Astro 210

Lecture 9

Feb. 5, 2018

Announcements

- **HW3 due online in PDF, Friday 5:00 pm**
- HW grading under way; you'll be notified when done
- **register** your iClicker; link on course webpage
- first Planetarium shows **Today** and **Wednesday**
  online: **reservations**, schedules, directions, **report form**

Last Time: Newton Explains Kepler

Q: how did Newton go about explaining Kepler's Laws?

Q: are these really pre*dictions or* post*dictions?*

Q: what are the allowed Newtonian orbits?
Q: how can we classify them?
Q: what is their range of applicability–where/when/to what do they apply?
Q: what's special about parabolic orbits?

Last Time: Newton Explains Kepler

Newton: solve $\vec{F} = m\vec{a} = m\ddot{\vec{r}}$ with $\vec{F} = -GMm/r^2\ \hat{r}$
gives back Kepler's laws, and so

- agrees precisely with observed planet orbits
- also explains how orbits arise from gravity
- and gives, e.g., circular speed: $\boxed{v_c = \sqrt{\frac{GM}{r}}}$
- and updates Kepler III: $a^3 = \left(\frac{GM}{4\pi^2}\right) P^2$

Newtonian gravity: possible orbits
line, circle, ellipse, parabola, hyperbola

Gravity and energy
Q: relation to bound and unbound orbits?

Escape Speed

At radius r, define **escape speed** $\boxed{v_{\text{esc}} = \sqrt{2GM/r}}$

- if launch from r with $v_{\text{launch}} < v_{\text{esc}}$
  then $TE < 0$: *fall back!* (elliptical orbit)
- if launch from r with $v_{\text{launch}} > v_{\text{esc}}$
  then $TE > 0$: *escape "easily"*: $v > 0$ at $r = \infty$
- if launch from r with $v_{\text{launch}} = v_{\text{esc}}$ exactly
  then $TE = 0$ exactly, *"just barely" escape*

So: escape speed is *minimum speed* needed to leave
a gravitating source

Example: escape speed from surface of earth
$v_{\text{esc}}(R_{\text{Earth}}) = 11$ km/s $= 25{,}000$ mph!
predict the future: if toss object with $v < 25,000$ mph, falls back
but if $v > 25,000$ mph *Q: example?* never returns!

Unbound Orbits: Hyperbolæ

finally, the "generic" unbound orbit:

hyperbola

$$r(\theta) = \frac{(1-e^2)a}{1+e\cos\theta} \tag{1}$$

$e > 1$, $a < 0$! $\rightarrow$ total energy $TE > 0$
speed $v > 0$ at $r = \infty$: nonzero speed far from M

Recall: at large r, hyperbola $\rightarrow$ *straight line*
But Newton says: $d\vec{v}/dt = -GM/r^2\ \hat{r}$
so as $r \rightarrow \infty$, then $d\vec{v}/dt \rightarrow 0$
$\Rightarrow$ gravity negligible, $\vec{v} \rightarrow$ const: *free body*=straight line!

orbit of unbound "flyby":
distant nearly free body $\rightarrow$ passing: pulled toward M
$\rightarrow$ distant deflected nearly free body

Newtonian Orbits: Scope and Implications

Newtonian orbits apply to much more than Sun+planets:
motion outside *any* isolated, spherically symmetric mass

Including:

- all objects orbiting Sun
- planet and Moon systems
- planets around other stars
- binary star systems

Also note: *if not other forces act*
that is: move around single, spherical source

- *bound orbits always remain bound*
- *unbound orbits always remain unbound*

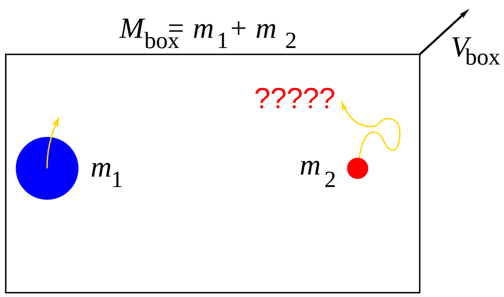
Two-Body Problem

Thus far: cheated! (i.e., simplified)
(1) neither Sun nor planets "nailed down", and
(2) Newton III $\rightarrow$ planets exert net force on Sun
$\Rightarrow$ (3) Sun moves too! (but larger M, so less accel.)

How to analyze 2-body system?

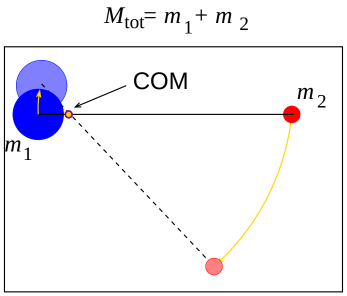
imagine a box, with mass M, with no net forces on it (floating in space).
Q: how would it move?
Q: what if the box has pieces in it–still same answer?

box of mass M_{box}: without forces, moves inertially
i.e., as free body $\longrightarrow$ constant velocity $\vec{V}_{\text{box}}$



now open box: contains two pieces, mass m_1 and m_2
no matter what pieces do, box still has constant $\vec{V}_{\text{box}}$

now imagine moving with same velocity as box:
so to you, box is at rest
if at one time, know where the two particles are (draw)
∞ then later if particle 1 has moved (draw new position) then:
Q: can you say anything about where particle 2 has to be?



define center of mass (COM):

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \tag{2}$$

a mass-weighted average

Can show: $\ddot{\vec{R}} = 0 \rightarrow \dot{\vec{R}} = const.$

$\rightarrow$ can pick inertial frame where $\dot{\vec{R}} = 0$

in 2-body problem: convenient to choose $\vec{R} = 0$
origin of coordinates

Planet & Sun as a Two-Body System

in Sun-planet system: center of mass is "fixed" (free body)
but Newton III says that since Sun pulls on planet
then planet pulls back on Sun
$\longrightarrow$ *both* accelerate
$\longrightarrow$ *both* orbit around center of mass
Q: but what's the difference in the motions?
Q: who's more correct: Copernicus or Ptolemy?

Testing Newton's Gravity

Moons of Jupiter: obey Kepler's laws
$\longrightarrow$ Jupiter's gravity works like Sun's, Earth's

1830's: Uranus observed orbit did *not* follow predictions
of Newtonian solar system model
$\Rightarrow$ the death Newton's gravity?

recall: theory must explain *all* data, not just some!
so despite Newton's great job with planets, moons
even *one clear failure is enough*

Q: so do we have to throw out Newtonian gravity?
Q: why hesitant to throw out?
Q: if not abandon, what's another solution to the problem?

iClicker Poll: Uranus Discrepancy

1830's Problem: *measured* Uranus orbit doesn't match predictions of Newtonian Gravity *theory*

Vote your conscience!

Which seems more likely to you?

A Newton's gravity theory *correct*, but not all gravity sources had been included

B Newton's gravity theory *incorrect* (or at least incomplete)

Q: what experiment/observation would tell which is right?

maybe haven't included all sources of gravity?
maybe unknown/unseen object causes U's deviations?
$\Rightarrow$ a new planet?

if unknown object, could predict where should be
did this, looked. saw:
`www: Neptune`
1846: Neptune found at right position
$\triangleright$ *predicted* by Newton's gravity ("dark matter")

very impressive! victory snatched from jaws of defeat!
triumph of Newtonian dynamics and gravity

many other confirming observations
`www: binary star orbits`

Heliocentric vs Geocentric Finale

What is the main lesson, for the practice of science, of the geocentric vs heliocentric shift?

Note:
not asked *content* of science (don't say lesson=heliocentric) but rather the *practice*—what does it tell us here and now about how to do science?

Geocentric vs Heliocentric: Lessons

For me, a big lesson is **Humility!**
naive to think: "Greeks = dumb, us = smart"
rather a sobering reminder: sometimes, same observations
can be explained in radically different ways

also: can have bias not even aware of
shapes how view world, seems reasonable to everyone
humbling! examples in QM, relativity

what's more...probably going on still today!
remember: all astronomy, all science ultimately tentative
In this course: my guess: $\sim 80\%$ stand test of time
but don't know which 20% is wrong...so have to learn it all!

that said, not everything up for grabs or matter of taste...
confidence/uncertainty varies tremendously
My Wagers

Director's Cut Extras

Gravity and Electrostatics: Family Resemblance

Note: *this discussion is optional*, not tested or part of homework!
but answers some great questions from class

Note the amazing similarities: for pairwise interactions

$$\vec{F}_{\text{grav}} = -\frac{GMm_{\text{test}}}{r^2}\hat{r} \qquad \vec{F}_{\text{electrostat}} = +\frac{Qq_{\text{test}}}{r^2}\hat{r}$$

$$\vec{g} = \frac{\vec{F}_{\text{grav}}}{m_{\text{test}}} = -\frac{GM}{r^2}\hat{r} \qquad \vec{E} = \frac{\vec{F}_{\text{electrostat}}}{q_{\text{test}}} = \frac{Q}{r^2}\hat{r}$$

Newtonian Gravity **Electrostatics**

- both inverse square laws
- both add forces via superposition
- **formally equivalent** $\rightarrow$ same technology for both!

Gravity and Electrostatics: Gauss' Law Siblings

Electrostatics:
for a *distribution of charge*, with charge density ρ_{charge}
inverse square law generalizes to **Gauss' Law**

$$\nabla \cdot \vec{E} = 4\pi\rho_{\text{charge}}$$

Gravity: for *distribution of mass*, w ith mass density ρ_{mass}
Gauss' Law also applies!

$$\nabla \cdot \vec{g} = -4\pi G\rho_{\text{mass}}$$

example: gravity inside spherical, non-point, mass distribution
$\vec{g}(r) = -GM_{\text{enc}}(r)/r^2\ \hat{r}$
with enclosed mass $M_{\text{enc}}(r) = 4\pi \int_0^r \rho(r)\ r^2 dr$