Astro 596/496 NPA

Lecture 5

January 25, 2019

Announcements:

- Preflight 1 due
- Problem Set 1 posted on Compass
  uses solar system abundance data posted on course links

Last Time: nuclear binding energy

Q: highest B/A? lowest?

Q: implications for solar system abundances?

nuclear masses and structure: liquid drop + quantum effects

consider atoms: *Q: implications of shell structure?*

Q: so for nuclei?

Nuclear Shell Model

in atoms:

quantum states → electronic shells → periodic behavior

for certain "magic Z": closed shell → tightly bound electrons

⇒ unusually stable atoms (e.g., noble gases)

in nuclei:

also quantum states

expect shell behavior, but not necessarily same numerology

for each nucleon:

(1) approximate force by all other nucleons as a central potential

(2) Schrödinger's eq. → energy levels & occupation numbers

(3) filled levels → closed shell

→ very tight binding

occur for special values of N and Z

"magic numbers"

www: 3D harmonic oscillator levels and magic numbers

magic numbers:

$Z_{\text{magic}} = 2, 8, 20, 40, 82$

$N_{\text{magic}} = 2, 8, 20, 50, 82, 126$

www: solar abundances vs A and vs N

especially stable if doubly magic
i.e., both N and Z are magic: ^{4}He, ^{16}O, ^{40}Ca, ^{90}Zr, ^{208}Pb

Note: because $^4\text{He} \equiv \alpha$ doubly magic $\rightarrow$ very tightly bound
(1) light nuclei which have $N = Z =$ even
are tightly bound **"α" nuclei**: ^{12}C, ^{16}O, ^{20}Ne, ..., ^{40}Ca

www: solar system abundances

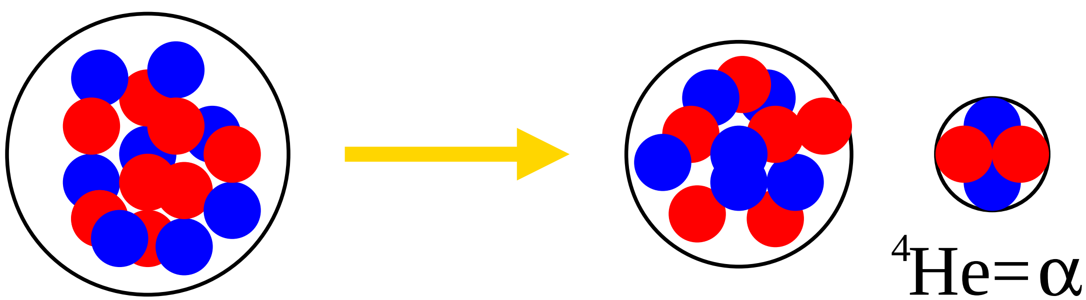
(2) $A = 5$ and $A = 8$ **unstable**: decay to $\alpha+$ nucleon and $\alpha + \alpha$
$\rightarrow$ **"mass gaps" at** $A = 5, 8$ essential for history of universe

Nuclear Decays

Some nuclei unstable
"parent" nuclei spontaneously decay to *"daughter"* nuclei

Alpha Decay

$\boxed{\alpha}$ alpha decay: release $^4\text{He} = \alpha$

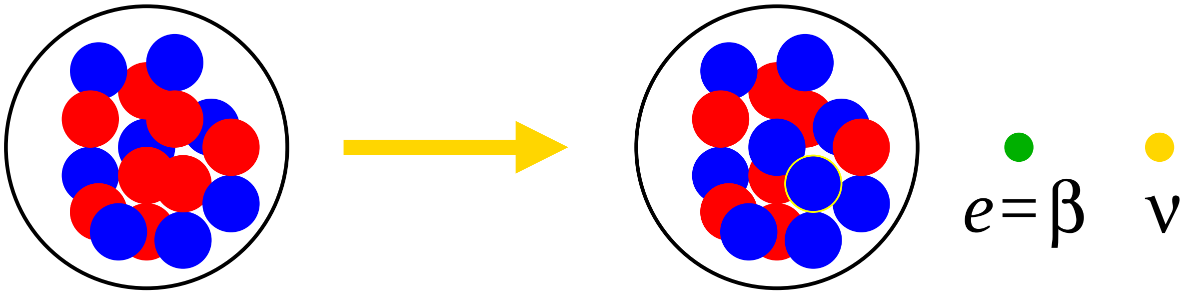


example: $^{238}\text{U} \rightarrow\ ^{234}\text{Th} + \alpha$

decay via **nuclear interaction**, *nucleons reshuffled*

Beta Decay

β beta decay: release electron $e = \beta$



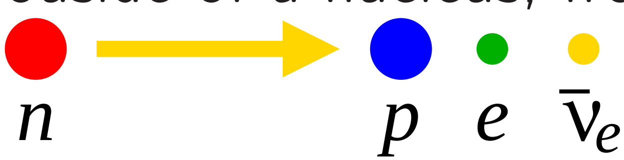
transformation $(N, Z) \rightarrow (N \mp 1, Z \pm 1) + e + \nu_e$

decay via **weak interaction:**

nucleon type changed, ν present

β^- example: **free neutron decay**

ouside of a nucleus, free neutrons are unstable!



$n \rightarrow p + e^- + \bar{\nu}_e$ with $\bar{\nu}_e$ = anti-neutrino

Beta Decay Varieties

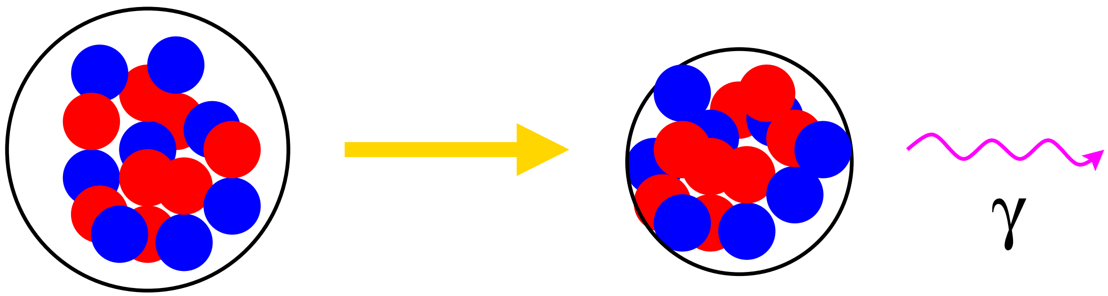
β^- example: tritium decay ${}^3_2\mathrm{H}^1 \rightarrow {}^3_1\mathrm{He}^2 + e^- + \bar{\nu}_e$
i.e., neutron decay within nucleus

β^+ example: ${}^{16}\mathrm{F} \rightarrow {}^{16}\mathrm{O} + e^+ + \nu_e$
i.e., have $p \rightarrow n + e^+ + \nu_e$
creates positron e^+ and ordinary (non-anti) neutrino

β-decay variant: *electron capture*
transformation $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$
requires electron, e.g., from innermost atomic orbits ("K-capture")
example: ${}^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e$ occurs in early Universe

Gamma Decay

γ gamma decay: de-excite, emit photon γ



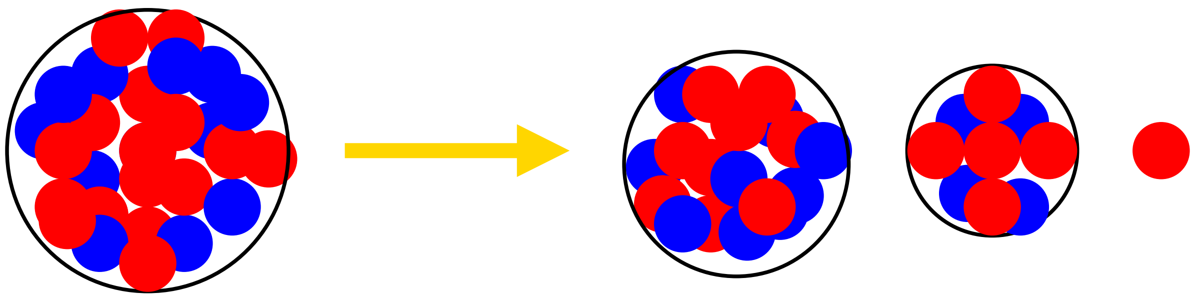
decay via **EM interaction**, (*same nucleus*)

example: excited ${}^{26}\text{Mg}^{*} \rightarrow {}^{26}\text{Mg}^{\text{g.s.}} + \gamma$

$E_\gamma = E_{\text{excited}} = 1.8$ MeV

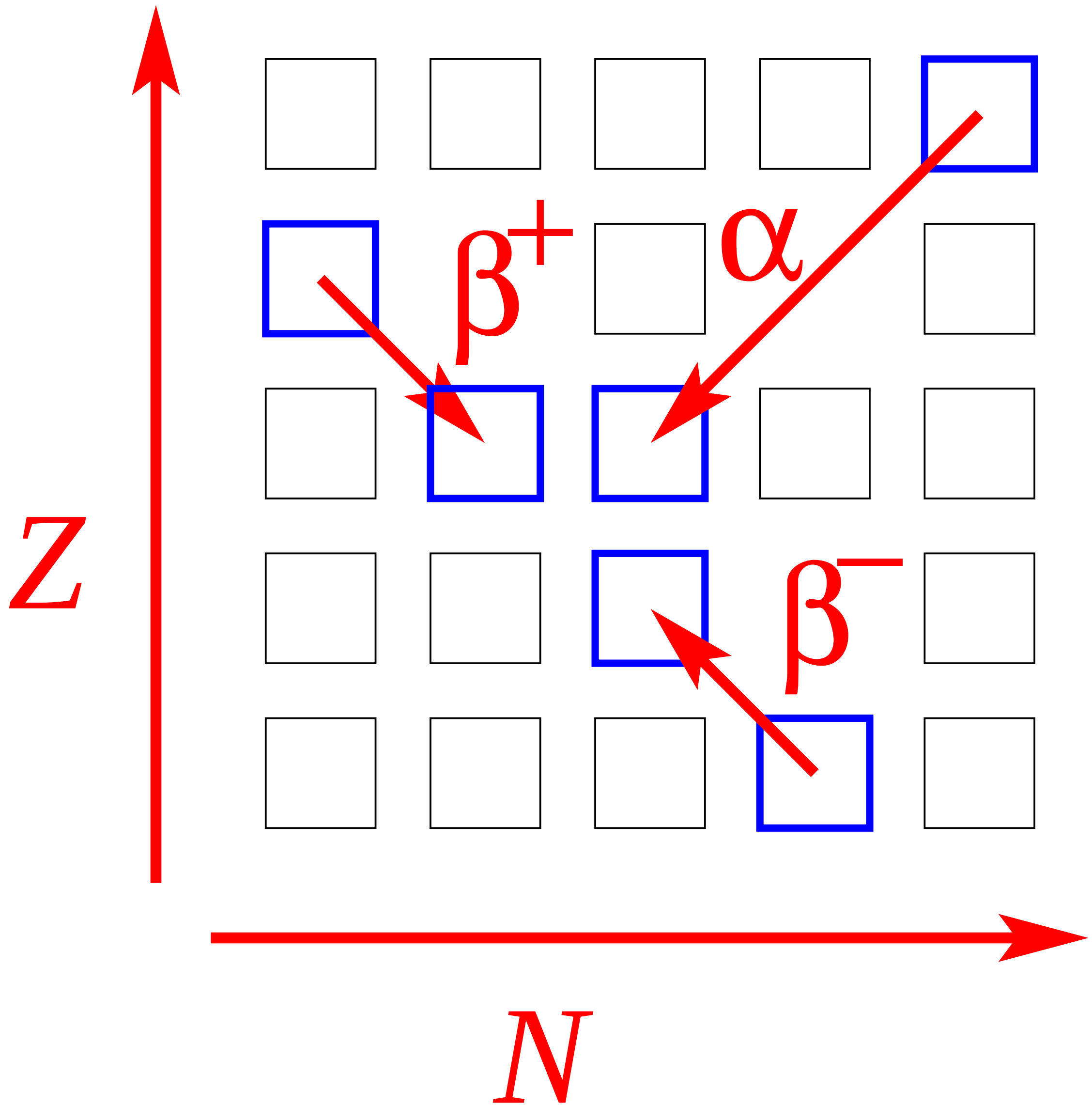
Nuclear Fission

fission: nucleus splits



only occurs for heaviest nuceli–actinides U, Pu
daughters have distribution of masses and number
`www: Chart of Nuclides`

Q: which decays change A?
Q: which decays change Z?
Q: chart of nuclides paths for α? β^-? β^+? fission?


β+
α
Z
β−
N

Radioactive Decay Rate

const decay probability P per unit time:

$$\frac{dP}{dt} = \lambda = const \tag{1}$$

and so

$$\frac{dn}{dt} = -n\,\frac{dP}{dt} = -\lambda n \tag{2}$$

decay lifetime ("*mean life*") $\tau = 1/\lambda$

solution: $\boxed{n = n_0 e^{-t/\tau}}$

also write $n = n_0 2^{-t/t_{1/2}} = n_0 (e^{\ln 2})^{-t/t_{1/2}} = n_0 e^{-t \ln 2/t_{1/2}}$

"*half-life*" $t_{1/2} = \tau \ln 2$

`www: supernova 1987A brightness vs time = ‘‘lightcurve’’`