65 \times 10^{-7} \frac{9}{9-4} \text{m} = 656 \text{nm}$
Q: what color is this? `www: Balmer spectrum`
`www: Sun spectrum;` Hα $\rightarrow$ the Sun contains hydrogen!

Prince Louis-Victor de Broglie

not only light behaves like particle & wave
but also matter:
$\longrightarrow$ matter waves exist!?!

what is λ?
for photons, λ and $p = E/c$ related:

$$\lambda = \frac{c}{f} = \frac{c}{E/h} = \frac{hc}{E} = \frac{h}{p} \tag{2}$$

recall $h = 6.6 \times 10^{-34}$ Joule $\cdot$ s = Planck's constant

de Broglie hypothesis/guess: same holds for matter

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{3}$$

i.e., matter has wave properties
expect to show up on lengthscales $\sim \lambda$

Q: so why doesn't a baseball diffract out of your hand?

A Quantum Baseball?

regulation mass $m = 5$ oz $= 0.14$ kg
easy toss: $v \sim 1$ m/s
$\longrightarrow$ momentum $p = mv \sim 0.14$ kg m/s
$\longrightarrow$ de Broglie wavelength

$$\lambda_{\text{deB,baseball}} = \frac{h}{p} = 5 \times 10^{-33} \text{ m} \lesssim 10^{-14} \times \text{size of proton} \quad (4)$$

wave properties and hence quantum effects unobservably small!
$\longrightarrow$ expect baseballs to exhibit classical (Newtonian) behavior
$\longrightarrow$ can't blame fielding errors on quantum mechanics!

Q: in what circumstances would quantum effects not be small?
i.e., for what objects is λ_{deB} larger?

Bohr model of the atom

quantum structure of atom: e orbits are matter waves
"semiclassical"−mixes Newtonian & quantum ideas

- de Broglie waves → standing waves in atom
- e orbits circular
- only certain radii, speeds allowed ("quantized states") → only certain allowed energies
- during e transitions between states, photon emitted → photon energies quantized → spectral lines

Bohr Atom: Quantum Electrons Orbit Nucleus

Ingredients:

- circular orbits
- electrons have de Broglie wavelengths $\lambda = h/p = h/m_e v$

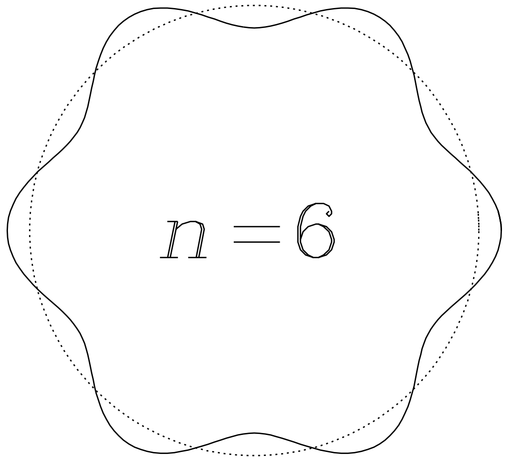
- standing waves:

Demo: slinky

e orbit path length

an integer multiple of λ:

$$2\pi r = n\lambda = n\frac{h}{m_e v} \quad (5)$$



$\longrightarrow$ for each n, radii and speeds related

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- Coulomb force provides centripetal accel:

Q: remind me–what is Coulomb force?

Coulomb force: electrical attraction between opposite charges
an inverse square law! same structure as gravity!

$$F_{\text{Coulomb}} = \frac{q_1 q_2}{r^2} = \frac{e^2}{r^2} \tag{6}$$

(cgs charge units: $e^2_{\text{cgs}} = k e^2_{\text{SI}} = e^2_{\text{SI}}/4\pi\varepsilon_0$)

Coulomb provides electron's centripetal acceleration:

$$m_e a_c = F_{\text{Coulomb}} \tag{7}$$

$$m_e \frac{v^2}{r} = \frac{e^2}{r^2} \tag{8}$$

another relation between r and v
$\rightarrow$ two equations, two unknowns $\rightarrow$ solution exists

The Bohr Atom: Results

Bohr: fit *integer* number $n \geq 1$ standing waves
into Coulomb-controlled circular orbits
⇒ only certain definite radii/speeds/energies allowed
⇒ "quantized" orbits

allowed radii:

$$r_n = n^2 \frac{\hbar^2}{e^2 m_e} \tag{9}$$

allowed speeds:

$$v_n = \frac{1}{n} \frac{e^2}{\hbar} \tag{10}$$

Q: so physically, higher n means?

Q: how is this similar to and different from ordinary non-quantum ("classical") circular Kepler motion?

Bohr orbit energies:

$$\begin{aligned} E_n &= \frac{1}{2} m_e v_n^2 - \frac{e^2}{r_n} && (11) \\ &= -\frac{1}{n^2} \frac{e^4 m_e}{2\hbar^2} \propto \frac{1}{n^2} && (12) \end{aligned}$$

recall: negative energy $\rightarrow$ *bound* orbits

$\rightarrow$ electron bound to nucleus, takes energy to remove

diagram: energy level structure

Q: which level is most tightly bound?

Q: what about photon energies (lines)?

In transition, γ energy is **difference** between states:
if go from $n_{\text{hi}} \rightarrow n_{\text{lo}}$, with $n_{\text{hi}} > n_{\text{lo}}$, photon energy is

$$\begin{aligned} E_\gamma &= E_{n_{\text{hi}}} - E_{n_{\text{lo}}} && (13) \\ &= -\frac{e^4 m_e}{2\hbar^2}\left(\frac{1}{n_{\text{hi}}^2} - \frac{1}{n_{\text{lo}}^2}\right) && (14) \\ &= \frac{e^4 m_e}{2\hbar^2}\,\frac{n_{\text{hi}}^2 - n_{\text{lo}}^2}{n_{\text{hi}}^2 n_{\text{lo}}^2} && (15) \end{aligned}$$

photon wavelength:

$$\lambda_\gamma(n_{\text{hi}} \rightarrow n_{\text{lo}}) = \frac{hc}{E_\gamma} = hc\frac{2\hbar^2}{e^4 m_e}\frac{n_{\text{hi}}^2 n_{\text{lo}}^2}{n_{\text{hi}}^2 - n_{\text{lo}}^2} \quad (16)$$

define: Rydberg $R = \frac{4\pi c\hbar^3}{e^4 m_e} = 1.1 \times 10^7\ \mathrm{m}^{-1}$

$$\Rightarrow \lambda_\gamma(n_{\mathrm{hi}} \to n_{\mathrm{lo}}) = \frac{1}{R}\ \frac{n_{\mathrm{hi}}^2 n_{\mathrm{lo}}^2}{n_{\mathrm{hi}}^2 - n_{\mathrm{lo}}^2} \tag{17}$$

put $n_{\mathrm{lo}} = 2$: drop to 1st excited state

$$\Rightarrow \lambda_\gamma = \frac{1}{R}\ \frac{4 n_{\mathrm{hi}}^2}{n_{\mathrm{hi}}^2 - 4} = 3.6 \times 10^{-7}\ \frac{n^2}{n^2 - 4}\ \mathrm{m} \tag{18}$$

$\Rightarrow$ Balmer's result! explained by quantum mechanics!

Lyman series: $n_{\mathrm{lo}} = 1$
Balmer series: $n_{\mathrm{lo}} = 2$
Paschen series: $n_{\mathrm{lo}} = 3$