Astro 210

Lecture 7

Jan. 31, 2018

Announcements

- **HW2 due online in PDF, Friday 5:00 pm**
- HW1 grading under way; you'll be notified when done
- **register** your iClicker; link on course webpage
- first Planetarium shows Mon Feb 5 and Wed Feb 7
  info online: **reservations**, schedules, directions, report form

Last Time:

Kepler's laws

Q: which are?

Q: why a Big Deal?

Q: when does a planet move fastest? slowest? Q: what planet has shortest orbit period? longest?

Kepler's Laws: Twitter Version

1. planet orbits are ellipses, with Sun at one focus
2. orbits sweep equal areas in equal times
3. $P_{\mathrm{yr}}^2 = a_{\mathrm{AU}}^3$

these completely and precisely characterize planet orbits

Phases of Venus

Galileo's telescope showed: Venus has phases

Ptolemaic and Copernican models make *different* predictions

`www: Venus phase animations: geocentric vs heliocentric`

Q: Ptolemy has freedome for where to put Sun, what does this do?

`www: observed phase cycle`

Experimentum Crucis: The Critical Test

Definite and different predictions: • Ptolemaic model: *Venus phase always $<$ half illuminated* • Copernican model: *Venus shows all phases*

Galileo: **full cycle of phases seen!**

observations contradicted Aristotle/Ptolemy
supported Copernicus
"paradigm shift" (Kuhn)
radical change in outlook/conceptual framework

Galileo brilliant but also arrogant and politically naive

→ offended powerful people, including the Pope, a former ally

tried in Inquisition and forced to recant geocentric view

- his work, Copernicus, Kepler banned until 1832
- official semi-apology ("mistakes were made") 1992(!)

complex situation: crackdown as much political as theological

Note:

1. really not at all obvious to people that Earth orbits Sun
2. the paradigm shift was difficult and threatening

New World Order

Copernican Revolution (17th Century): we're one typical planet among many
not center of solar system

With earth removed from center of universe,
Aristotle's division of terrestrial and heavenly
no longer made sense as physics

$\Rightarrow$ need to re-examine "natural motion"
$\Rightarrow$ search for force that keeps planets in place
$\Rightarrow$ Galileo's **experiments**

Galileo: Dynamics & Gravity

Galileo not only great astronomer
but also a great physicist
paved way for Newton's dynamics by study of
two special cases of motion

1. **"free body"** – *no* external influences
natural motion: coast in straight line, const speed
$\longrightarrow$ retain current state of motion
$\longrightarrow$ bodies have **inertia**

2. **"free fall"** – when only influence is *gravity*
Galileo recognized another key motion
Demo: Tower of Pisa expt
∞ $\longrightarrow$ constant acceleration **indep of mass**!
$a = g, \; g = 9.8\,\text{m/s}^2$

Galilean free fall: constant acceleration $a = g$

So speeds change linearly with time

$v = v_0 + gt$; if $v_0 = 0$, $v = gt$

Distance traveled is quadratic in time:

$$d = \int_0^{t_f} dt\, v(t) = \int_0^{t_f} dt\, gt = \frac{1}{2} g t_f^2 \quad (1)$$

Ex how long does it take to drop from table to floor?

$d \sim 1\text{m} \Rightarrow t^2 = 2d/g = 2 \times 1\text{m}/9.8\,\text{m/s}^2 \sim 0.2s^2 \Rightarrow \boxed{t \sim 0.45 \text{ s}}$

Sir Isaac Newton 1642–1727 English

Newton's Laws of Motion – "T-Shirt Review"

Newton I

a free body = no net force (i.e., no acceleration)
motion: constant velocity $\rightarrow$ same speed **and** direction

Mathematically:
displacement $\vec{r}$, velocity $\vec{v}$, acceleration $\vec{a}$ are **vectors**
⋆ **displacement** $\vec{r} = (x, y, z)$, distance $r = |\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$
⋆ **velocity** $\vec{v} = d\vec{r}/dt$, speed $v = |\vec{v}|$
⋆ **acceleration** $\vec{a} = d\vec{v}/dt$, magnitude $a = |\vec{a}|$

Note: time derivative of vector $\vec{v}(t) = [v_x(t), v_y(t), v_z(t)]$
is $d\vec{v}/dt \equiv \dot{\vec{v}} = [\dot{v}_x(t), \dot{v}_y(t), \dot{v}_z(t)]$
where "overdot" $= d/dt$

Newton I

a free body = no net force (i.e., no acceleration)
motion: constant velocity $\longrightarrow$ same speed **and** direction

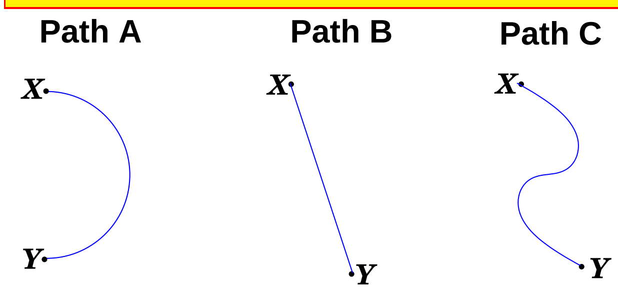
mathematically:
acceleration $\vec{a} \equiv \dot{\vec{v}} = 0$
$\Rightarrow$ velocity $\vec{v}_{\text{free}}(t) = \vec{v}_0 = const$

Newton I:
- encodes Galileo's "free body" behavior
- establishes existence of inertial frames

iClicker Poll: Acceleration

young James T. Kirk (remake version) drives from point X to Y
his motorcycle speedometer readings are unknown
maybe constant, maybe not

In which case(s) is it certain he accelerated?



A Path C only

B Paths A and C

C Paths A, B, and C

D if speed kept constant, all paths can be unaccelerated

Newton II

acceleration is proportional to force,
and inversely proportional to body's mass
$\Rightarrow a = F/m$ or $F = ma$
or $F = dp/dt$, with $p = mv$ (**momentum**)
or in 3-D:

$$\boxed{\vec{p} = m\vec{v}} \tag{2}$$

$$\boxed{\vec{F} = d\vec{p}/dt} \tag{3}$$

Newton II is machine to *predict the future!*
Q: why? how? what needed for Newtonian fortunetelling?

Fortunetelling (and Archæology!) with Newton II:

$\dot{\vec{v}} = \vec{F}/m$: force changes speed

$\rightarrow$ after time interval δt, velocity changed by $d\vec{v} = \vec{F}\delta t/m$

$\rightarrow$ carries particle to new position

$\rightarrow$ where it feels new force

$\rightarrow$ which changes speed

...lather, rinse, repeat

So: if we know

- present position & speed (initial conditions)

then we can predict the future and reconstruct the past:

- determine the nature of the forces
- apply Newton II and turn mathematical crank
- solve particle trajectory for all time–past, present, future!

Newton III

"action-reaction"

apply to: forces between two interacting objects

the rule:
when one body exerts force on another
the other body exerts force of equal magnitude
but opposite direction on the one

$$\vec{F}_{12} = -\vec{F}_{21}$$
$$1 \text{ on } 2 = -2 \text{ on } 1$$

note magnitudes same: $|F_{12}| = F_{12} = F_{21}$

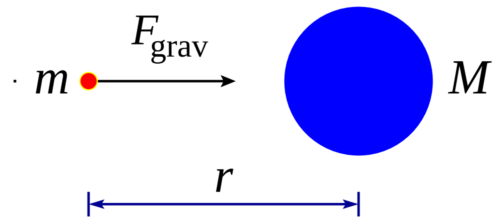
`www: jumpshot`

Newton Gravitation

Newton's **Law of Gravitation**
a force, gravity, exists between any two objects having mass
depends on masses M, m and
distance $\vec{r}$ between centers

coordinates: centered at M;
then force on m is

$$\vec{F} = -G\frac{Mm}{r^3}\vec{r} = -G\frac{Mm}{r^2}\hat{r}$$



where $r = |\vec{r}|$, and $\hat{r} = \vec{r}/r$ is a radial unit vector

G is Newton's constant: universal–applies everywhere!
but has to be determined experimentally *Q: how?*
expt: $G = 6.67 \times 10^{-11}$ m^3/kg s^2

Q: find acceleration on earth surface?

gravitational acceleration on surface of earth $= \oplus$
for body of "test" mass m:

$$a = \frac{F}{m} = \frac{1}{m} G \frac{m M_\oplus}{R_\oplus^2} = G \frac{M_\oplus}{R_\oplus^2} \tag{4}$$

$$= 6.7 \times 10^{-11} \mathrm{m^3/kg s^2} \frac{6.0 \times 10^{24} \mathrm{kg}}{(6.4 \times 10^6 \mathrm{m})^2} = 9.8 \mathrm{m/s^2} \tag{5}$$

Note:

- test mass m cancels! as Galileo found experimentally!
- that is: inertial mass = gravitational coupling

  Not obvious! no reason why need to be identical

- mass m and weight F different things

iClicker Poll: Weightlesses in Space?

Consider an astronaut orbiting Earth on the Space Shuttle
Is she weightless?

A yes

B no

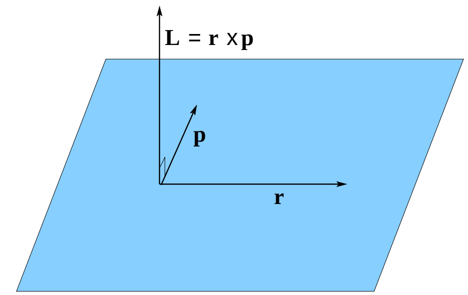
C depends on whether the rockets are firing

Note larger issue:
cosmic context requires rethinking “homegrown” intuition

Angular Momentum

For point mass, **angular momentum** defined as:

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} \quad (6)$$



i.e., using cross product

look at time change:

$$\frac{d}{dt}\vec{L} = m\dot{\vec{r}} \times \vec{v} + m\vec{r} \times \dot{\vec{v}} \quad (7)$$
$$= m\vec{v} \times \vec{v} + m\vec{r} \times \vec{a} \quad (8)$$
$$= \vec{r} \times \vec{F} = \vec{\tau} \quad \text{torque} \quad (9)$$

angular counterpart of Newton II:

- net (linear) force changes linear momentum
- net twisting force = torque changes angular momentum

Gravity and Angular Momentum

angular momentum changed by net torque

$$\frac{d}{dt}\vec{L} = \vec{r} \times \vec{F} \tag{10}$$

when force is due to **gravity**, torque:

$$\vec{\tau} = \vec{r} \times \vec{F} = -G\frac{mM}{r^3}\vec{r} \times \vec{r} = 0 \tag{11}$$

so if force is gravity, then

$$\frac{d}{dt}\vec{L} = 0 \tag{12}$$

and thus $\vec{L} = const$:

angular momentum is **conserved!**

Q: what about gravity force gauranteed this?

What Keeps the Earth in Orbit?

circular orbit $\rightarrow$ centripetal accel.

angular speed $d\theta/dt = \omega = 2\pi/P = const$

$$\vec{a}_c = -\omega^2 \vec{r} = -\frac{v^2}{r}\ \hat{r}$$

diagram: show $\vec{v}$, $\vec{r}$, $\vec{a}$

Newton II: acceleration demands net force

but Newton gravity supplies a force!

$\rightarrow$ Newtonian gravity is crucial and necessary ingredient
for understanding the dynamics of planetary motion
but have to see how the detailed predictions
compare with observation

Program:

- assume Newtonian gravity controls planetary motion
- that is, for any planet let $\vec{F}_{\text{net}} = \vec{F}_{\text{Sun-planet}}$
- input this into Newton's laws
- turn mathematical cranks $\rightarrow$ predict orbits
- compare predictions with observation

Solutions: Orbits

For attractive inv. sqare force, orbits are cross sections of cone:

- circle
- ellipse
- parabola
- hyperbola
- line

Circle eccentricity $e = 0$

at each point:

$F = ma = mv_{\rm c}^2/r$

$\Rightarrow GMm/r^2 = mv_{\rm c}^2/r$

$\Rightarrow$ circular orbits have speed $v_{\rm c} = \sqrt{\frac{GM}{r}}$

example: find circular speed 1 AU from Sun

$v_{\rm c} = 3 \times 10^4$ m/s

Kepler from Newton

Kepler I: Orbits are ellipses

Newton: bound orbits due to gravity are ellipses: check!

Kepler II: Equal areas in equal times

Newton: consider small time interval dt

move angle $d\theta = \omega dt$

sweep area

diagram: top view: path, $d\theta, \vec{r}, \vec{v}, \vec{v}_t$

$$dA = \frac{1}{2} r^2 d\theta = \frac{1}{2} r^2 \omega dt \tag{13}$$

but $\omega = v_\theta / r$, where $\vec{v_\theta} \perp \vec{r}$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{14}$$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{15}$$

finally, $r v_\theta = |\vec{r} \times \vec{v}| = |\vec{L}|/m$
Q: why?, so

$$dA = \frac{1}{2} \frac{L}{m} dt \tag{16}$$

Woo hoo! were' home free! *Q: why?*

But $L = const$ for radial force ($\vec{r} \times \vec{F} = 0$)
so

$$\frac{dA}{dt} = \frac{L}{2m} = const \tag{17}$$

Kepler II! $\rightarrow$ comes from ang. mom. cons.!