Astro 596/496 NPA
Lecture 15
February 20, 2019

Announcements:

- **Preflight 3** due Friday
  Part (a) is individual
  Part (b) is group discussion of alternate universe

Last time: cosmic contents and the Early Universe

Q: what dominates cosmic expansion at late times? why?

Q: early times?

Y. Zel'dovich:

"The Universe is the poor [wo]man's accelerator."

Q: in what sense is this true? opportunities? challenges?

Particle Physics: Conservation Laws, Take I

Rules for transitions from initial to final states
$\Rightarrow$ scattering, reactions, decays

- total **energy and momentum** (i.e., 4-momentum) is conserved

use relativistic definitions, e.g., include rest mass; then
$\sum E_i = \sum E_f$
$\sum \vec{p}_i = \sum \vec{p}_f$
e.g., $n \rightarrow \nu$ $\otimes$ ...since $m_n \neq m_\nu$

- **electric charge**: $\sum Q_i = \sum Q_f$

e.g., $p + p \rightarrow p + n$ $\otimes$

- **baryon number**: so far, *baryon* $= n$ *or* p

  $B(n) = B(p) = +1$

  $B(\bar{n}) = B(\bar{p}) = -1$

  for nucleus, $B_i = A_i \Rightarrow n_{B,i} = A_i n_i$

  conservation: $\sum B_i = \sum B_f$

  e.g., $p + p \rightarrow p + p + n$ $\otimes$

  but $p + p \rightarrow p + p + p + \bar{p}$ OK

- **lepton number**: so far, *lepton* $= e$ *or* ν_e

  $L(e^-) = L(\nu_e) = +1$

  $L(e^+) = L(\bar{\nu}_e) = -1$

  conserved in Weak interaction: reactions obey $\sum L_i = \sum L_f$

  check: $n \rightarrow p + e^- + \nu_e$ $\otimes$

  $n \rightarrow p + e^- + \bar{\nu}_e$ OK

  $e^+ e^- \rightarrow \nu_e$ $\otimes$

  $e^+ e^- \rightarrow \nu_e \bar{\nu}_e$ OK

Conservation Laws and Particle Interactions

"Everything not forbidden is compulsory" – Murray Gell-Man

any reactions and decays obeying all conservation laws
must have nonzero probability to occur
but that probability (cross section/lifetime) may be small

for all reactions and decays $i \to f$, define Q value

$$Q = \sum_i m_i c^2 - \sum_f m_f c^2 \tag{1}$$

reactions: ≥ 2 bodies $\to$ ≥ 2 bodies
exothermic if $Q > 0$, min "threshold" energy if $Q < 0$

decays: 1 body $\to$ ≥ 2 bodies *Q: why?*
can only occur if $Q > 0$ *Q: why?*
but if $Q > 0$ channel exists, decay will happen

BIG BANG NUCLEOSYNTHESIS

Gateway to the Early Universe

Big Bang Nucleosynthesis: Introduction

(Kolb & Turner Ch. 4; Olive, Steigman & Walker; BDF)

Big Bang Nucleosynthesis = BBN = Primordial nucleosynthesis

Basic idea:

follow weak, nuke reactions in expanding universe

initially: nuclei "ionized" to n, p only

when T low enough: $n, p \to$ "ground state"

To get in mood:

- What is appropriate T (E) scale for nuclear "recombination"?
- At this T, what is non-rel, rel?
- U. expansion is dominated by?

Big Bang Nucleosynthesis Stage: Early Universe

BBN energy scale for nuclear “recombination:”
when $T \sim$ *nuke binding*—— ~ 1 *MeV*

BBN epoch: for $T \sim 1$ MeV $\sim 10^{10}$ K
- scale factor $a \sim 10^{-10}$ (!)
- redshift $z \sim 10^{10} \gg z_{\rm eq} \sim 10^4$: much before matter/rad eq

$\Rightarrow$ universe is deep in *radiation-dominated era*

note this is consistent with your PS2 lower limit!

At these temperatures:
- What particles alive? decayed?
- What is role of dark matter, cosmo constant?

BBN Actors: Nucleons, Pairs, and Neutrinos

since $T \gtrsim 1$ MeV $> m_e$, pairs $e^{\pm}$ abundant & relativistic!

- "radiation" species: $\gamma, e^{\pm}, \nu\bar{\nu}$
- "matter" species: $m_n, m_p \gg T$: neutrons, protons non-relativistic
- dark matter, cosmo constant (dark energy) presumably present but we *assume* non-interacting, and unimportant

  but maybe not! can see what happens if so, and probe DM/DE!

Abundance evolution:

- While nuclei "ionized," is n or p more abundant?
- When "recombine," what is "ground state"?

when only n, p: expect roughly similar abundances
but: since $m_n > m_p$, higher "cost" for neutrons
→ should have $n/p < 1$: neutrons less abundant

when T low enough, $n, p \rightarrow$ "nuclear ground state":
set by *maximum available binding energy*
- *globally*, max B/A for ^{56}Fe,
  but not enough time to reach this state
- *among lightest nuclides*, max B/A at ^{4}He
  → highest binding energy of light nuclei

so when light nuclei form, final products = **primordial nuclides**:
- $^{4}\mathrm{He}=\boxed{2p2n}$: limited by the available n
- H: leftover "unpartnered" p

but incomplete nuke "burning" leaves
- traces of D, ^{3}He, ^{7}Li

That's it! But now the job is: understand BBN in detail
Q: what is needed to calculate abundance changes vs time

BBN Prologue: Densities and Temperatures

to understand BBN, we will need :

- **reaction rates** per species $i = n, p, d, {}^3\mathrm{He}, \ldots$

$$\Gamma_{\mathrm{per}\, i}(ij \to k\ell) = n_j \langle \sigma_{ij \to k\ell} v \rangle$$

  as well as lab-measured decay rates for radioactive nuclei

- **cosmic expansion rate**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} \rho_{\mathrm{rad}}$$

thus we need for all times and thus temperatures $T(a)$

- particle number densities $n_i(T)$
- the total cosmic radiation energy density $\varepsilon_{\mathrm{rad}}(T) = \rho_{\mathrm{rad}} c^2$

for all species, relativistic and non-relativistic

Statistical Mechanics: Dimensional Analysis

consider a *thermal bath* of *ultra-relativistic bosons* b

- temperature T
- particle mass $m_b c^2 \ll kT$
- equal numbers of b and $\bar{b}$ (or $b = \bar{b}$)

  i.e., no net b abundance, which means $|\mu_b|/T \ll 1$

full derivation: Director's Cut Extras – check it out!

quick and dirty: *dimensional analysis*

Ultra-Relativistic Dimensional Analysis

for a relativistic boson species at T, we want:

- number density n_b
- energy density ε_b
- pressure P_b

scales in the problem:

- kT, but not $m_b \ll T$
- QM relevant: $\hbar$
- special relativity relevant: c

Q: how to construct number density n_b, ε_b, P*?*

Hint: $\hbar c \approx$ 200 MeV fm has dimensions [energy × length]

Hint: you already know the answer for a famous boson!

Q: which one? does dimensional analysis work?

Ultra-Relativistic Thermal Particles

dimensional analysis: kT, $\hbar$, c form one *length*

$$\ell = \frac{\hbar c}{kT} = \frac{\hbar}{p_T} \tag{2}$$

the thermal de Broglie length

from this we estimate **number density**

$$n \sim \ell^{-3} \sim \left(\frac{kT}{\hbar c}\right)^3 \tag{3}$$

energy density

$$\varepsilon \sim kT\ell^{-3} \sim \frac{(kT)^4}{(\hbar c^3)} \tag{4}$$

pressure has dimensions of energy density, so

$$P \sim \varepsilon \tag{5}$$

of course we know thermal photons result: blackbody radiation!

Ultra-Relativistic Thermal Bosons: Exact Results

for boson species with g internal states (helicity etc)

$$\begin{aligned} n_{\rm rel,b} &= \frac{g}{2\pi^2\hbar^3c^3}\int_0^\infty dE\frac{E^2}{e^{E/kT}-1} \\ &= g\frac{\zeta(3)}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \propto T^3 \qquad (6) \end{aligned}$$

$$\begin{aligned} \rho_{\rm rel,b}c^2 &= \frac{g}{2\pi^2\hbar^3c^3}\int_0^\infty dE\frac{E^3}{e^{E/kT}-1} \\ &= g\frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3} \propto T^4 \qquad (7) \end{aligned}$$

where

$$\zeta(3) = \sum_{n=1}^{\infty}\frac{1}{n^3} = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots = 1.20206\ldots \qquad (8)$$

photons: $g = 2$ polarizations

Q: what if anything changes for ultra-relativistic fermions?