Astro 596/496 NPA

Lecture 12

February 13, 2019

Announcements:

- **Problem Set 2** due Friday
- Office Hours right after class, or by appointment

Last time:

- Hubble's Law *Q: namely? characteristic scales?*
- cosmic scale factor $a(t)$

  Q: what is it? physical significance? units? value today?

`www: expansion animation`

  Q: connection between a and Hubble's law?

cosmic lengths evolve according to

$$\vec{\ell}(t) = \underbrace{a(t)}_{\substack{\text{scale factor}\\ \textit{time varying}}} \quad \underbrace{\vec{\ell}_0}_{\substack{\text{present distance}\\ \textit{fixed once and for all}}} \tag{1}$$

where we are free to choose $a(t_0) = 1$ today, and $\ell_0 = \ell(t_0)$ is present value ("comoving coordinate")

leads to

$$\vec{v}(t) = \frac{d\vec{r}}{dt} \equiv \dot{\vec{r}} = \vec{r}_0 \dot{a} = \frac{\dot{a}}{a} \, a\vec{r}_0 \equiv H(t)\,\vec{r}(t) \tag{2}$$

$\Rightarrow$ Hubble law!

now interpret "Hubble parameter"

as **expansion rate** $H(t) \equiv \dot{a}/a$

Redshifts

quick-n-dirty: wavelengths are lengths! ..it's right there in the name!
$\longrightarrow$ expansion stretches photon $\lambda \Rightarrow$ $\lambda \propto a$

if *emit* photon at $t_{\rm em}$, then at later times

$$\lambda(t) = \lambda_{\rm emit} \frac{a(t)}{a(t_{\rm em})} \tag{3}$$

if *observe today* at t_0

$$\lambda_{\rm obs} = \lambda(t_0) = \frac{\lambda_{\rm em}}{a_{\rm em}} \tag{4}$$

measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1}{a_{\rm em}} - 1 \quad \Rightarrow \quad a_{\rm em} = a(z) = \frac{1}{1+z}$$

Scale factor $\leftrightarrow$ redshift

$$a = \frac{1}{1+z}$$
$$z = \frac{1}{a} - 1$$

Example: highest spectroscopically confirmed redshift GN-z11 galaxy with $z = 11.09^{+0.08}_{-0.12}$!

`www: GN-z11 recordholder`

For this galaxy:

$\rightarrow$ scale factor $a = 1/(1+11.1) = 0.083$

interparticle (intergalactic) distances 8.3% of today!

$\rightarrow$ galaxies 1+11.1=12.1 times closer

squeezed into volumes $(12.1)^3 = 1770$ *times smaller!*

Q: expansion effect on photon energies?

Redshifts and Photon Energies

in **photon** picture of light: $E_\gamma = hc/\lambda$

so in cosmological context photons have

$$E_\gamma \propto \frac{1}{a} \quad (5)$$

photon energy decreases ("redshifts") with cosmic expansion

Consequences:

- ▷ *Q: photon energy density* $\varepsilon(a)$*?*
- ▷ if thermal radiation,
  Q: $T \leftrightarrow \lambda$ *connection?*
  Q: expansion effect on T*?*

Relativistic Species

Photon energy density: $\varepsilon = E_\gamma \, n_\gamma$
avg photon energy: $E_\gamma \propto a^{-1}$
photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)
$\Rightarrow$ for relativistic species $\equiv$ **radiation** $\boxed{\varepsilon_{\rm rad} \propto a^{-4}}$

Thermal (blackbody) radiation:
Wien's law: $T \propto 1/\lambda_{\rm max}$
but since $\lambda \propto a \rightarrow$ then $\boxed{T \propto 1/a}$

Consequences:

- $\varepsilon_{\rm rad} \propto T^4$: *Boltzmann/Planck!*
- T *decreases with time* $\rightarrow$ *U cools as it expands!*
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but "garden variety" quasar: $z = 3$
  "feels" $T = 8$ K (effect observed!)

Cosmodynamics

$a(t)$ gives expansion history of the Universe
which in turn tells how densities, temperatures change
$\longrightarrow$ given $a(t)$ can recover all of cosmic history!

but...

How do we know $a(t)$?
Q: What controls how scale factor $a(t)$ *grow with time?*

Cosmodynamics Computed

cosmic dynamics is evolution of a system which is

- gravitating
- homogeneous
- isotropic

Complete, correct treatment: General Relativity

→ take GR! ...or Cosmology next semester

quick 'n dirty:

Non-relativistic (Newtonian) cosmology

pro: gives intuition, and right answer

con: involves some ad hoc assumptions only justified by GR

Inputs:

- arbitrary cosmic time t
- cosmic mass density $\rho(t)$, spatially uniform

- cosmic pressure $P(t)$: in general, comes with matter
  but for non-relativistic matter, P not important source of
  energy and thus mass ($E = mc^2$) and thus gravity
  so ignore: take $P = 0$ for now (really: $P \ll \rho c^2$)

thus: *gravity is only force*
all cosmic matter is in "*free fall*"

Construction:

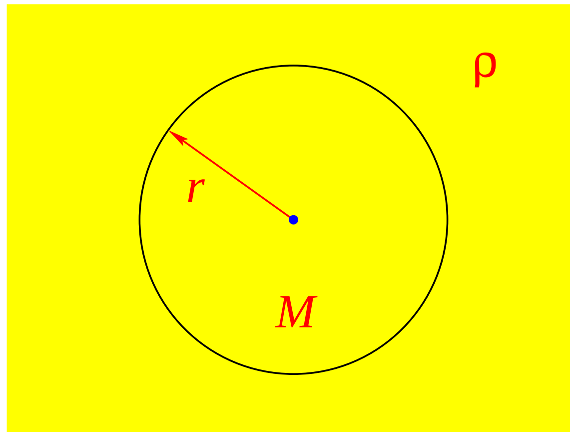
pick arbitrary point $\vec{r}_{\rm center} = 0$,

surround by comoving sphere, radius $r(t)$

that moves in order to always enclose

some arbitrary but fixed mass

$$M(r) = \frac{4\pi}{3}\, r^3 \rho = const \qquad (6)$$



consider a point on the sphere

Q: is it accelerated?

Q: what is $\ddot{\vec{r}} = ?$

Newtonian Cosmodynamics

a point on the sphere feels acceleration

$$\ddot{\vec{r}} = \vec{g} = -\frac{GM}{r^2}\hat{r} \tag{7}$$

with pressure $P = 0$

multiply by $\dot{\vec{r}}$ and integrate:

$$\dot{\vec{r}} \cdot \frac{d}{dt}\dot{\vec{r}} = -GM\frac{\hat{r} \cdot d\vec{r}/dt}{r^2} \tag{8}$$

$$\frac{1}{2}\dot{r}^2 = \frac{GM}{r} + K = \frac{4\pi}{3}G\rho r^2 + K \tag{9}$$

Q: physical significance of K? of it's sign?

Q: what happens when we introduce scale factor?

Friedmann (Energy) Equation

introduce scale factor: $\vec{r}(t) = a(t)\vec{r}_0$

"energy" eqn: Friedmann eq.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\kappa c^2}{R^2 a^2} \quad (10)$$

full GR gives $K = r_0^2(\kappa c^2/R^2)$
curvature term with parameters

- κ sets cosmic geometry
  $\kappa = 0$: "flat" Euclidean geometry
  $\kappa = +1$: positively curved "spherical" geometry
  $\kappa = -1$: negatively curved "hyperbolic" geometry
- const R is lengthscale: "curvature" of U.

`www: cosmic geometries`

Relativistic Gravitation

Einstein:

- mass and energy are equivalent $E = mc^2$
- mass is source of gravity
- so *energy is also source of gravity*

for each cosmic component or "species" i,
with energy density ε_i
equivalent mass density given by $\rho_i c^2 = \varepsilon_i$

total mass-energy density is

$$\rho = \sum_i \rho_i = \sum_i \frac{\varepsilon_i}{c^2} \tag{11}$$

this is what enters into Friedmann

The Mighty Friedmann (Energy) Equation

fundamental equation of the Standard Cosmology:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\kappa c^2}{R^2 a^2} \tag{12}$$

Q: why is it so important?

Q: what's a variable?

Q: what's a parameter?

Dissecting Friedmann

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\kappa c^2}{R^2 a^2} \tag{13}$$

variables change with time

a: cosmic scale factor

ρ: total cosmic mass-energy density

parameters constant, fixed for all time

$\kappa = \pm 1$ or 0: sign of "energy" (curvature) term

R: characteristic lengthscale, GR $\rightarrow$ curvature scale

Q: how does expansion of U depend on contents of U?

Q: how does expansion of U not *depend on contents of U?*

Q: what about acceleration–$\ddot{a}$?

Friedmann Acceleration Equation

Newtonian analysis gives $\ddot{a}$ for $P \rightarrow 0$
In full GR: with $P \neq 0$, get Friedmann **acceleration** eq.

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho + 3P/c^2)} \tag{14}$$

Pressure and Friedmann
⋆ in "energy" ($\dot{a}$) eq.: P *absent*, even in full GR
⋆ in acceleration eq., GR $\rightarrow P$ present, *same* sign as ρ
adds to "active gravitational mass"
Q: why? Q: contrast with hydrostatic equilibrium?

Friedmann energy eq is "equation of motion" for scale factor
i.e., governs evolution of $a(t)$.
To solve, need to know:

- is there curvature? $\leftrightarrow$ is $\kappa = 0, 1, -1$?
- ρ dependence on a

Worked Example: An Empty Universe

consider an "empty" universe:

- $\rho = 0$, or really $G\rho \ll c^2/R^2 a^2$
- $\kappa = -1$ *Q: why?*

Friedmann is nontrivial:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{c^2}{R^2 a^2} \tag{15}$$

solve:

$$\frac{\dot{a}}{a} = \frac{c}{Ra} \tag{16}$$

$$a(t) = \frac{ct}{R} \tag{17}$$

Q: sketch $a(t)$ for this model? physical interpretation? fate?

imagine we live in such a universe, and measure H_0

Q: what is R? t_0? $H(t)$? $\ddot{a}$?

An Empty Universe is a Coasting Universe

for an "empty", negatively curved universe
Friedmann: $H^2 = (\dot{a}/a)^2 = c^2/R^2a^2$
evaluate today when $a_0 = 1$: $H_0 = c/R$
so curvature radius $R = c/H_0 = d_{\mathrm{H}}$: Hubble length!

Friedmann solution:

$$a(t) = \frac{ct}{R} = H_0 t = \frac{t}{t_0} \tag{18}$$

- *fate*: as $t \to \infty$, scale factor $a \to \infty$ *expand forever!*
- again $a_0 = 1$ gives age $t_0 = H_0^{-1}$: Hubble time!
- $H(t) = \dot{a}/a = 1/t$: rate slow with time
  so $v = r/t$: *this is the Milne (explosion) universe!*
- $\ddot{a} = 0$: zero acceleration – *"coasting" cosmology*

Q: $a(t)$ with matter added? How to add matter? radiation?