Astro 596/496 NPA
Lecture 4
January 23, 2019

Announcements:

- Preflight 1 posted
  Due next Friday before class
  for PF1, readings are *suggested* – can skim
  Director's Cut Extras below has notes
  on notation in Asplund paper

Last Time: **abundances**
Q: top solar system elements?
nuclear masses
Q: approximate mass of nucleus with (N, Z)?
Q: convenient expression for exact mass?

Mass Defects

mass excess or **mass defect** of nuclide i:

$$\Delta_i = (m_i - A_i m_u)c^2$$

residual between true mass and $Am_{\rm u}$ approximation
where: $\Delta(^{12}{\rm C}) = 0$ via $m_{\rm u}$ definition

compare:

$$m(^{238}{\rm U}) \;=\; 221742.9559 \text{ MeV} \tag{1}$$
$$\Delta(^{238}{\rm U}) \;=\; 47.3077 \text{ MeV} \tag{2}$$

same information content!

`www: Chart of the Nuclides` Δ_i entries

Q: what would it mean if all mass defects = 0?
Q: mass defects ≠ 0 for all but ^{12}C–implications??

Nuclear Binding Energy

if nucleons had same mass m_{u} and *did not interact*
then pile of A_i nucleons has mass $m_i = A_i m_{\mathrm{u}}$ exactly
and we'd measure $\Delta_i = 0$
but no interactions = no biding = nucleons would disperse
no nuclei would exist! we wouldn't exist! yikes!

Instead **nucleons do interact** via nuclear force!
- bound together in nuclei
- must input energy to rip nuclei apart!

Q: how to quantify binding?

Nuclear Binding Energy

binding energy: energy needed to rip nucleus $\rightarrow$ nucleons

for nuclide i, define:

$$B_i = \text{parts} - \text{whole} \quad (3)$$
$$= \left(Z_i m_{\mathrm{H}} c^2 + N_i m_n c^2\right) - m_i c^2 \quad (4)$$
$$= Z_i \Delta_p + N_i \Delta_n - \Delta_i \quad (5)$$

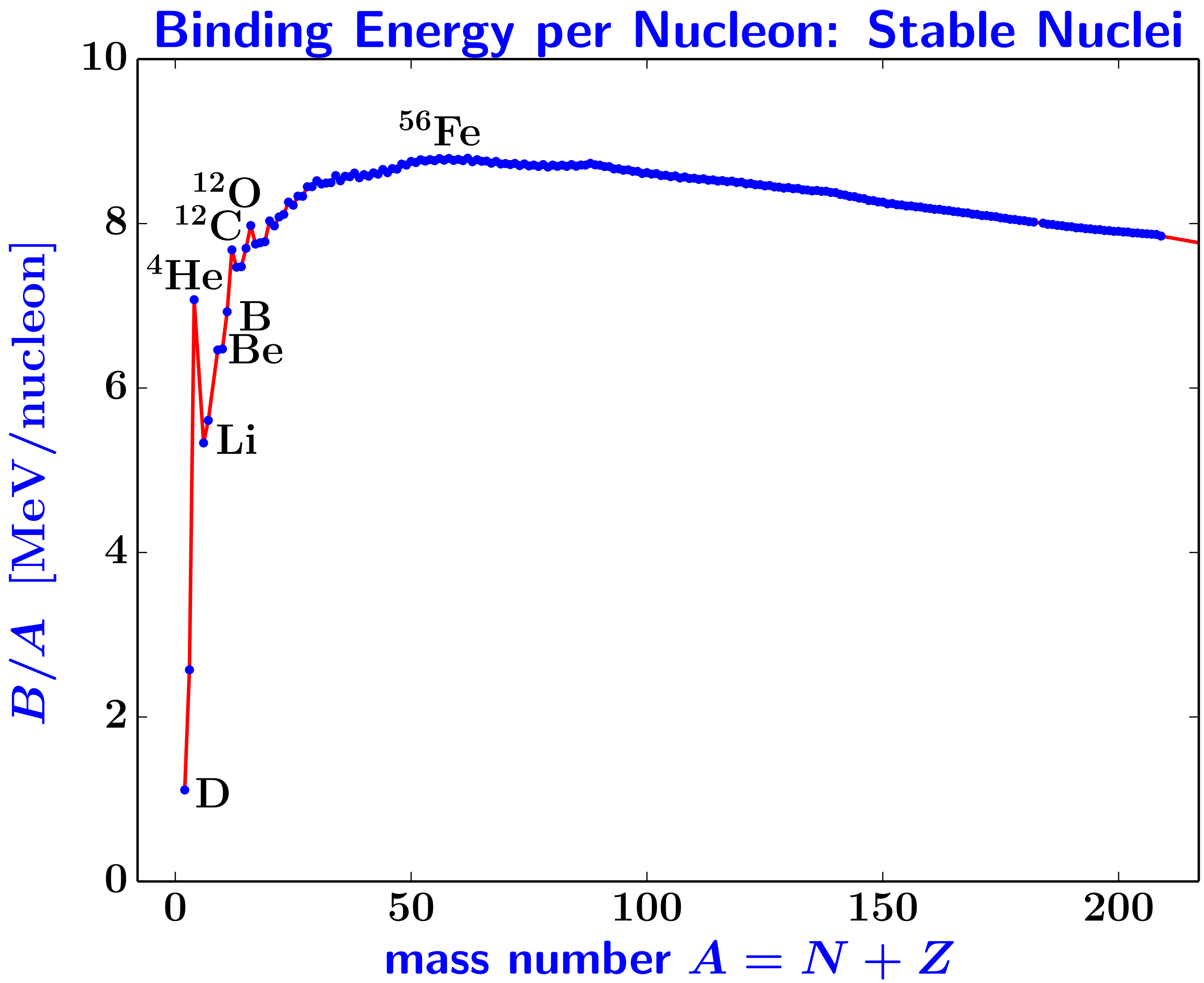
- mass defect Δ_i encodes binding energy B_i
- stability requires $B_i > 0$, i.e., whole $<$ sum of parts (compare hydrogen atom: $m_{\mathrm{H}} = m_p + m_e - 13.6 \text{ eV}/c^2$)

Binding Energy Patterns

note that larger nuclei have large B_i, but shared among more nucleons

consider: **binding energy per nucleon** B/A
Q: what does this represent physically?

`www: Chart of nuclides` B/A patterns? implications?



 Q: what strikes you?

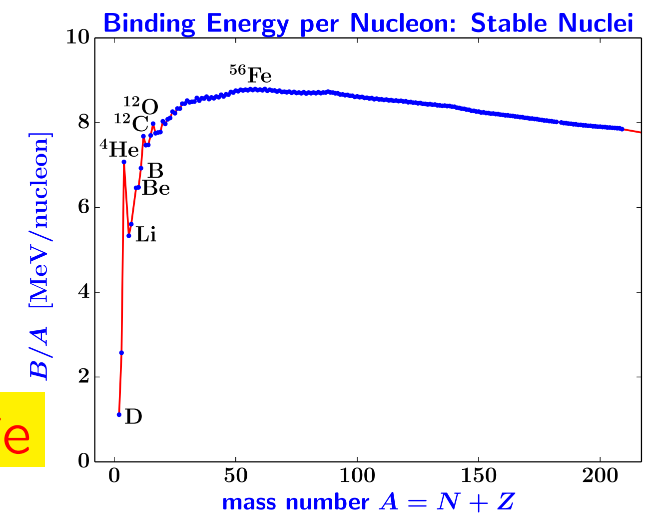
Binding Energy: Trends and Consequences

Overall nuclear binding energy features in Chart of Nuclides:

- highest binding along valley of stability
  $\Rightarrow$ stable isotopes are the most tightly bound

For stable nuclei:

- sharp rise in B_i/A_i at low A
- local max at ^{4}He
- *no stable nuclei at* $A = 5, 8$
- lowest B/A for D, LiBeB
- *max* B/A *for middle masses:* **peak at ^{56}Fe**



with this in mind, revisit solar abundances:

`www: solar system abundances`

Q: significance?

The Nuclear Fingerprint in Solar Abundances

Many observed features in solar abundances
reflect observed features in nuclear binding energy curve

for example:

⋆ D, Li, Be, B are “fragile”: weakly bound–low B/A
but these also have very low abundances

⋆ binding has broad peak around ^{56}Fe
...where abundances show a broad peak

See nuclear binding energy effects in solar system
(and stellar) abundances:

$\Rightarrow$ confirms: abundance pattern controlled by *nuclear physics*!

Nuclei vs Atoms

Useful to compare/contrast nuclear vs atomic interactions and structures

Q: controlling forces/interaction(s) in each?

Q: nuke/atomic similarities?

Q: nuke/atomic differences?

Properties of the Nuclear Force

EM force completely understood
via Maxwell's equations and QED
for static point charges: simple potential $V(r) = Q/r$
- central force
- always either attractive or repulsive

Nuclear force not fully understood, much more complicated
(see, e,g., Krane, *Nuclear Physics*)
- nuclear interaction and nucleons not fundamental
  but manifestations of strong interactions among quarks
  analogy: EM force vs molecular interactions
- nucleon-nucleon ($N - N$) potential
  not known from first principles: empirical

The Nuclear Force

- $N - N$ interaction *attractive* a "large" distances $\gtrsim 1$ fm,
  strongly *repulsive* at short distances $\lesssim 1$ fm
  $\rightarrow 0$ at ~ 1 fm

- $N - N$ interaction strongly *spin-dependent*
  case $A = 2$: dineutron $\boxed{nn}$ and diproton $\boxed{pp}$ do no exist!
  duetron $^2\text{H} = d = np$ exists, but only as
  $J = 1$ $(p \uparrow n \uparrow)$, *not* $J = 0$ $(p \uparrow n \downarrow)$
  $\Rightarrow$ have $\vec{s}_1 \cdot \vec{s}_2$ terms in $N - N$ potential

- $N - N$ potential has *non-central* "tensor" term
  anisotropic, angle average $= 0$

- $N - N$ force *charge symmetric*:
  $n - n$ interaction $= p - p$ aside from Coulomb effects

- $N - N$ nearly *charge independent*: $V_{nn} \simeq V_{pp} \simeq V_{np}$

Nuclear Richness $\equiv$ Complexity

A Look Ahead to Particle Physics

nucleons *not* fundamental particles
but *bound states* of quarks and gluons
$\Rightarrow$ nuke force really an interaction among complex objects
(baryons, mesons) with substructure

Analogy: *Chemistry*
ultimately controlled by E&M,
but via *atoms*: many-body quantum structures
in principle, can calculate atomic/molecular
structure, reactions, scattering *ab initio*
but in practice exceedingly difficult.
Yet can do chemistry anyway:
notice patterns & useful approximations

take similar approach to nuke physics.

Mass Formula

goal: understand bulk nature of nuclei
binding energy curve, nature of valley of stability

want to know binding energy of nuclides:

$$B(A, N, Z) = ? \tag{6}$$

approach: make a rough model of nucleus, use to find functional form; then use mass data to fill in parameters:
"semi-empirical mass formula"
a.k.a., "semi-unbelievable mass formula"
(pioneered by on Wiezäcker $\rightarrow$ "wise-acre mass formula")

Binding Energy Bulk Effects: "Liquid Drop Model"

identify binding energy effects:

$$BE(A, N, Z) = E_{\text{vol}} + E_{\text{surf}} + E_{\text{Coul}} + E_{\text{sym}} + E_{\text{pair}} + E_{\text{shell}}$$

- **volume energy**: $E_{\text{vol}} \propto V \propto A$:
write $E_{\text{vol}} = b_{\text{vol}} A; \quad b_{\text{vol}} \simeq 15.5$ MeV
not $E_{\text{vol}} \propto \#$ pairs $= A(A-1) \sim A^2$
$\Rightarrow$ "saturation" due to short-range nuke force

- **surface effect**: fewer neighbors to bind with
penalty $\propto$ area: $E_{\text{surf}} \propto -r^2$
$\rightarrow E_{\text{surf}} = -b_{\text{surf}}\ A^{2/3}; \quad b_{\text{surf}} \simeq 16.8$ MeV

- **Coulomb repulsion**: reduces binding
$E_{\text{Coul}} \sim -Z(Z-1)e^2/r$
$\rightarrow E_{\text{Coul}} = -b_{\text{C}}\ Z(Z-1)A^{-1/3}; \quad b_{\text{C}} \simeq 0.72$ MeV

Quantum Effects on Nuclear Binding

liquid drop model ignores quantum effects:
nucleons are Fermions: **Pauli principle applies**
separately fill neutron and proton states

• **Symmetry:** Pauli favors filling n and p levels equally
optimal for $N = Z$, so asymmetry penalty:
$E_{sym} \propto -|\text{excess}| \sim (Z - N)^2/A$
$\Rightarrow E_{sym} = -b_{sym}\ (Z - N)^2/A; \quad b_{sym} \simeq 23$ MeV

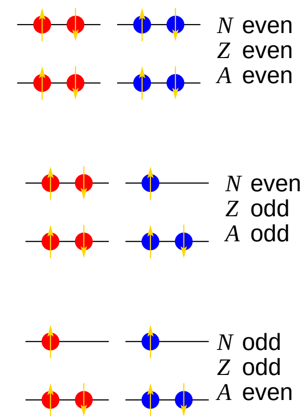
•**Pairing:** Pauli $\rightarrow$ identical nucleons pair off
with opposite spins

Q: what are the possibilities?
Q: what configurations most favored? least?

Pairing: separately fill levels for n and p

for each nucleon type:

- paired state most tightly bound
- possible if even number of that nucleon type
  but odd number per type $\rightarrow$ unpaired nucleon



$$E_{\text{pair}} = \begin{cases} +\delta & N \text{ even - } Z \text{ even} \quad (A \text{ even}) \\ 0 & \text{odd-even} \quad (A \text{ odd}) \\ -\delta & N \text{ odd - } Z \text{ odd} \quad (A \text{ even}) \end{cases} \tag{7}$$

where $\delta = b_p A^{-3/4}$; $\quad a_p \sim 34$ MeV

consequences:

(1) *stable odd-odd nuclei rare and weakly bound*

(2) *odd A less bound than even A*

Q: implications for solar system abundances?

`www: solar system abundances--isotopic, elemental`

Nuclear Physics Encoded in Solar Abundances II:

The Odd-Even Effect

Recall: plotting abundance vs A
"zig-zag" is odd-even A effect
$\Rightarrow$ more confirmation that
nuclear physics controls solar abundances

Note: odd-even effect seen in both *elemental*
and *isotopic* abundance pattersn
But chemistry can only effect elemental patterns
$\longrightarrow$ underlying cause is nuclear and not chemical!

Nuclear Shell Model

in atoms:

quantum states $\longrightarrow$ electronic shells $\longrightarrow$ periodic behavior

for certain "magic Z": closed shell $\longrightarrow$ tightly bound electrons

$\Rightarrow$ unusually stable atoms (e.g., noble gases)

in nuclei:

also quantum states

expect shell behavior, but not necessarily same numerology

for each nucleon:

(1) approximate force by all other nucleons as a central potential

(2) Schrödinger's eq. $\longrightarrow$ energy levels & occupation numbers

(3) filled levels $\longrightarrow$ closed shell

$\longrightarrow$ very tight binding

occur for special values of N and Z

"magic numbers"

`www: 3D harmonic oscillator levels and magic numbers`

magic numbers:

$Z_{\text{magic}} = 2, 8, 20, 40, 82$

$N_{\text{magic}} = 2, 8, 20, 50, 82, 126$

`www: solar abundances vs` A `and vs` N

especially stable if doubly magic

i.e., both N and Z are magic: ^{4}He, ^{16}O, ^{40}Ca, ^{90}Zr, ^{208}Pb

Note: because $^4\text{He} \equiv \alpha$ doubly magic $\rightarrow$ very tightly bound

(1) light nuclei which have $N = Z =$ even

are tightly bound **"α" nuclei**: ^{12}C, ^{16}O, ^{20}Ne, ..., ^{40}Ca

`www: solar system abundances`

(2) $A = 5$ and $A = 8$ **unstable**: decay to $\alpha+$ nucleon and $\alpha + \alpha$

$\rightarrow$ **"mass gaps" at** $A = 5, 8$ essential for history of universe

Director's Cut Extras

Nuclear Density and Degeneracy

consider nuclear density:
empirically–nearly constant for all nuclei
number density $n = A/\text{Vol} = 0.17\ \text{fm}^{-3}$,
or $\rho = M/\text{Vol} \simeq 0.2\ \text{GeV fm}^{-3} \simeq 3 \times 10^{14}\ \text{g cm}^{-3}$

nucleons are fermions:
must obey Pauli principle $\rightarrow$ important if *degeneracy* occurs
but *does* it occur?

Q: how to estimate if nuclei are degenerate?

to test for degeneracy $\Rightarrow$ estimate Fermi energy
i.e., energy that nucleon "gas" would have
if packed together as much as Pauli allows

if degenerate: $xp_x \sim \hbar$ and also y, z:
if packed into length x, minimal momentum is
"Fermi momentum" $p_F \sim \hbar/x$

so for nuclus with size $r \sim 1A^{1/3}$ fm (1 fm $= 10^{-13}$ cm)
Fermi momentum $p_F \sim h/r \sim 2\pi\hbar/r$
Fermi energy $E_F = p_F^2/2m_u \sim 20 - 40\ A^{-2/3}$ MeV

Q: what should this be compared with?
Q: what do we conclude?

compare to actual nuclear energy level spacings

`www: energy level diagram for` ^{12}C

we find

$$E_F > E_{\text{nuke level}} \sim 1 \text{ MeV} \quad (8)$$

$$E_F > E_{\text{EM}} \sim 1.4 Z^2 \text{MeV}/r_{\text{fm}} \quad (9)$$

i.e., typical nucleon energies are *below* Fermi level

$\Rightarrow$ to zeroth order, the nucleus is a degenerate gas of nucleons confined by the strong force

Note: since $n \sim A/r^3 =$ constant, nuclear radius scales as

$r \simeq 1.2 A^{1/3}$ fm

Tips for PF1 Optional Reading (Asplund et al 2005)

Abundance Notation

very commonly used

$$[A/B] \equiv \log_{10}\left[\frac{(A/B)_{\text{observed}}}{(A/B)_{\text{solar system}}}\right] \tag{10}$$

e.g.,: if a star has $\text{Fe/H}_\star = 0.01\text{Fe/H}_\odot$, then $[\text{Fe/H}]_\star = -2$
note: $[A/B]$ is a *logarithmic* measure of abundance
i.e., $[A/B]$ is a "**d**ecimal **ex**ponent
$\rightarrow$ really dimensionless, but "units" sometimes called "`dex`"

also used

$$[A] = 12 + \log_{10}(A/\text{H}) = \log_{10}(10^{12} A/\text{H}) \tag{11}$$

e.g., since $(\text{Fe/H})_\odot = 3 \times 10^{-5}$, then $[\text{Fe}] = 7.5$
also dimensionless logarithmic measure,
"units" sometimes also called "`dex`"