Astro 210
Lecture 40
April 30, 2018

Announcements:

- Congratulations! You are done with HW!
- Final Exam: May 7, 8:00 am - 11:00 am
  info on Moodle
- Solar Observing: alternative exercise posted
  for those unable to attend
  due Friday at 5pm

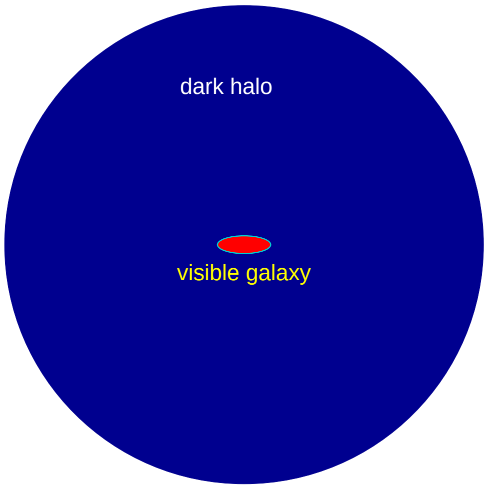
Last Time: Galaxies

Q: what's a galaxy? main types ("morphologies")?

Q: internal dynamics?

Q: role of dark matter in galaxy structure and dynamics?

Galaxies are Made of Dark Matter



Last time:
all galaxies have flat rotation curves
matter extends out far beyond visible galaxy

- **galaxy masses dominated by dark matter**
- **dark halo** extends far beyond visible galaxy
  forms gravitational potential well
  ordinary matter falls in, form visible galaxy

COSMOLOGY

Physical Cosmology

Modest goals:
scientific understanding of the

- origin
- evolution
- contents
- structure
- future

of the Universe

To be a science: must have empirical evidence
$\longrightarrow$ need observable data to reveal/test the above

Q: like what?

A Universe of Galaxies

Galaxies are the building blocks of the Universe
similar to stars as building blocks of galaxies
and atoms as building blocks of matter

Data: galaxy survey = map of galaxies = map of the Universe
`www: Typical survey data: 2dFGRS`

Q: what do you notice?

Q: features on large scales? small scales?

Structure of the Universe

Galaxy distribution in space:

- small scales: clumpy
- large scales: smooth

Einstein's **Cosmological Principle**

on large scales, Universe is

1. **homogeneous**:
   galaxies fill space with (nearly) uniform density
2. **isotropic**:
   universe looks same in all directions

Q: What's a U that is isotropic but not homogeneous?

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iClicker Poll: Guess the Galaxy Motions

measure velocity respect to us for many galaxies
i.e., in galaxy spectra, look for shifts in lines

What are proportions of approaching=blueshift vs receding=redshift?

A number approaching $>$ 100 number receding

B number approaching $>$ 10 number receding

C number approaching $\approx$ number receding

D number receding $>$ 10 number approaching

E number receding $>$ 100 number approaching

Motions of Galaxies

Sloan Digital Sky Survey: $\approx 1,000,000$ galaxy spectra
only 16 galaxy blueshifts (many spurious)
essentially *all galaxies show redshift*
all galaxies move away from us!

Edwin Hubble (1929)
galaxy distance d and speed v related
`www: Edwin's original data; modern data`
data show: $v \propto d$, or

$$v = Hd \tag{1}$$

Q: in simple terms, what does this say about galaxy motion?

Hubble's Law

Hubble says: farther $\longrightarrow$ faster

more specifically: **Hubble's Law**

$$v = Hd \quad (2)$$

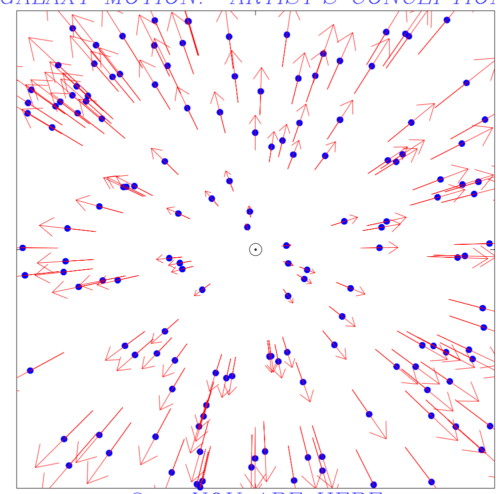
H: "Hubble constant" (sometimes written H_0)
Q: units of H?
value: $H_0 \approx 70$ km/(s · Mpc)

draw random (homogeneous) dots
Q: add velocity vectors–what's the pattern?

What does velocity pattern suggest?
Q: logical possibilities?

The Meaning of Hubble's Law

GALAXY MOTION: ARTIST'S CONCEPTION



⊙ = YOU ARE HERE

observe: all galaxies move away from us!

galaxy motion highly organized

not random! not like atoms in gas

⟶ cries out for interpretation!

Egoist view: we are at center of Universe!

but:

- MW typical galaxy, not special
- center un-Copernican

Einstein view:

no center! ...and no edge! Universe is homogeneous!

the universe is expanding

highfalutin mathematics:
Hubble's law a very special mathematical form
in fact, the **only** form that has the following property:
photocopy universe

Hubble's law universal:
in expanding U, **all** galaxies
find $v = Hd$
$\longrightarrow$ don't need to live in a special place to
find Hubble's law

Revolution Re-Re-Revisited

Copernican Revolution I (17th Century):
Earth is one typical planet among many
not center of solar system

Copernican Revolution II (earth 20th Century):
Sun is one typical star among many
not center of Milky Way Galaxy

Copernican Revolution III (1920's):
Milky Way is one typical galaxy among many
Universe much larger than previously thought

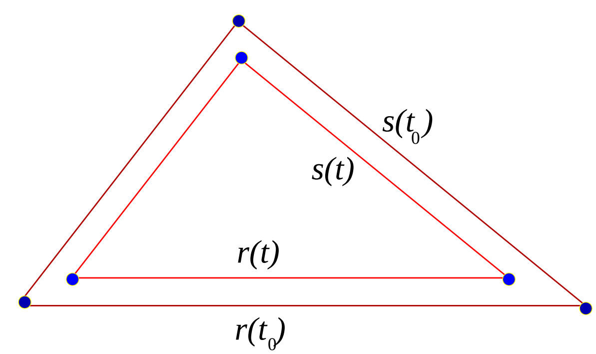
Copernican Revolution IV (1929):
we live in an expanding universe
which has no center at all!

Expanding Universe: Theory

recall General Relativity (Einstein Gravity):
⋆ space (& time) dynamic!
⋆ Universe is dramatic example of this

consider arbitrary triangle defined by 3 observers at t_0
Hubble law → observers in relative motion
→ at later time t, larger triangle

the claim:
later △ always *similar to* original △
Q: what are similar triangles?
Q: why must similarity hold?
Q: connections among r's and s's?



similar: triangle angles preserved

$\Rightarrow$ *side ratios preserved*,

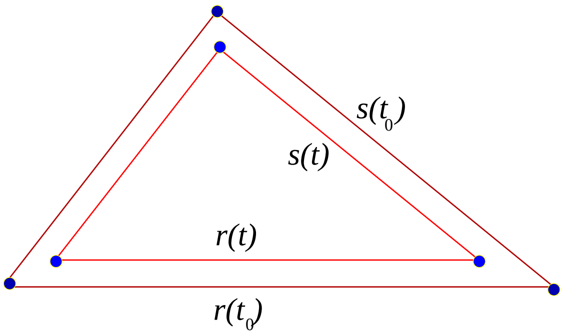
so must have

$$\frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)}$$

holds for any triangle,

so side *length ratio depends only on time t:*

$$a(t) = \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)}$$



Q: what does this imply about cosmic motion?

We have shown:
Cosmo Principle demands *any length* $r(t)$ evolves as

$$r(t) \propto a(t) \tag{3}$$

and so without loss of generality we may write

$$\boxed{r(t) = a(t)\ r_0} \tag{4}$$

where we choose $\boxed{a(t_0) = 1}$ today, and
$r_0 = r(t_0)$ is *present value* of length ("comoving coordinate")

$a(t)$ must be universal **cosmic scale factor**
can depend only on time
and at any t: a has same value everywhere in space

This is huge!
Q: why? What have we proven?

Explaining Hubble: Expansion

on cosmic lengthscales, space uniformly expands
stretches "like rubber sheet"

Describe mathematically: write $d(t) = a(t)r$
$d(t)$: physical separation
r: fixed coords on rubber sheet
$a(t)$: measures expansion – "scale factor" or stretch-o-meter
put present age of Universe: $t = t_0$ today
then $a(t_0) = a_0 = 1$

Director's Cut Extras

How does the Universe expand?

- need to find $a(t) \to$ encodes cosmic expansion history
- How to find $a(t)$?
  similar to Newtonian physics we know and love:
  find law of motion that relates motion to gravity sources

consider two points (observers) in Universe at t

- (arbitrary) central point at $r = 0$
- a test mass m at distance $d(t) = a(t)\, r$

find energy, motion of test mass

$\to$ tells motion of any arbitrary point in universe

$\to$ cosmic dynamics!

what is energy, motion of test mass?

- sees spherical "enclosed" cosmic mass
  $M = \rho V_{\text{sphere}} = 4\pi/3\ d^3 \rho$
- has energy $E_{\text{tot}} = KE + PE = const$ (< 0 if bound)

$$\frac{1}{2}mv^2 - \frac{GMm}{d} = const \quad (5)$$

$$m\left[\frac{1}{2}(Hd)^2 - \frac{4\pi G}{3}\rho d^2\right] = const \quad (6)$$

$$H^2 - \frac{8\pi G}{3}\rho = \frac{2const}{md^2} = -\frac{\kappa}{a^2} \quad (7)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{\kappa}{a}{}^2 \quad (8)$$

Friedmann equation for evolution of $a(t)$

Cosmodynamics

Friedmann is cosmic "equation of motion"
kinda a cosmic "$F = ma$" (really, cosmic energy equation)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{\kappa}{a^2} \tag{9}$$

Which means that the evolution of the universe

- expansion rate $H = \dot{a}/a$, and thus
- cosmic history $a(t)$, are controlled by

are controlled by the contents of the universe

- density ρ (can change with time)
- "energy" (really–curvature) constant $\kappa = const$

A Matter-Only Universe

Simplest situation (simplest universe!): matter only

- $\rho = \rho_{\text{matter}}$ only (no funny business!)
  then in sphere d, mass $M = 4\pi d^3 \rho/3 = const$
  so $\rho \propto a^{-3}$: as U expands, ρ decreases!
- total energy (curvature) zero: $\kappa = 0$

Evolution Solved

Assemble the pieces and turn the crank:

$(\dot{a}/a)^2 = 8\pi G/3\ \rho_0 a^{-3}$

$\dot{a}/a \propto a^{-3/2}$

$a^{1/2} da \propto dt$

integrate: $\int_0^a a^{1/2} da \propto \int_0^t dt$

$a^{3/2} \propto t$

$a \propto t^{2/3}$

put $t = t_0$ today

want $a(t_0) = 1$

So finally arrive at solution for matter-only Universe: $a(t) = (t/t_0)^{2/3}$

Q: fate of such a universe? i.e., what happens when $t \to \infty$?

What does it mean?
For matter-only universe, $a(t) = (t/t_0)^{2/3}$

fate: a always increases with time
$\rightarrow$ universe expands forever
as $t \rightarrow \infty$, then $a \rightarrow \infty$
universe becomes very low density!

what is t_0 = age of Universe?
since $H = \dot{a}/a = 2/3t$ always!
today: $t_0 = 2/3 \; 1/H_0 \simeq 10$ billion years

Q: is this reasonable?
compare: Earth 4.6 billion yrs
oldest stars: 10–12 billion yrs