Astro 210

Lecture 6

Jan 29, 2018

Announcements

- **HW2 due online in PDF, Friday 5:00 pm**
- **HW1 extended until 11:59pm today**
- **register** your iClicker; link on course webpage
- first Planetarium shows Mon Feb 5 and Wed Feb 7
  info online: **reservations**, schedules, directions, report form
- if this is your first class: see me afterward!

Last time: a tale of two cosmologies

- **Geocentric**

Q what's that? how does it explain sunrise? retrograde?

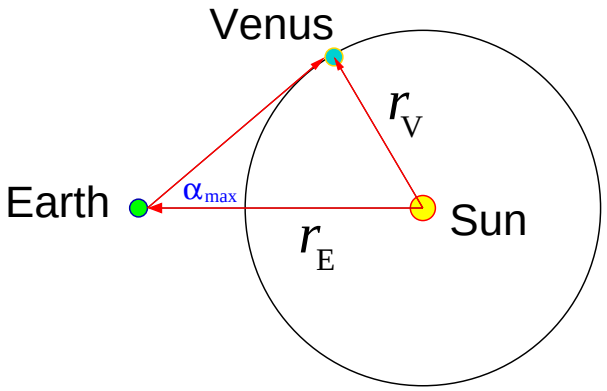
- **Heliocentric**

Q what's that? how does it explain sunrise? retrograde?
bonus: gives relative scale of Solar System *Q: how?*
New unit: AU, sometimes also written au *Q: what's that?*

Today: geocentric vs heliocentric cagematch!

Copernican Model: Solar System Proportions

Venus: *maximum angle from Sun* = max "elongation"
observed as $\alpha_{\max} = 46^\circ$



from diagram: right triangle, Earth-Sun distance is hypotenuse
$\Rightarrow \sin\alpha_{\max} = r_V/r_E$
$\Rightarrow r_V = r_E/\sin\alpha_{\max} = 0.72\, r_E$

New unit: "astronomical unit" = average Earth-Sun distance

$$1 \text{ AU} = 1 \text{ au} \equiv r_E = 1.50 \times 10^8 \text{ km}$$

- Earth (average) orbit radius: 1 AU
- Venus orbit: 0.72 AU

Copernicus: What's New and What's Not

- planets still on spheres
- Copernicus sill used epicycles!
- predictions not better than in Ptolemy's model
- $\longrightarrow$ geometrically equivalent
- Copernicus' model not generally accepted
  and Ptolemaic–Copernican disagreement though to be metaphysical, *unanswerable* question

Q: so how do we decide which is right?

Tycho Brahe 1546-1601: Danish Astronomy Extraordinaire

in youth: observed "nova stella" (supernova)

`www: Tycho sketch`

→ heavens are not eternal but changeable ("corruptible")!

observed Sun, Moon, planets for 20 years:
no telescope, but still careful, accurate data

Tycho not a good number cruncher
→ like any good professor: made grad student do the work!

Johannes Kepler 1571–1630: Harmony of the Worlds

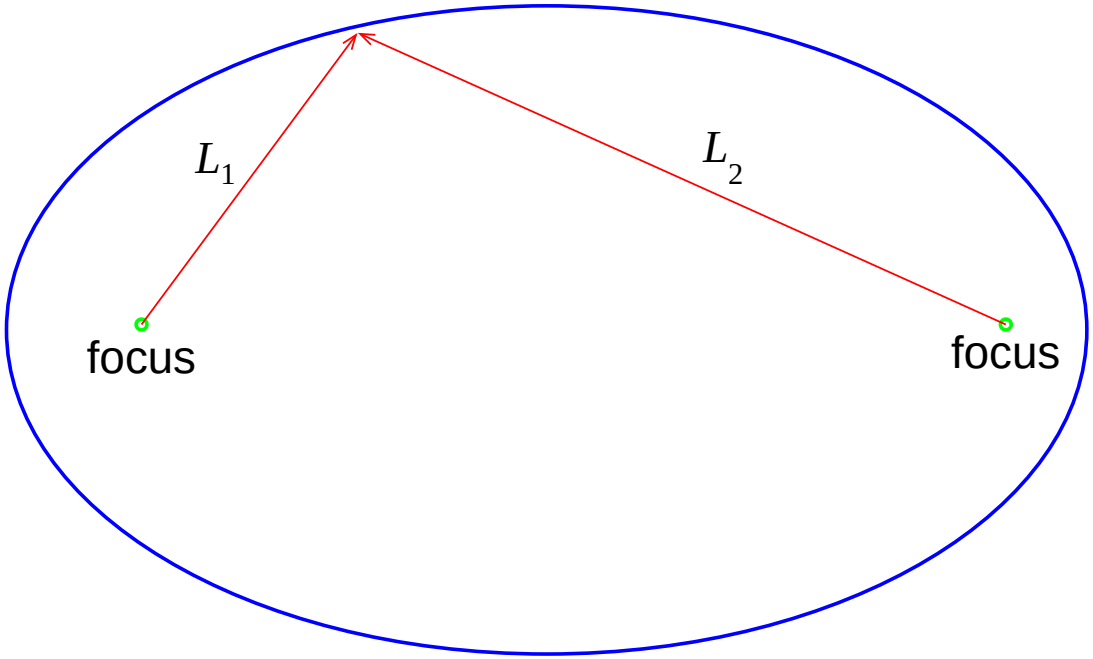
Analyzed Tycho's precision data for **20 years**(!)
$\longrightarrow$ especially Mars motions

at first: used *heliocentric model* with *circular orbits*
but observations didn't quite agree
a small error (few arc min!) remained...took seriously

after years of trial & error:
- abandoned circular orbits
- completely & accurately described planet orbits

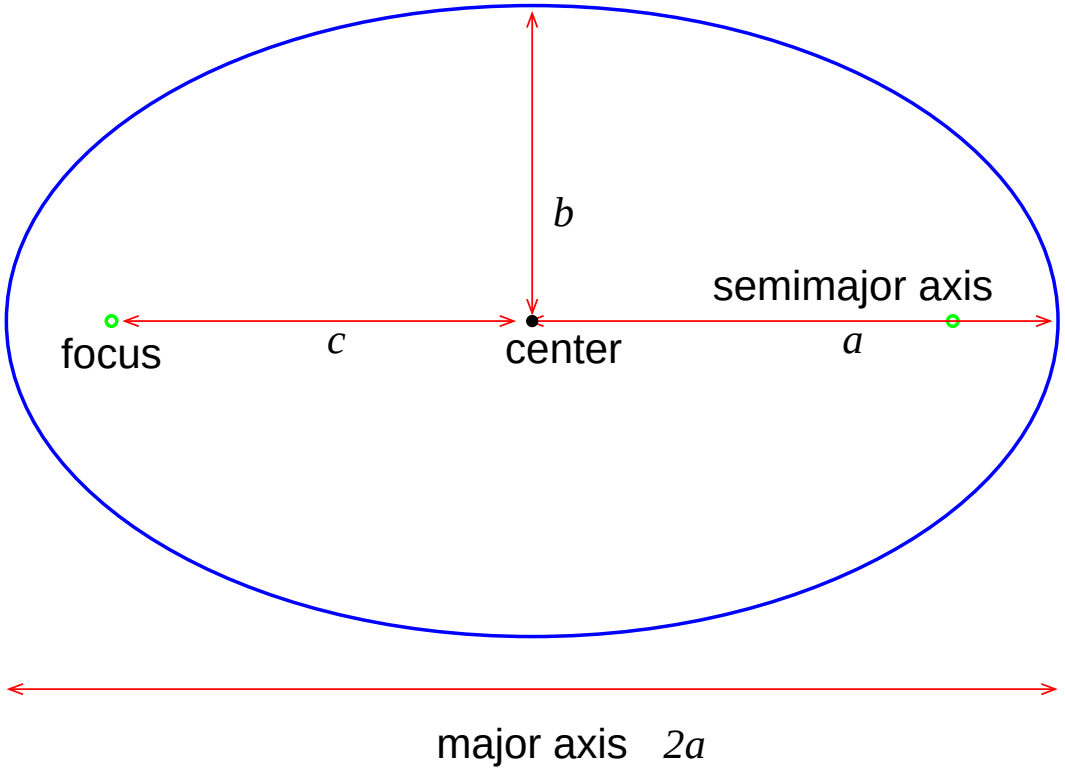
Kepler I: Law of Ellipses

each planet's orbit is an **ellipse**
with the **sun at one focus**



$$L_1 + L_2 = constant$$

Ellipse Anatomy



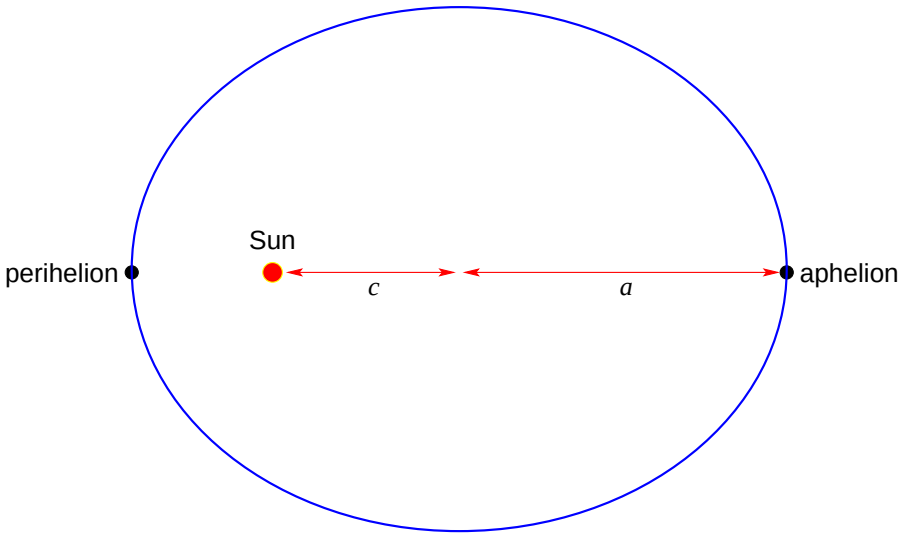
- two foci
- semi-major axis $\boxed{a}$
- focal length c
- semi-minor axis $b = \sqrt{a^2 - c^2}$

any ellipse fully characterized by:

∞ a and eccentricity $\boxed{e = c/a}$

Q: what do we get for $e = 0$*?* $e = 1$*?* `www: eccentricity demo`

Kepler I: orbit is **ellipse** with **sun at one focus**



Orbit anatomy

aphelion: *farthest* point from Sun

perihelion: *closest* point to Sun

Q: what is aphelion distance in terms of a *and* e*?*

Q: If the Sun's at one focus, what's in the other focus?

Q: Sun, Moon, planet geometry in 3-D and on celestial sphere?

Q: What does Kepler I not *say about orbits?*

Aphelion/perihelion distance:

$$r_{\text{ap/peri}} = a \pm c = a \pm a\frac{c}{a} = (1 \pm e)a \qquad (1)$$

At the other focus: nothing! (sorry!)

ellipses are **planar** objects:

- In 3D: Moon-Earth, Sun-Earth, and Sun-Planet orbits
  are each confined to its own plane
  not necessarily all coplanar (and are not!), but close
- on sky = celestial sphere: great circles

Note: Kepler I only gives orbit *shape*
but says *nothing* about how orbit evolves in time
$\rightarrow$ need more info to fully describe orbit, hence...

Kepler II: Law of Equal Areas

a straight line from the planet to the sun
sweeps out equal areas in equal times

diagram: sketch areas

this amounts to telling about speed of planet

iClicker Poll: Kepler II and Planet Speed

When does a planet move the *fastest*?

A When it is closest to the Sun

B When it is farthest from the Sun

C Trick question! In vacuum of space, planet speeds must be constant

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`www: area animation`

Q: This still doesn't fully characterize an orbits–why not?

Kepler I gives orbit shape in space
Kepler II gives orbit evolution over time

but haven't yet connected the two:
how does spatial character (e.g., semimajor axis a)
relate to time character (e.g., period P)?

Need one last law...

Kepler III: Connecting Space and Time

planet orbit period P and a are related:

$$P^2 \propto a^3$$

$\Rightarrow P^2/a^3 = const$

constant is same value for all planets around Sun!
so any planet's orbit obeys

$$\frac{P^2}{a^3} = \frac{P^2_{\text{Earth}}}{a^3_{\text{Earth}}} \tag{2}$$

and thus we can choose to write

$$\left(\frac{P}{P_{\text{Earth}}}\right)^2 = \left(\frac{a}{a_{\text{Earth}}}\right)^3 \tag{3}$$

Q: in other words?

Kepler III: The Mighty Equation

For *any object orbiting Sun* (not just planets!):

$$P_{\text{yrs}}^2 = a_{\text{AU}}^3 \tag{4}$$

Q: ok for earth?

where P written in years, a in AU

Very powerful! e.g.:

Asteroids exist with orbits inside 1 AU (and some cross 1 AU!!)

`www: inner solar system objects--in real time!`

iClicker Poll: Kepler III

Kepler III: $P_{\text{yrs}}^2 = a_{\text{AU}}^3$

Consider an asteroid with an orbit entirely inside 1 AU

Is its period longer or shorter than a year?

A $P > 1$ yr, no matter eccentricity e

B $P < 1$ yr, no matter what e

C can't answer without knowing e

Kudos to Kepler

Several points worth noting...

⋆ An amazing discovery–mathematics underlies the workings of the cosmos!

⋆ Keplers laws remain accurate to this day–indeed, in slightly generalized form will show up in many (most!) situationswhere motions are controlled by gravity

⋆ Yet note what we still don't have:
an understanding of *why* Kepler's laws hold
→ that is, what is the *mechanism* that makes
planets move this way
...for that, need to wait for Kepler's successors...

Galileo Galilei

First to use telescope in Astronomy

`www: Galileo shows scope to Duke`

contributions:

- mountains on the moon
- moons of Jupiter
- sunspots

None of these directly contractic the geocentric model
but all are contrary to its underlying philosophy

⋆ heavenly objects are imperfect

⋆ a clear example of a heavenly motion not centered on Earth