Astro 596/496 NPA
Lecture 11
February 11, 2019

Announcements:

- **Problem Set 2** due Friday

Q1(a): consider a classical particle with reduced mass μ
and total energy E
in a Coulomb potential $V_{\mathrm{C}}(r) = Z_1 Z_2 e^2/r$
"turnaround radius" is closest approach

Last time: began cosmology
Cosmological Principle *Q: what's that? who so powerful?*
cosmic kinematics

- redshift z *Q: definition? units? sign?*
- *Q: galaxy redshift vs blueshift percentages?*

Cosmic Kinematics

1920's: Hubble, Slipher: all galaxies' spectral lines shifted:

- galaxies move relative to us!
- essentially all galaxies show shift to *red*:

$\lambda_{\text{obs}} > \lambda_{\text{lab}} = \lambda_{\text{rest}}$

Define: **redshift**

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} \quad (1)$$

if interpret as Doppler (for non-relativistic $v \ll c$, $z \ll 1$)

$$v \approx cz$$

Edwin Hubble (1929)

`www: Hubble PNAS paper`

`www: original, old-school Hubble diagram`

speed-distance correlation: linear

$$v \propto r \tag{2}$$

Hubble: $\boxed{v = Kr}$

but isotropy implies *Q: what?*

Hubble's Law

Hubble: $v = Kr$

isotropy $\Rightarrow$ same K in all directions

modern: Hubble's Law

$$\boxed{\vec{v} = H\vec{r}} \tag{3}$$

Q: sketch?

at present: time $\boxed{t_0}$ ("sub-0 = today")

measure

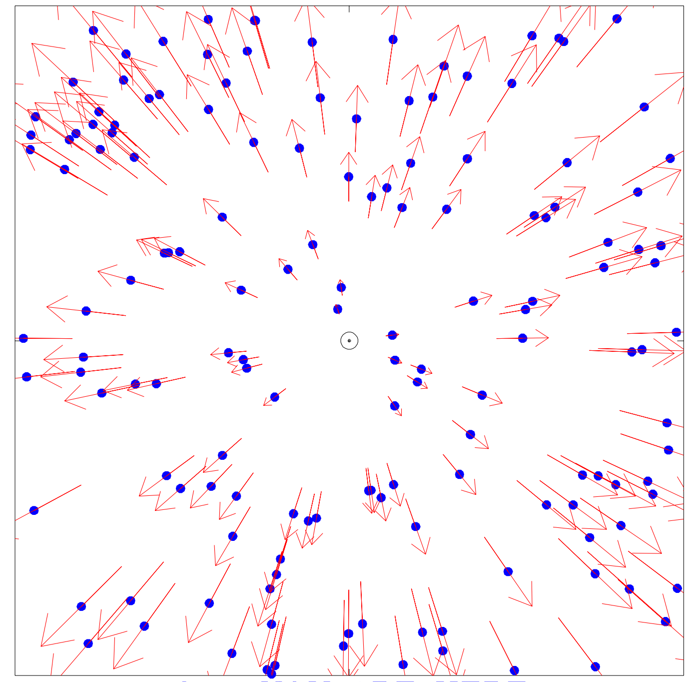
$$H_0 \equiv H(t_0) = 70 \pm 3 \ \mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{4}$$

Hubble parameter or Hubble "constant" *Q: why scare quotes?*

Q: what are dimensions of H?

Structure + Dynamics: Evolution

GALAXY MOTION: ARTIST'S CONCEPTION



⊙ = YOU ARE HERE

observe:

- U. homogeneous, isotropic
- Hubble law $\vec{v} = H\vec{r}$

  Q: restate in simple language?

Q: how reconcile?

at least 2 logical possibilities...

1. "Egoist" interpretation: we are at the center of U.

imagine: **explosion** at $t = 0$, centered on us at $r = 0$

- galaxies fly away with uniform distribution of speeds v_{gal}
- after explosion: coast as free particle

then $r_{\mathrm{gal}} = v_{\mathrm{gal}} t_{\mathrm{today}}$ fastest $\rightarrow$ farthest!

$\triangleright$ so $v_{\mathrm{gal}} = r_{\mathrm{gal}}/t_{\mathrm{today}} \equiv H_0 r_{\mathrm{gal}} \propto r_{\mathrm{gal}}$: recover Hubble's law!

can say more!

Q: age of universe in this model?

Q: maximum distance of debris? implications?

Characteristic Cosmic Scales

in "explosion model" can calculate age of Universe as

$t_{\rm today} = t_0 = t_{\rm H} = 1/H_0 = 14 \times 10^9$ yr $= 14$ Gyr ago

"Hubble time" useful timescale even to non-egoists!

sets scale of $\sim$ "expansion age" of U

limiting speed: c

fastest explosion debris goes farthest $v_{\rm max} = c = H r_{\rm max}$

at characteristic distance: "**Hubble length**"

$$r_{\rm max} = d_H = \frac{c}{H_0} = c t_H = 4200 \text{ Mpc} \quad (5)$$

useful lengthscale even to non-egoists!

→ sets $\sim$ size of *observable* U

Egoist/Explosion Model (Milne 1933) is logically possible! But...

Q: give a philosophical reason why we don't believe this?

Q: give a physical reason why this treatment can't be right?

Q: give an observational reason why we don't believe this?

Critiques of Cosmic Egoism

We are at the center of the universe?

Philosophically:

• *not Copernican* ("principle of mediocrity")

Physically:

• *haven't included gravity!*

Observationally:

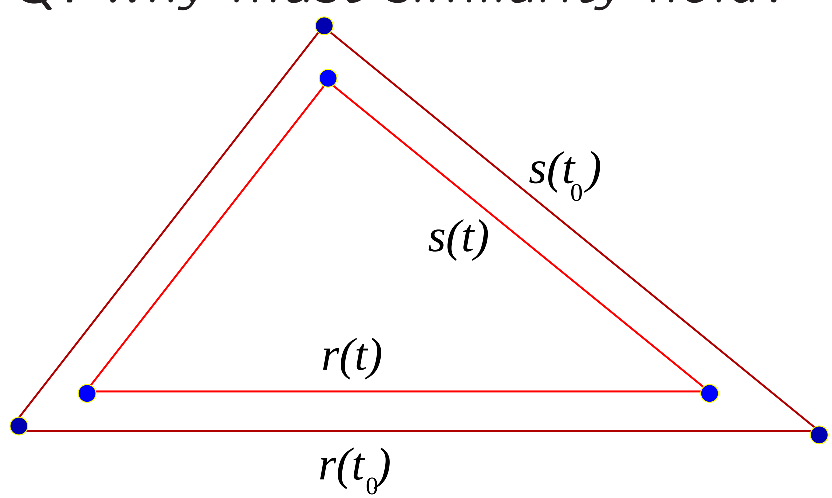
• *Milky Way and nearby galaxies don't look special*
not what expect from center of explosion
compare supernova $\rightarrow$ neutron star, black hole

...yet v pattern makes us look special...

Cosmo Principle Constrains Kinematics

consider arbitrary triangle defined by 3 observers at t_0
Hubble law $\longrightarrow$ observers in relative motion
$\longrightarrow$ at later time t, larger triangle

the claim:
later Δ always similar to original Δ
Q: what are similar triangles?
Q: why must similarity hold?



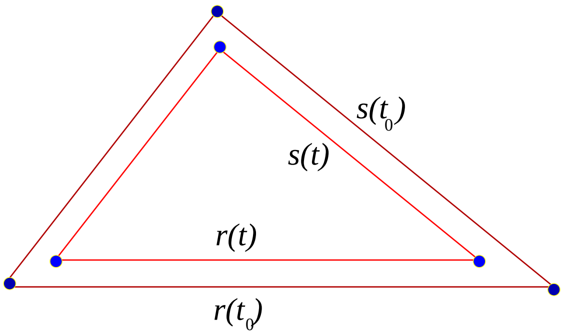
Q: connections among r's and s's?

similar: triangle angles preserved

$\Rightarrow$ *side ratios preserved*,

so must have

$$\frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)}$$



holds for any triangle,

so side *length ratio depends only on time t:*

$$a(t) = \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)}$$

Q: what does this imply about cosmic kinematics?

We have shown:

Cosmo Principle demands *any length* $r(t)$ evolves as

$$r(t) \propto a(t) \tag{6}$$

and so without loss of generality we may write

$$\boxed{r(t) = a(t)\, r_0} \tag{7}$$

where we choose $\boxed{a(t_0) = 1}$ today, and

$r_0 = r(t_0)$ is *present value* of length ("comoving coordinate")

$a(t)$ must be universal **cosmic scale factor**

can depend only on time

and at any t: a has same value everywhere in space

This is huge!

Q: why? What have we proven? What is character of motion?

The Cosmic Scale Factor

We have shown: Cosmo Principle demands
for two "particles" (possibly Galaxies!)
distance evolves according to

$$\vec{\ell}(t) = \underset{\substack{\text{scale factor}\\ \textit{time varying}}}{a(t)} \quad \underset{\substack{\text{present distance}\\ \textit{fixed once and for all}}}{\vec{\ell}_0} \tag{8}$$

where we are free to choose $a(t_0) = 1$ today, and
$\ell_0 = \ell(t_0)$ is present value ("comoving coordinate")

$a(t)$ must be universal **scale factor**
can depend only on time
but at any t same value everywhere in space

This is huge!
Q: why? What have we proven? What is character of motion?

Cosmic Expansion

the meaning of Hubble Law: Take 2

2. Einstein interpretation:
will see: General Relativity + Cosmo Principle demand

Universe is expanding

all galaxies receding from all others
bold, strange idea!

Q: consistency check–what must expansion explain?

Expansion: Einstein $\rightarrow$ Hubble

`www: expanding universe animation`

for two arbitrary observers (e.g., "galaxies")
scale factor gives distances
$\vec{r}(t) = \vec{r}_0 a(t)$
so velocity is: note: "overdot" is time deriv $\dot{x} \equiv dx/dt$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} \equiv \dot{\vec{r}} = \vec{r}_0 \dot{a} = \frac{\dot{a}}{a}\, a\vec{r}_0 \equiv H(t)\, \vec{r}(t) \tag{9}$$

$\Rightarrow$ Hubble law!
now interpret "Hubble parameter"
as **expansion rate** $\boxed{H(t) \equiv \dot{a}/a}$

we have shown:

if A sees Hubble's law, then so do (arbitrary) B and C

thus: if *any* observer measure Hubble's law

then *all* observers will measure Hubble's law!

so: Hubble law implies

$\rightarrow$ *all* galaxies recede according to same law

$\rightarrow$ no need for center, space has no special points

Moreover: Hubble law is *only* motion

which preserves homogeneity and isotropy

i.e., *any* other motion breaks cosmo principle

...but Hubble law is exactly what's observed!

Q: implications–present, past, future values for a?

present: at t_0, $a(t) = 1$
expanding, so

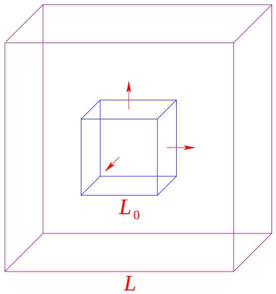
past: $a(t) < 1$
future: $a(t) > 1$

e.g., at some time in past $a = 1/2$
"galaxies twice as close"

Q: how do cosmic volumes depend on a?
e.g., *Q: when $a = 1/2$?*

Expansion and Areas, Volumes

consider a cube, galaxies at corners
present side length L_0
at any time: length $L(t) = a(t)\, L_0$



cube is "comoving" w/ expansion
• *volume* $V = L^3 = L_0^3 a^3 = V_0\ a^3$, thus $V \propto a^3$
• area of a side: $A = L^2 = A_0\ a^2$, thus $A \propto a^2$

`www: raisin cake analogy`
`www: balloon analogy`
Q: what is tricky, imperfect about each analogy?

Cosmic Expansion and Cosmic Contents

Looking ahead: n and p are examples of **baryons**
assign baryon numbers $B_n = B_p = +1$ and $B_e = B_\gamma = B_\nu = 0$
baryon number is conserved in strong/weak/EM reactions:
initial $\sum_i B_i = \sum_f B_f$ *final*

usually this means: total nucleon number conserved

Q: cosmic baryon number density n_{B} *dependence on* a*?*

Q: nonrelativistic mass ("matter") density ρ_{M} *dependence on* a*?*
Q: implications for early universe?

baryon number is *conserved*

...except at *very* high energies/early times?

so in some volume V:

baryon number $\mathcal{N}_{\rm B} = n_{\rm B} V = const$ fixed

but $V \propto a^3$, so: $\boxed{n_{\rm B} \propto a^{-3}}$

similarly, in nonrelativistic limit:

energy conservation $\rightarrow$ mass conservation $\Rightarrow$ $\boxed{\rho_{\rm M} \propto a^{-3}}$

definition: to cosmologist, **matter** $\equiv$ *non-relativistic* matter

today: $\rho_{\rm matter}(t_0) \equiv \rho_{\rm M,0}$

at other epochs (while still non-relativistic): $\rho_{\rm M} = \rho_{\rm M,0}\ a^{-3}$

in Early U: high densities $\rightarrow$ high reaction rates

$\rightarrow$ processes which are unimportant (slow) now

could have been fast

Q: what about expansion effect on relativistic particles–e.g., photons?

Director's Cut Extras

More Formally, for the GR Cognescenti

relativistic "interval" between nearby "fundamental observers"
$s = (t, \vec{x}) = (t, r, \theta, \phi)$
and $s' = s + ds = s + (dt, \vec{dx}) = s + (dt, dr, d\theta, d\phi)$:
Roberson-Walker line element:

$$\begin{aligned} ds^2 &= dt^2 - a(t)^2 dx^2 \\ &= dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \end{aligned}$$

$\vec{x}$ is fixed, time-indep comoving coord

1. any FO has fixed comoving coords: $d\vec{x}_{\rm fo} = 0$
   $\Rightarrow$ FO time elaspes as $ds = dt$:
   $\Rightarrow$ t is cosmic time

1. two nearby FO at same t ($dt = 0$):
   separated by physical distanc $d\ell = a(t)dx$
   $\Rightarrow$ cosmic distances $\ell(t) = a(t)x$