Astro 596/496 NPA

Lecture 9

February 4, 2019

Announcements:

- **Preflight 2 due Friday**
  includes crowdsource group discussion

- **Some rest for the weary**
  no class meeting this Wednesday Feb 6
  use time for Preflight, especially cosmology

PF2 Discussion: Solar Neighborhood Abundances

PF2: group discussion of abundances in solar neighborhood stars
data from Bensby, Feltzing, & Oey (2014), aka *BFO*

keep in mind:

- observed abundances reflect stellar photosphere ("surface")
- these stars (like the Sun) don't mix core and outer envelope
  Q: so what do observed abundances measure?

- as our Galaxy evolves, cycling *gas* $\leftrightarrow$ *stars*

- at stellar death, results of nuclear processing ejected and mix with interstellar gas: composition changes with time
  Q: what should go up? what should go down?
  Q: what if all stars make same elements? different?

- BFO plot elements vs "metallicity" Fe/H
  Q: what does this measure physically?

- observer notation: for elements A and B in a star

$$[A/B] = \log_{10} \frac{(A/B)_{\rm obs}}{(A/B)_{\odot}} \qquad (1)$$

Q: meaning of [Fe/H] $= 0$*?* -1*?* -2*? of* [O/Fe] $= 0$*?* $+0.5$*?*

Last Time: Reactions–Cross Sections and Rate

Q: physical significance of a cross section σ? units?
Q: when is σ the geometric cross section? when not?

for reaction $a + b \rightarrow c + d$
Q: reaction rate $\Gamma_{\mathrm{per}\,b}$ per b? per a? units? Q: reaction rate per volume?

Q: what a and b have thermal velocity distributions?

Q: how are n vs charge particle cross sections different?

Last Time: Reactions–Cross Sections and Rate

for $ab \to cd$:
cross section gives effective area "seen" by reactants
defined by reaction rate per volume (at fixed relative v)

$$\frac{d\mathcal{N}_{\rm rxns}}{dV\ dt} = r_{ab\to cd} = \frac{1}{1+\delta_{ab}} n_a\ n_b\ \sigma_{ab\to cd}(v)\ v = \Gamma_{{\rm per}\,a} n_a = \Gamma_{{\rm per}\,b} n_b$$

if a and b are nonrelativistic at T: *thermonuclear rate*

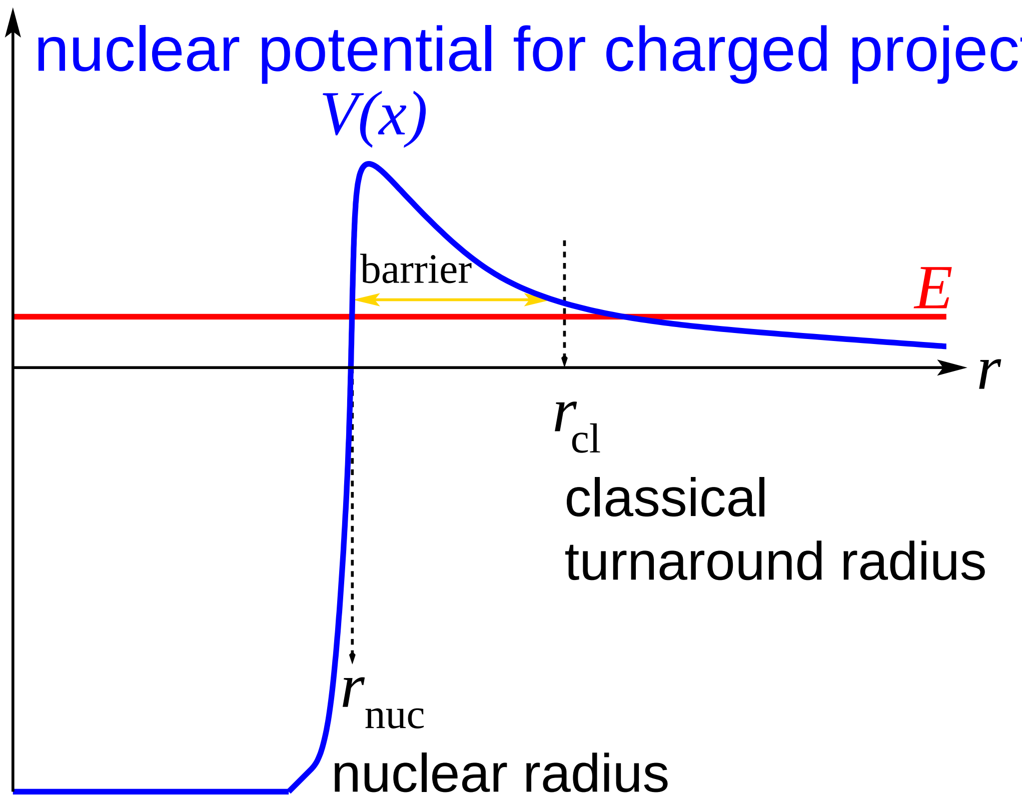
$$\langle r_{ab\to cd}\rangle = \frac{1}{1+\delta_{ab}} n_a n_b \langle \sigma_{ab\to} v\rangle \quad (2)$$

$$= \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty dE\ E\ \sigma(E) e^{-E/kT} \quad (3)$$

$\langle \sigma_{ab\to} v\rangle$ *suppressed for $E > kT$!*

- *neutrons* – no problem! $\sigma \sim 1/v$ at low energy
- *charged nuclei* – huge problem! Coulomb repulsion at energy! reactions can't even proceed without quantum tunneling!

Classical vs Quantum Probes of Nuclear Potential



Charged Particle Reaction Rates: S-Factor

probability for tunneling under Coulomb

$$P \propto e^{-2\pi Z_1 Z_2 e^2/\hbar v} = e^{-2\pi\eta} = e^{-bE^{-1/2}} \tag{4}$$

Also: *geometrical factor: cross section* $\sigma \propto \lambda_{\text{deB}}^2$,
$\lambda_{\text{deB}} = \hbar/p$ de Broglie wavelength
$\Rightarrow \sigma \propto 1/p^2 \propto 1/E$

expect σ functional form

$$\boxed{\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta} = \frac{S(E)}{E} e^{-bE^{-1/2}}}$$

$S(E)$: "astrophysical $S-$factor"

- $S(E)$ encodes nuclear contribution to reaction
- $S(E)$ often slowly varying with E
  Q: if so, σ behavior at large E? small E?
- σ and S-factor for ^{3}He$(\alpha,\gamma)^7$Be `www: data plotted`

Thermonuclear Rates

So: thermonuclear rates reduce to:

$$\begin{aligned} \langle \sigma v \rangle &= \langle \sigma v \rangle_T \\ &= \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty dE \; S(E) e^{-E/kT - bE^{-1/2}} \end{aligned}$$

Procedure:

(1) astro theory/obs identifies needed reaction

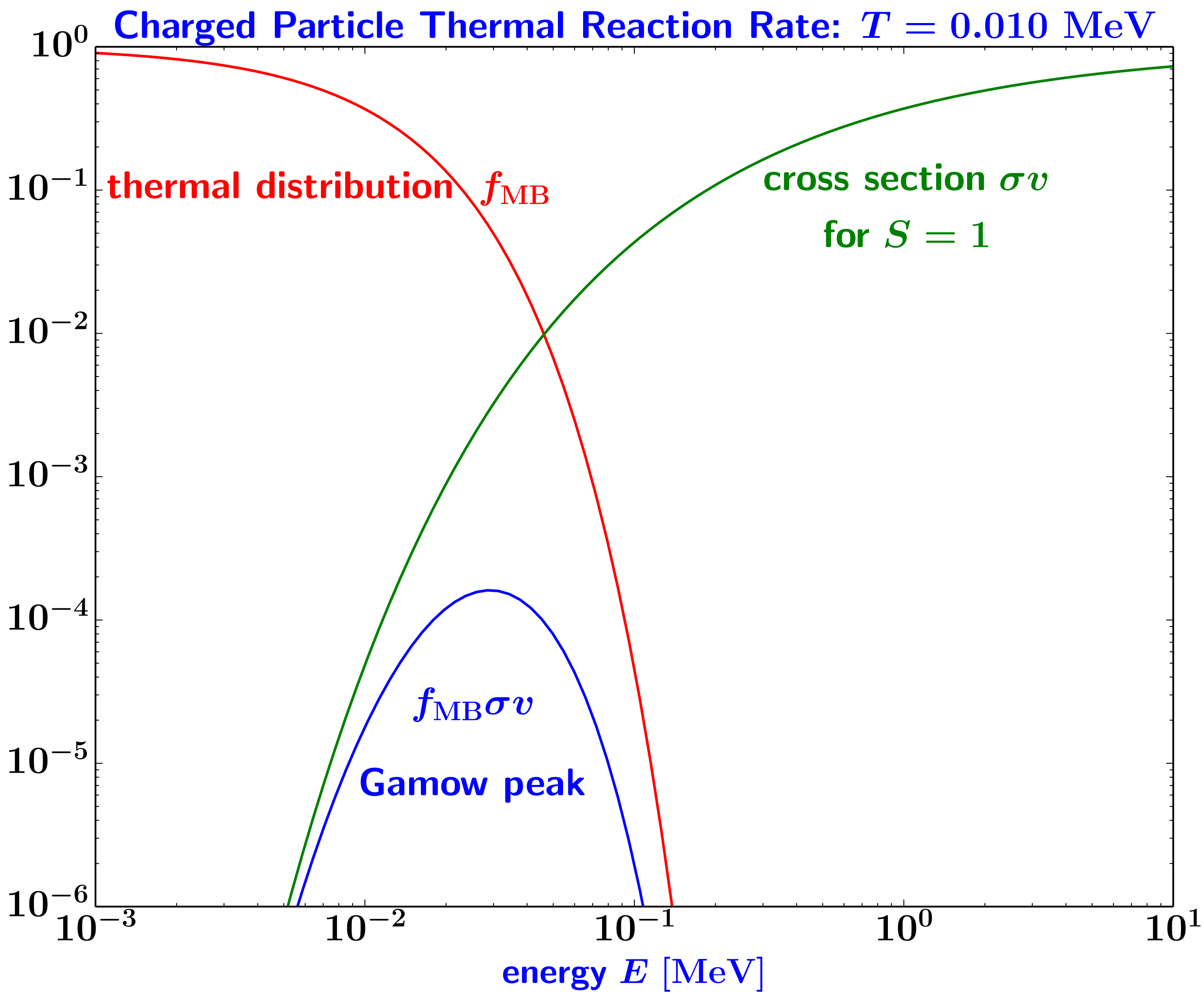
(2) nuclear expt: measure $\sigma(E) \to S(E)$

(3) find $\langle \sigma v \rangle$ vs T (usually numerically)

(4) fit result to function

∞ *Q: note exponential–behavior vs E? implications?*

Thermonuclear Rate Integrand



The Gamow Peak

integrand $S(E)e^{-G(E)}$ peaks at/near *minimum* of exponential
$G(E) = E/kT + bE^{-1/2}$
min at $G' = 0$: "most effective energy" or "*Gamow Peak*"
$E_0 = (bkT/2)^{2/3}$, where

$$G_{\rm min} \equiv \tau = G(E_0) = 3(b^2/4kT)^{1/3} \quad (5)$$

$$= 4.25(Z_1^2 Z_2^2 A)^{2/3} \left(\frac{10^9 \text{ K}}{T}\right)^{1/3} \quad (6)$$

Q: behavior with T? interpretation?

expand exponential around peak energy E_0:

$$G(E) \approx \tau + \frac{1}{2} G''(E_0)\,(E - E_0)^2 \quad (7)$$

use expansion of exponential
$G(E) \approx \tau + \frac{1}{2} G''(E_0)\,(E - E_0)^2$
in thermonuclear integral (method of steepest descent)

Then we have

$$\begin{aligned} \langle \sigma v \rangle &\simeq \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} S(E_0) e^{-\tau} \int_{-\infty}^{\infty} dE\ e^{-(E-E_0)^2/2\Delta^2} \\ &= \frac{8}{9\sqrt{3}} \frac{\hbar}{\pi Z_1 Z_2 e^2 m} \tau^2 e^{-\tau} S(E_0) \\ &\propto T^{-2/3} e^{-a/T^{1/3}} \end{aligned}$$

Q: behavior at high T? low T? are these reasonable?

Mean Lifetimes

for reaction $i + j \rightarrow k + l$
define mean lifetime $\tau_i(ij)$ of i against reaction with j as

$$(\dot{n}_i)_{ij} = -\frac{n_i}{\tau_i(ij)} \tag{8}$$

or $\tau_i(ij) = \|n_i/\dot{n}_i\|$

But $\dot{n}_i = -r_{ij} = -n_i n_j \langle \sigma v \rangle_{ij}$
$\Rightarrow$ $\boxed{\tau_i(ij) = 1/n_j \langle \sigma v \rangle_{ij} = 1/\Gamma_{\text{per}\, i}(ij)}$

useful to write

$$\Gamma_{\text{per}\, i}(ij) = n_j \langle \sigma v \rangle_{ij} \simeq \frac{X_j}{A_j}\frac{\rho}{m_u}\langle \sigma v \rangle_{ij} = \frac{X_j}{A_j}\rho[ij] \tag{9}$$

where $[ij] = \langle \sigma v \rangle_{ij}/m_u = N_{\text{Avo}}\langle \sigma v \rangle_{ij}$ given in tabulations

Why would this be a useful form?

Partial Lifetimes: Examples

Reactions in the Sun

In the solar core today:

$T \simeq 16$ MK $= 1.6 \times 10^7$ K

density $\rho \simeq 150$ g cm^{-3}

$X_{\rm H} \simeq 0.33$

What is lifetime of a deuteron against $d(p,\gamma)^3$He?

$$\frac{1}{\tau_d(pd)} = X_{\rm H}\rho[dp{\rightarrow}\gamma\,^3{\rm He}]$$
$$\simeq (0.3)(150\,{\rm g\,cm^{-3}})(2 \times 10^{-10}\,{\rm cm^3\,s^{-1}\,g^{-1}})$$
$$\sim 10^{-8}\ {\rm s^{-1}}$$

or $\tau_p(pd) \sim 3$ yrs: "immediately" compare $^{16}O(p,\gamma)^{17}F$

Exponential factor $\tau(E_0) \sim 4\times$ larger!

$\tau_O(p^{16}O) \sim 10^{57}$ s $\gg$ age of Univ!

no ^{16}O burned in solar core (on main seq)

COSMOLOGY

Physical Cosmology

Modest goals:
scientific understanding of the

- origin
- evolution
- contents
- structure
- future

of the Universe

we will see:

⋆ known particle & nuke physics plays decisive role
⋆ open questions in cosmology probably (?) linked to
open questions in particle physics

Cosmography Units: Astronomical Distances

Charity begins at home: *Astronomical Unit* (AU)

- average Earth-Sun distance, known very precisely
- $r(\text{Earth} - \odot) \equiv 1\ \text{AU} = 1.49597870660 \times 10^{13}$ cm

parsec

- derives from trigonometric parallax measures of stars
- star with parallactic angle p lies at distance

$$r(p) = \frac{1\ \text{AU}}{\tan p} \approx \frac{1\ \text{AU}}{p} \tag{10}$$

for $p = 1$ arcsec $= 4.8 \times 10^{-6}$ rad, distance is

$$r(1\ \text{arcsec}) \equiv 1\ \text{parsec} \equiv 1\ \text{pc} = 3.0857 \times 10^{18}\ \text{cm} \approx 3\ \text{lyr} \tag{11}$$

Q: pc, kpc, Mpc, Gpc *characteristic scales for what?*

Typical Lengthscales: Cosmic Hierarchy

⋆ typical star-star separation in galaxies ~ 1 pc

⋆ typical (visible) galaxy size $\sim 1\text{kpc} = 10^3$ pc

⋆ (present-day) typical galaxy-galaxy separation
~ 1 Mpc $= 10^6$ pc

⋆ (present-day) observable universe ~ 1 Gpc $= 10^9$ pc

Q: Why is this a "hierarchy"?

Observational Cosmology: Zeroth-Order Picture

Cosmic Matter Distribution

observable cosmo "building blocks" – galaxies
$\approx$ all stars in galaxies

`www: Galaxy Survey: 2dFGRS`
Q: what do you notice?
Q: e.g., distribution on small, large scales?
Q: distribution in different directions?

The Universe to Zeroth Order: Cosmological Principle

Observations teach us that

- at any given cosmic time ("epoch")
- to "zeroth order":

the Universe is both

1. **homogeneous** average properties same at all points

2 **isotropic** looks same in all directions

"Cosmological Principle"

the universe is homogeneous & isotropic

first guessed(!) by A. Einstein (1917)

- no special points! no center, no edge!
- "principle of mediocrity"? "ultimate democracy?"

Q: do you need both?

Q: e.g., how can you be isotropic but not homogeneous?

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Example: Cosmo principle and galaxy properties

Q: if cosmo principle true, how should it be reflected in observations of galaxies at any given time?

Q: what does cosmo principle say about how galaxy properties evolve with time?

Cosmo principle and galaxy properties:
at any given time:

- **average** density of galaxies same everywhere

- *distribution* of galaxy *properties* same everywhere
  range of types
  range of colors
  range of luminosity L, mass M, ...
  ratios of normal/dark matter

These are very restrictive constraints!

- time evolution:
  must maintain large-scale homogeneity and isotropy
  but otherwise, **by itself** cosmo principle allows any changes!

Cosmo Principle hugely powerful & the "cosmologist's friend"
very strongly constrains possible cosmologies
$\rightarrow$ large-scale spatial behavior maximally simple